Electrical Engineering
Week 7

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Week 7 Agenda: Capacitors

- Physical Concepts
- Symbols
- i–v Behavior
- Fabrication
- Power and Energy
- Parallel and Series Combinations
- Steady–State Solutions
- Charge and Discharge

Big Picture

Devices

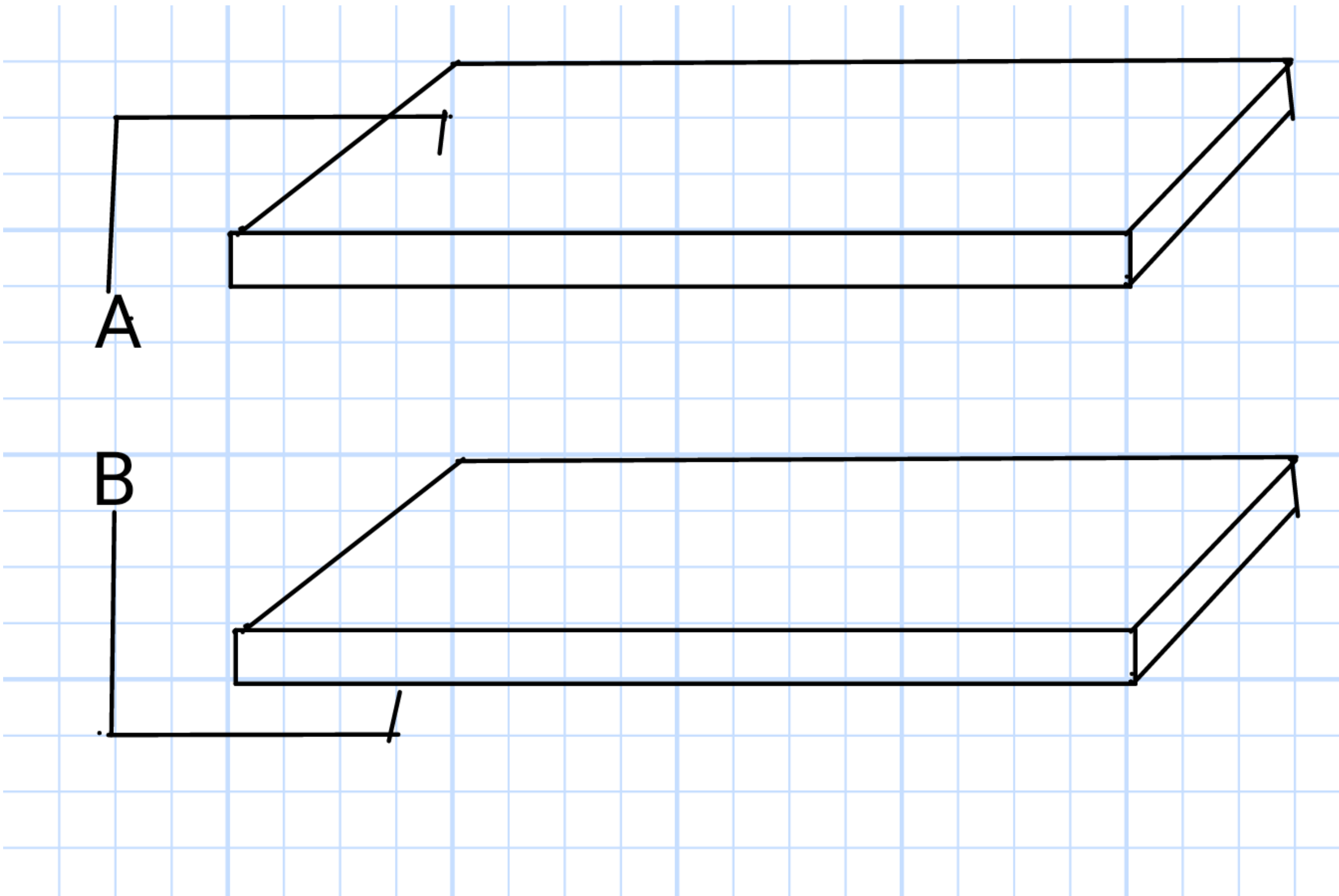
Resistors	Capacitors	Inductors
$v = iR$	$v = \frac{1}{C}\int i\ dt$	$i = \frac{1}{L}\int v\ dt$
R in Ohms	C in Farads	L in Henries
	Voltage Continuous	Current Continuous
	Open to DC	Short to DC

Circuits

RC or RL	LC	RLC
First Order DE	Second Order DE	2nd with Loss
Negative Exponentials	Sinusoids	Lossy Sinusoids

We can do interesting things with time–varying sources.

Concepts



$$q = \int i dt \qquad q = Cv \qquad i = \frac{d}{dt}(Cv) \qquad i = C\frac{dv}{dt}$$

Fluid Analogy

- Pump with intake and outlet closed
- Water flows and pressure difference increases
- Flow decreases as pressure builds
- Limit depends on strength of pump
- Provide a path and it will return to equilibrium
- Return will take time

The Math

$$i = C\frac{dv}{dt}$$

- Expect Exponentials (Negative)

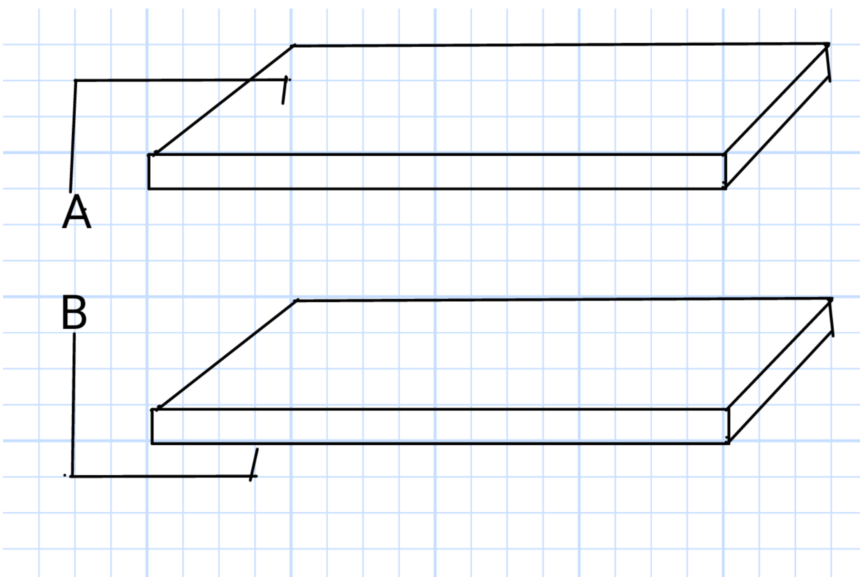
$$\frac{d\left(e^{-at}\right)}{dt} = -ae^{at}$$

- Expect “easy” solutions for sinusoidal sources

$$\frac{d\,\mathsf{sin}\,\omega t}{dt} = \omega\,\mathsf{sin}\,\omega t$$

- Expect to use Euler’s Formula a lot.

Values



$$i = C\frac{dv}{dt}$$

$$\frac{i}{dv/dt} = C$$

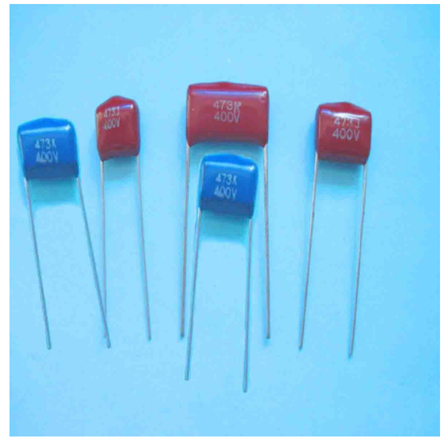
Typical:

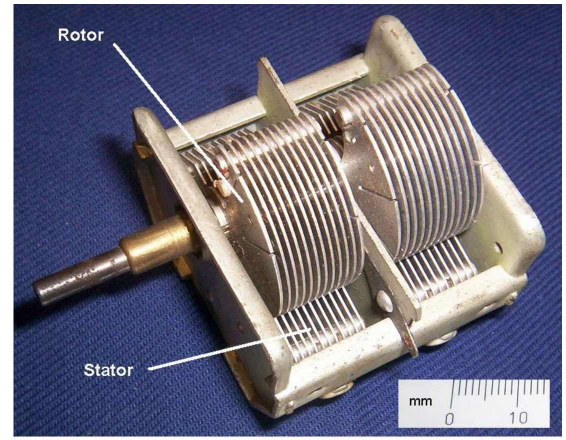
$$\frac{\text{mA}}{\text{V/ms}} = \frac{10^{-3}\text{Coulombs/s}}{\text{V}/\left(10^{-3}\text{s}\right)} =$$

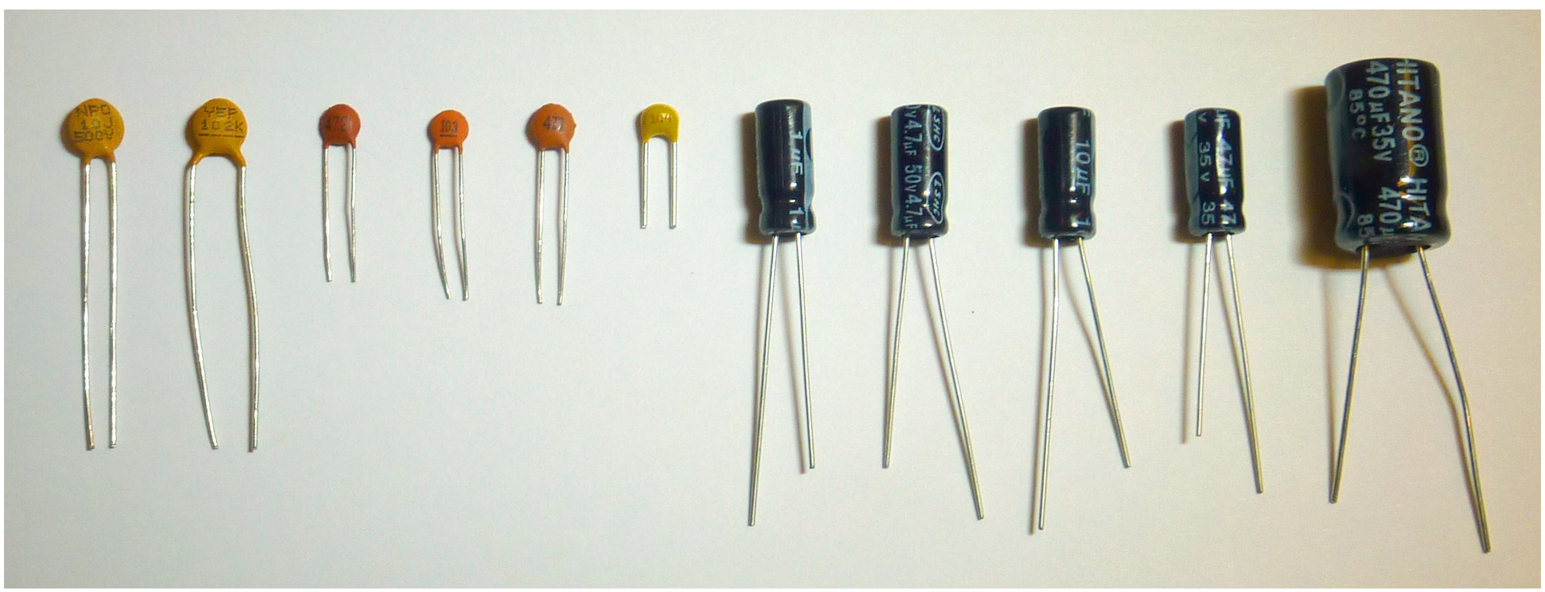
$$10^{-6}\frac{\text{Coulombs}}{\text{V}} = \mu\text{Farads}$$

Full range covers many orders of magnitude

Capacitors (1)

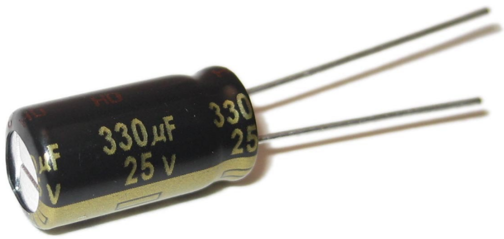






Capacitors (2)

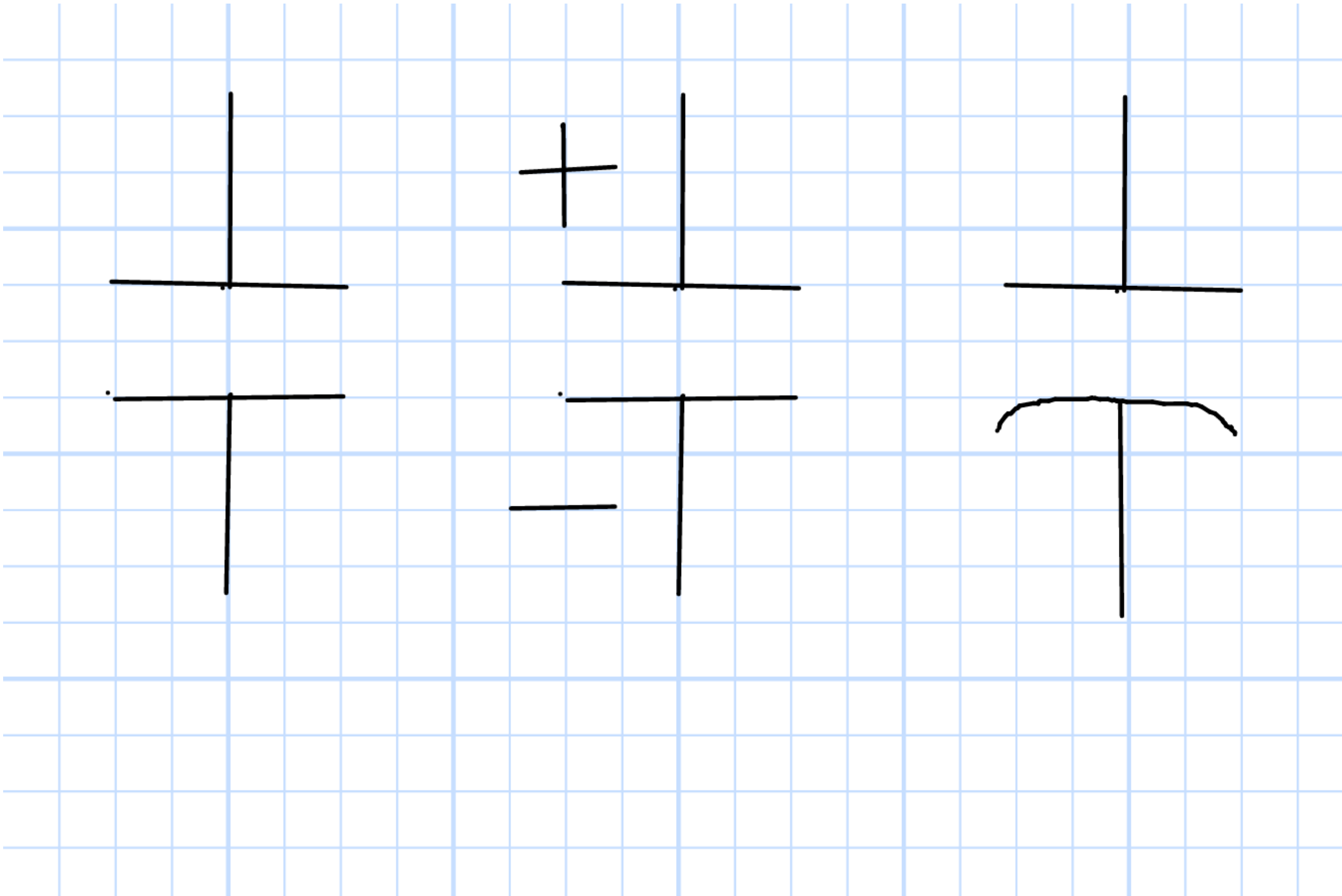
Electrolytics



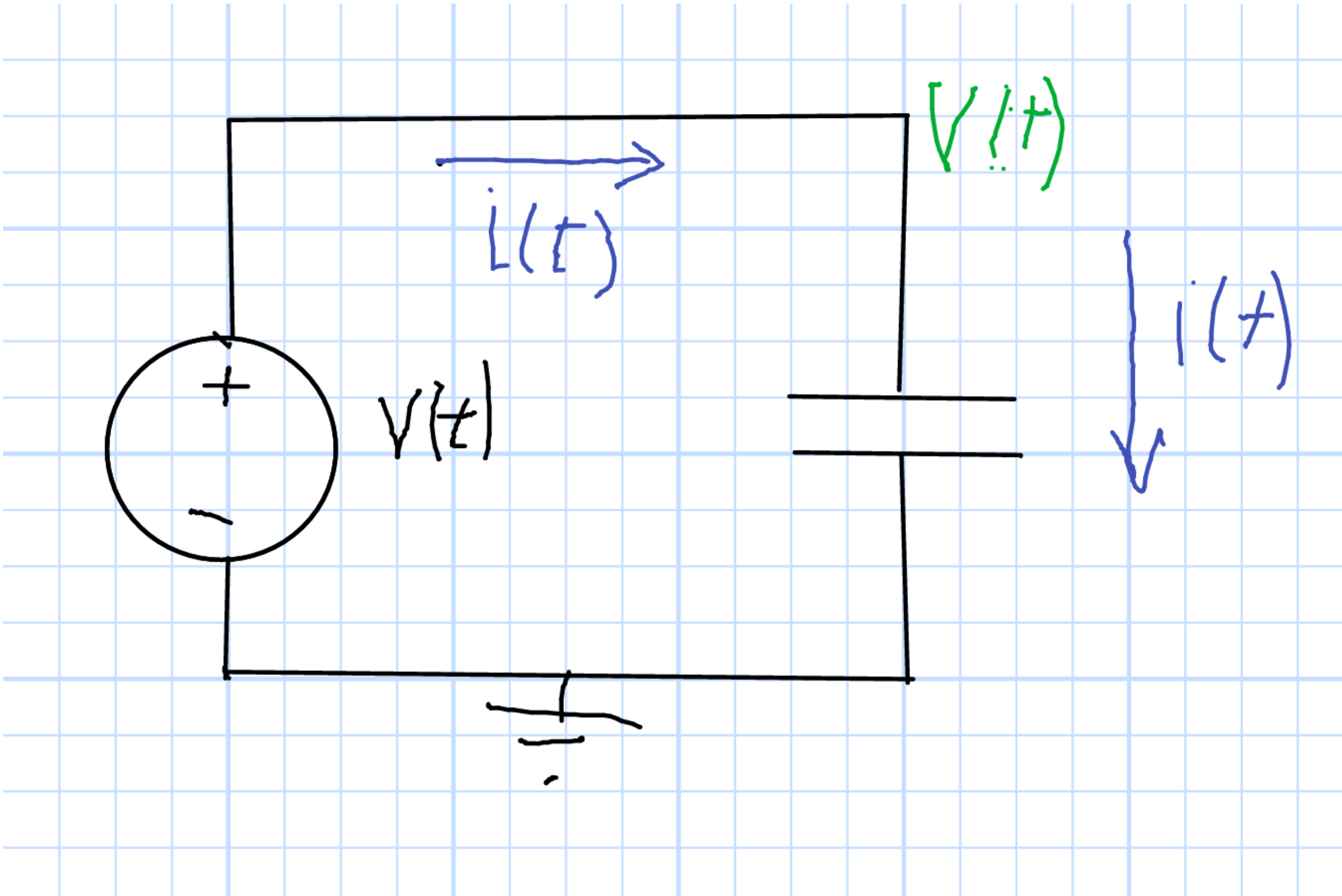
Big Capacitors

Principal Specifications: Capacitance (Farads), Maximum Voltage

Symbols



Voltage Source

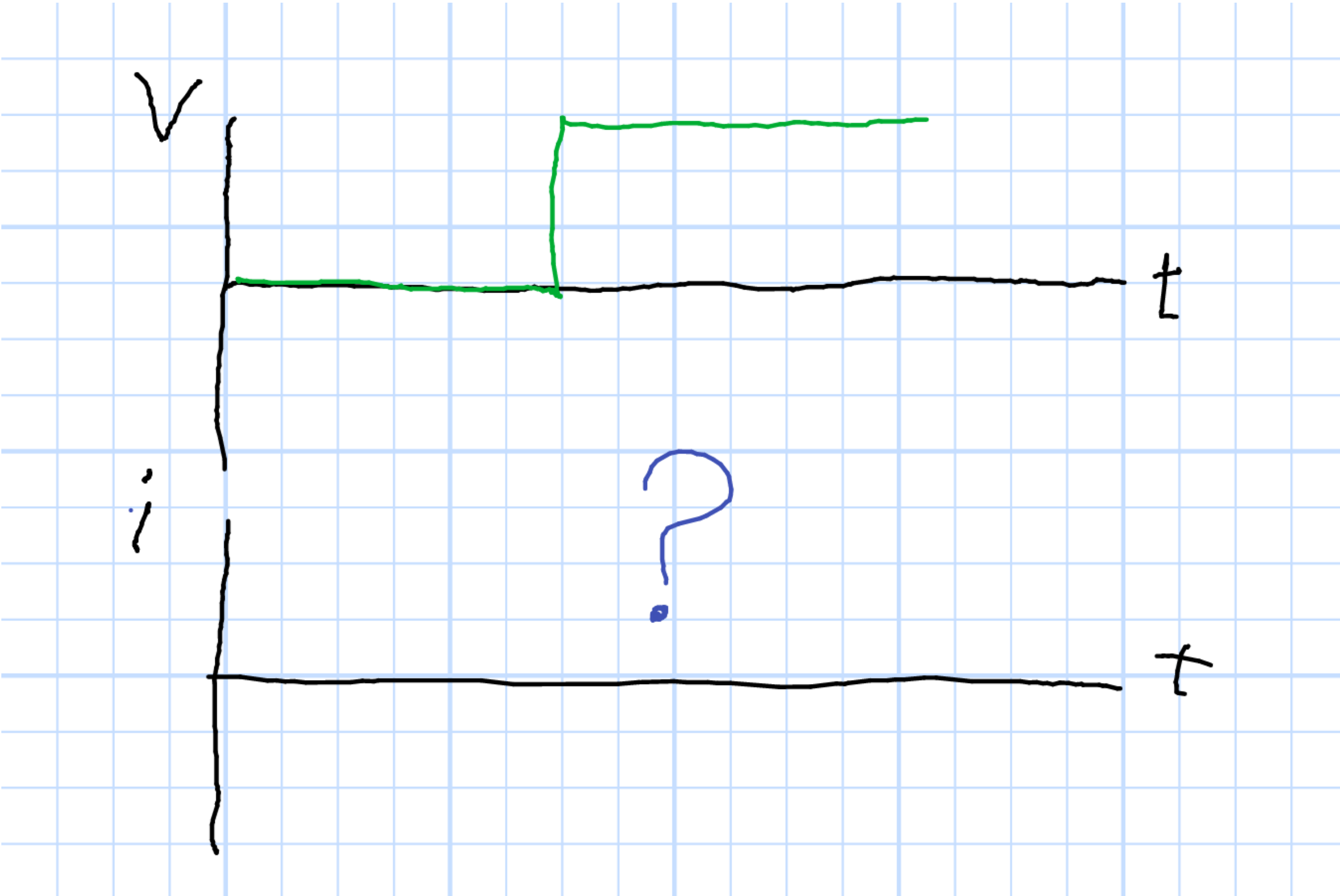


$$i = C\frac{dv}{dt}$$

Example

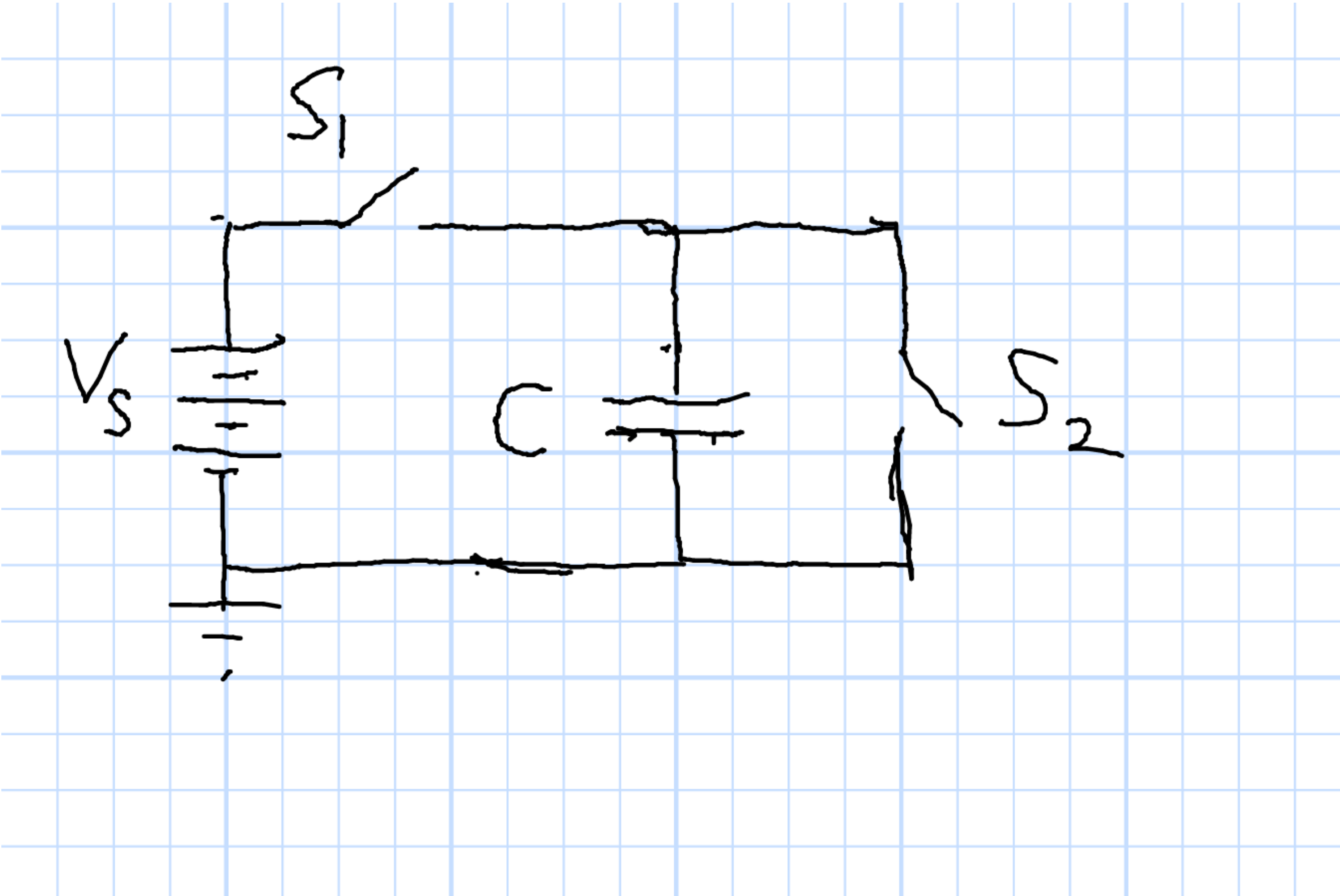
$$i = C\frac{dv}{dt}$$

What Will Happen?



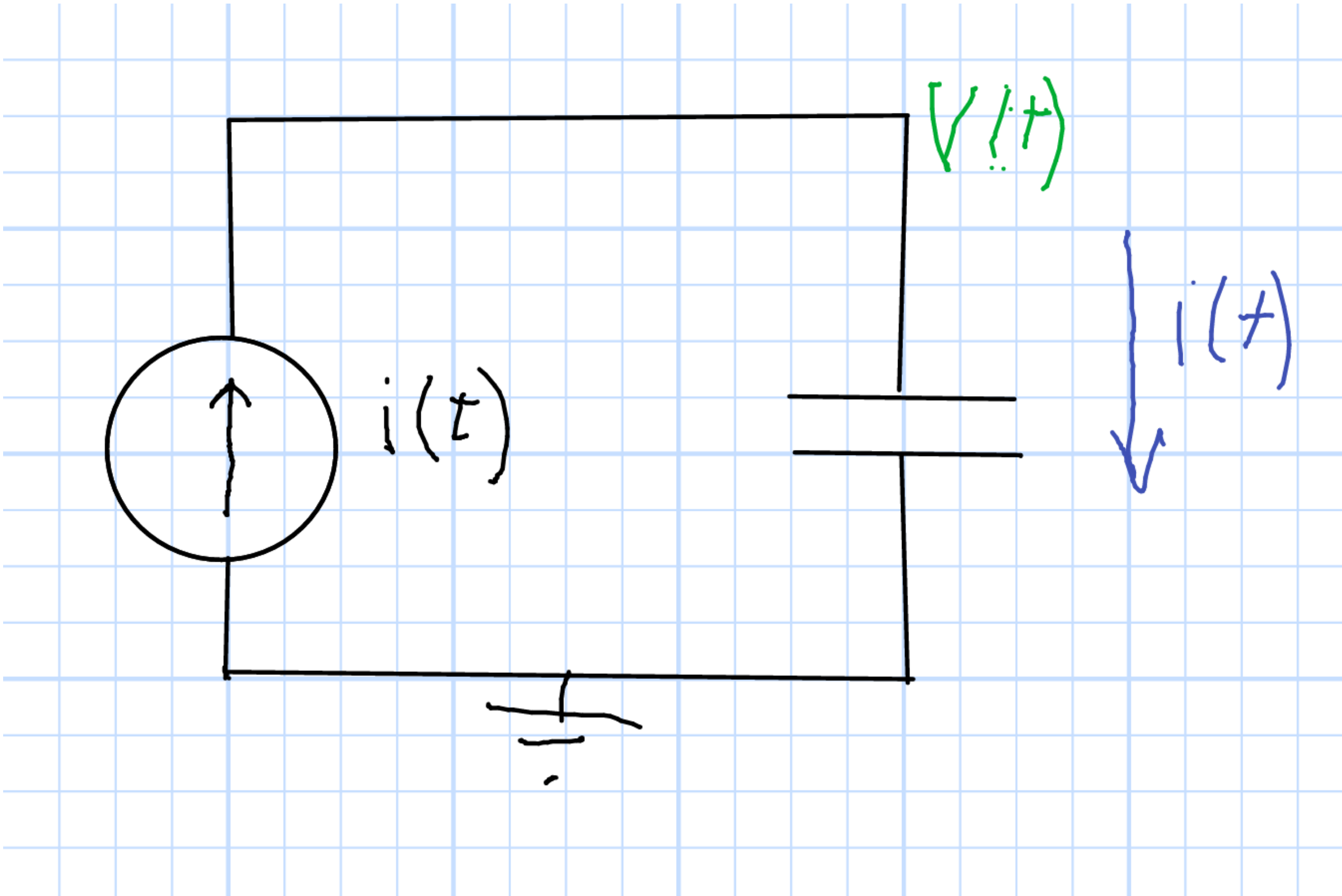
$i = C\dfrac{dv}{dt}$ finite i $\rightarrow$ Voltage Continuous in Time

What Will Happen?



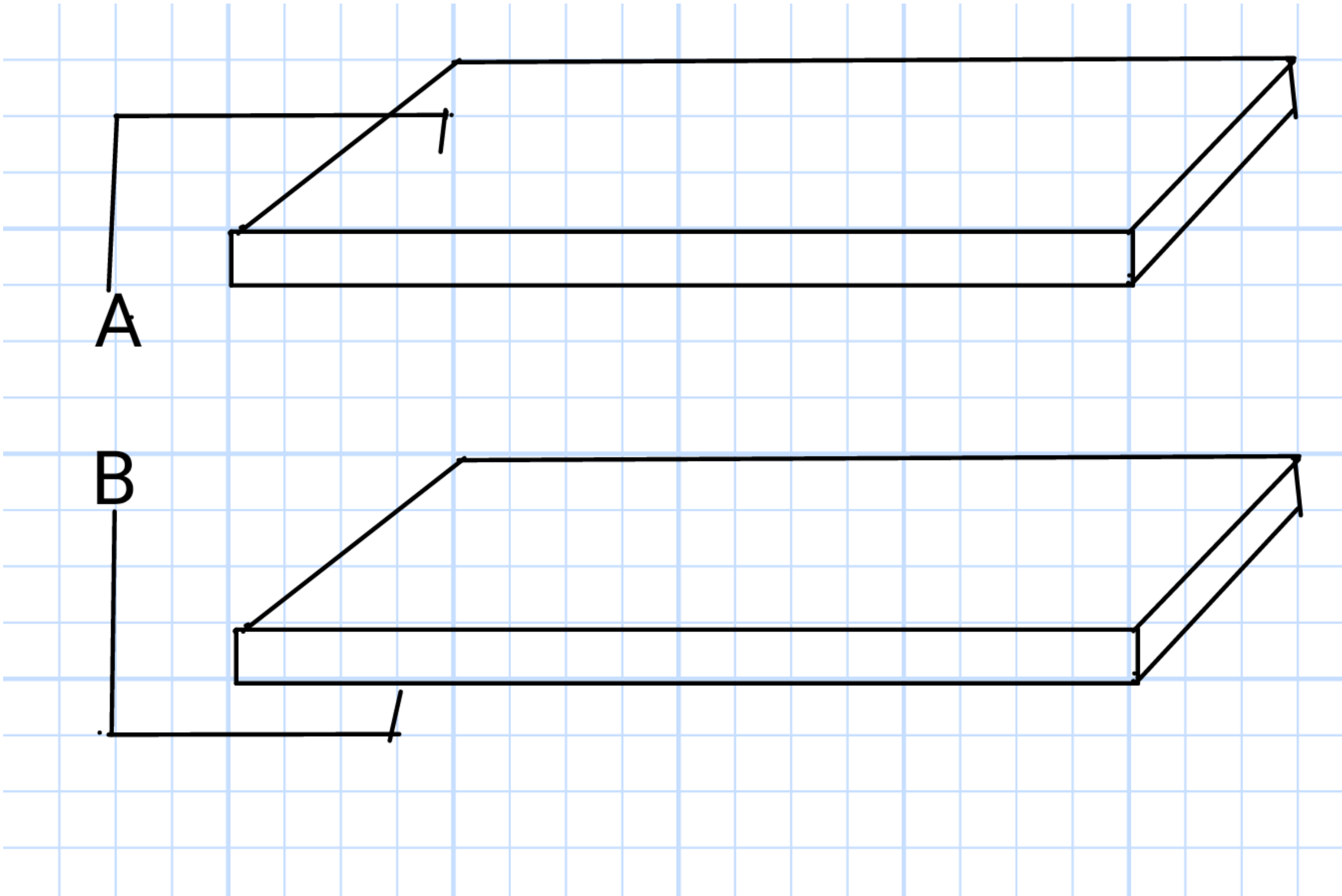
(1) Close S1 (2) Open S1 (3) Close S2

Current Source



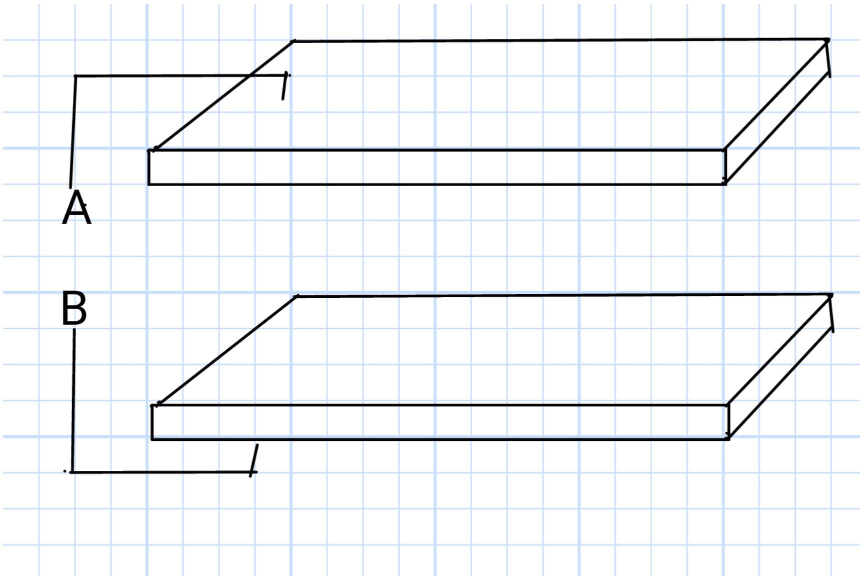
$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\,dt + v(t_0)$$

Fabrication



$$C = \frac{\epsilon A}{d}$$ ϵ is the Dielectric Constant

Equations



$$C = \frac{\epsilon A}{d}$$

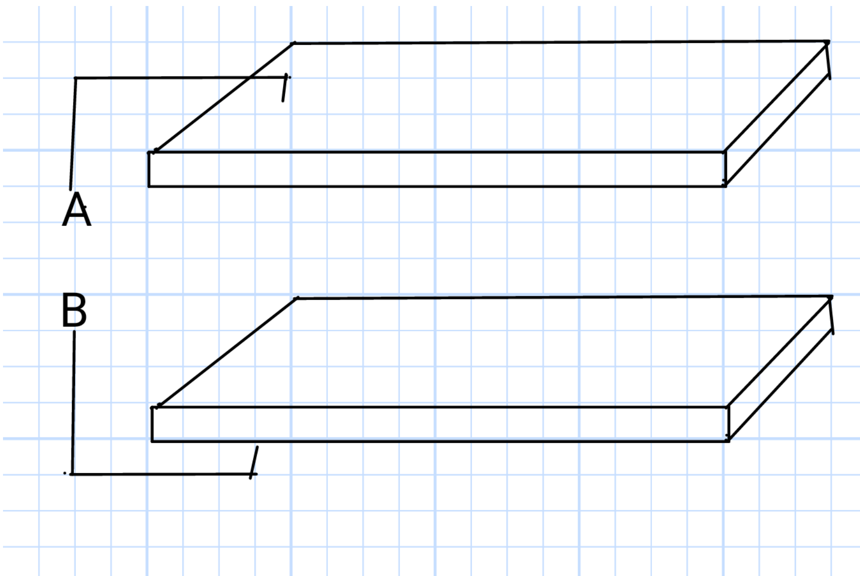
$$\epsilon = \epsilon_r \epsilon_0$$

$$\epsilon_0 = 8.85 \times 10^{-12} \mathrm{F/m}$$

Useful Term: Relative dielectric constant, ϵ_r

$\epsilon_r = 1$ for vacuum. Pretty close for air.

Example



$$C = \frac{\epsilon A}{d} = 10\mu\text{f}$$

$$\epsilon = 3.9\epsilon_0 \qquad \text{SiO}_2$$

$$\frac{A}{d} = 3 \times 10^5\text{m}$$

$$d = 10\mu\text{m} \qquad A = 3\text{m}$$

$$\frac{\text{High Voltage}}{\text{Small } d} \rightarrow \text{Breakdown}$$

Air: $\approx$ 30kV/cm

Glass: $\approx$ 100kV/cm

Interleave, Roll, or otherwise work to get more A

Use high ϵ_r

Use high Breakdown Voltage

Power and Energy

Power

$$p(t) = v(t)\, i(t) \qquad\qquad p(t) = v(t)\, C\frac{dv(t)}{dt}$$

Energy

$$w = \int p(t)\, dt = \frac{v^2 C}{2}$$

Example

$$w = \frac{(100\mathrm{V})^2\, 100\mu\mathrm{F}}{2} = 500\mathrm{mJ}$$

Another Cup of Coffee

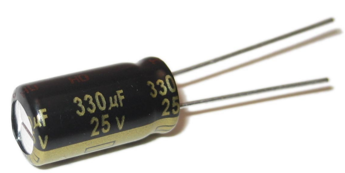
$$w = 42\text{kJ}$$

$$w = \frac{v^2C}{2} \qquad v^2C = 2w = 84,000$$

Examples

$$C = 1000\mu\text{F} \qquad v = 9.2\text{kV}$$

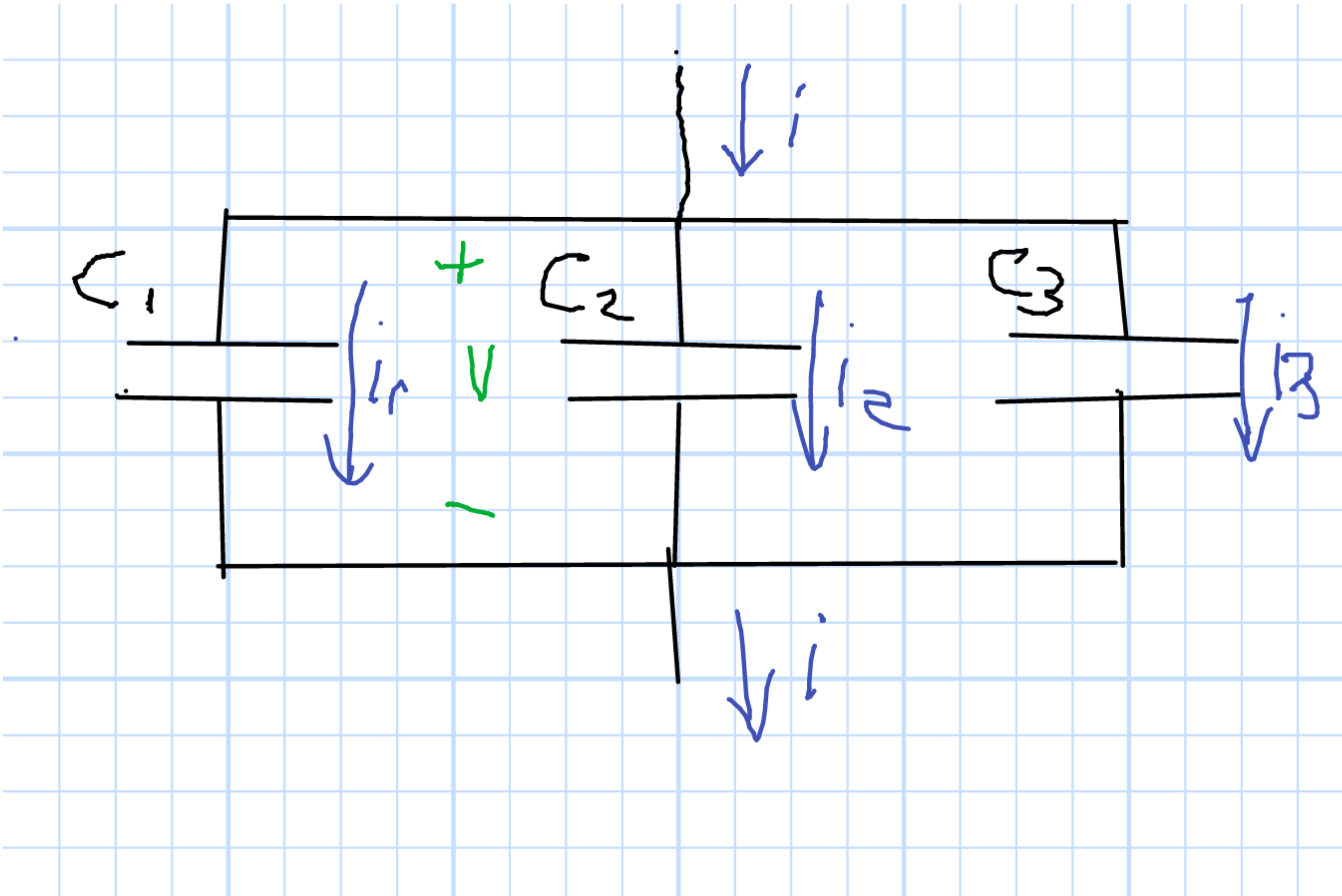
$$C = 100\text{mF} \qquad v = 920\text{V}$$



330μF $\qquad V_{limit} = 25$V

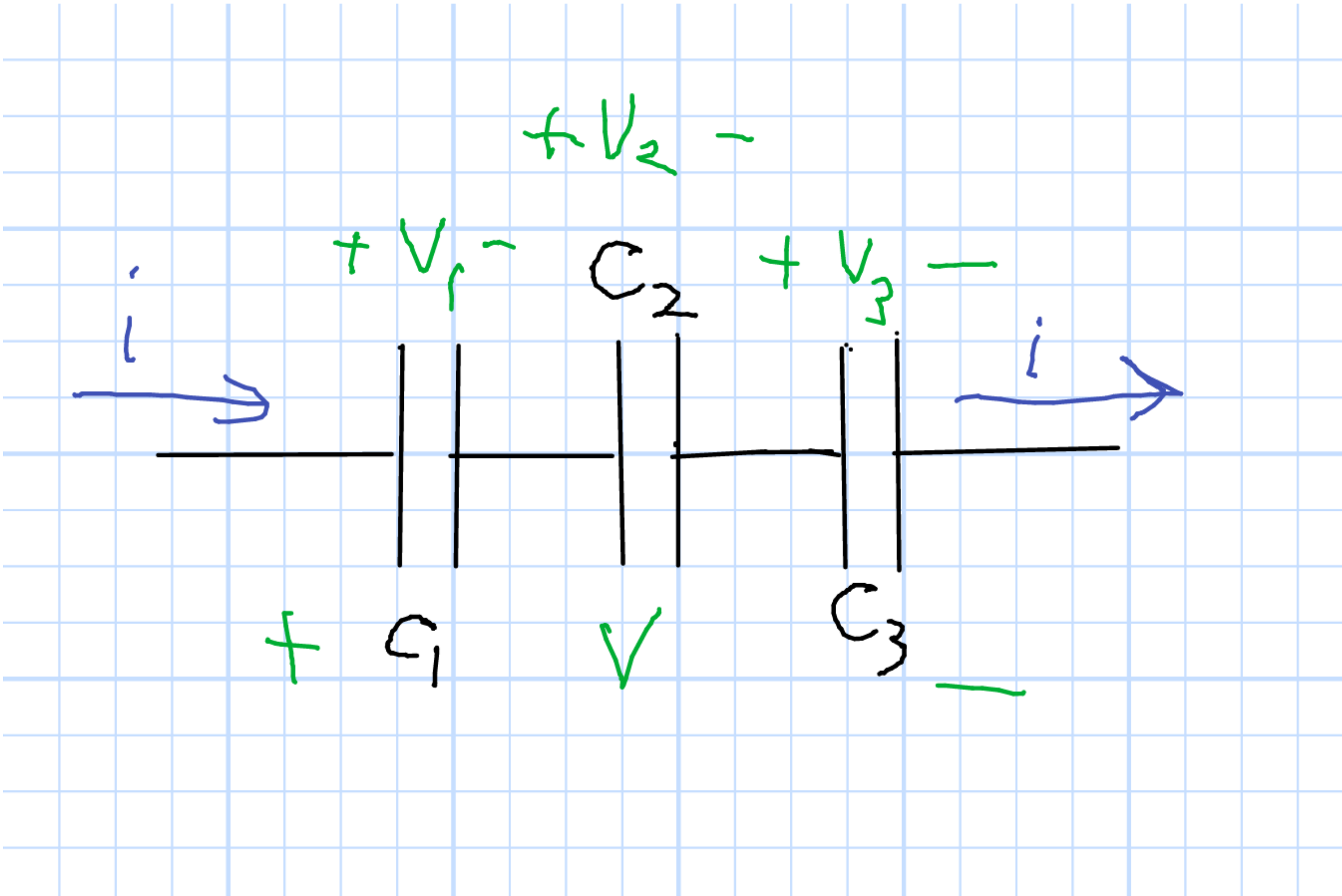
$\times$(3 or 3000)
with 40X to 400X voltage rating

Parallel Combinations



$$i_n = C_n \frac{dv_n}{dt} \qquad i = \sum i_n \qquad C = \sum C_n$$

Series Combinations

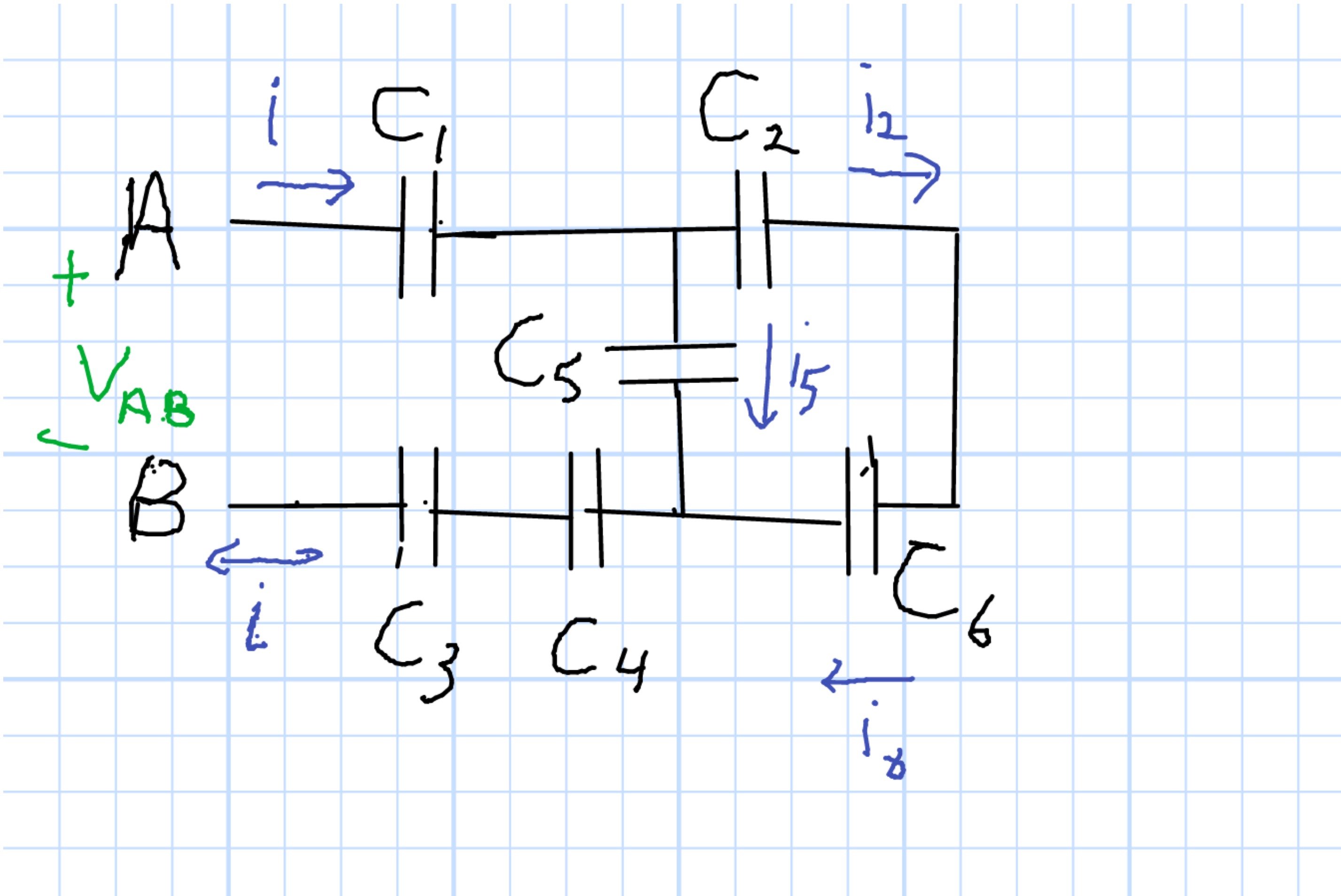


$$\frac{i}{C_n} = \frac{dv_n}{dt} \qquad \frac{dv}{dt} = \sum \frac{dv_n}{dt} \qquad \frac{1}{C} = \sum \frac{1}{C_n}$$

Parallel/Series Summary

	Series	Parallel
Voltage Sources	$v = \sum v_n$	Contradictory
Current Sources	Contradictory	$i = \sum i_n$
Resistors	$R = \sum R_n$	$\frac{1}{R} = \sum \frac{1}{R_n}$
Capacitors	$\frac{1}{C} = \sum \frac{1}{C_n}$	$C = \sum C_n$

Example Problem

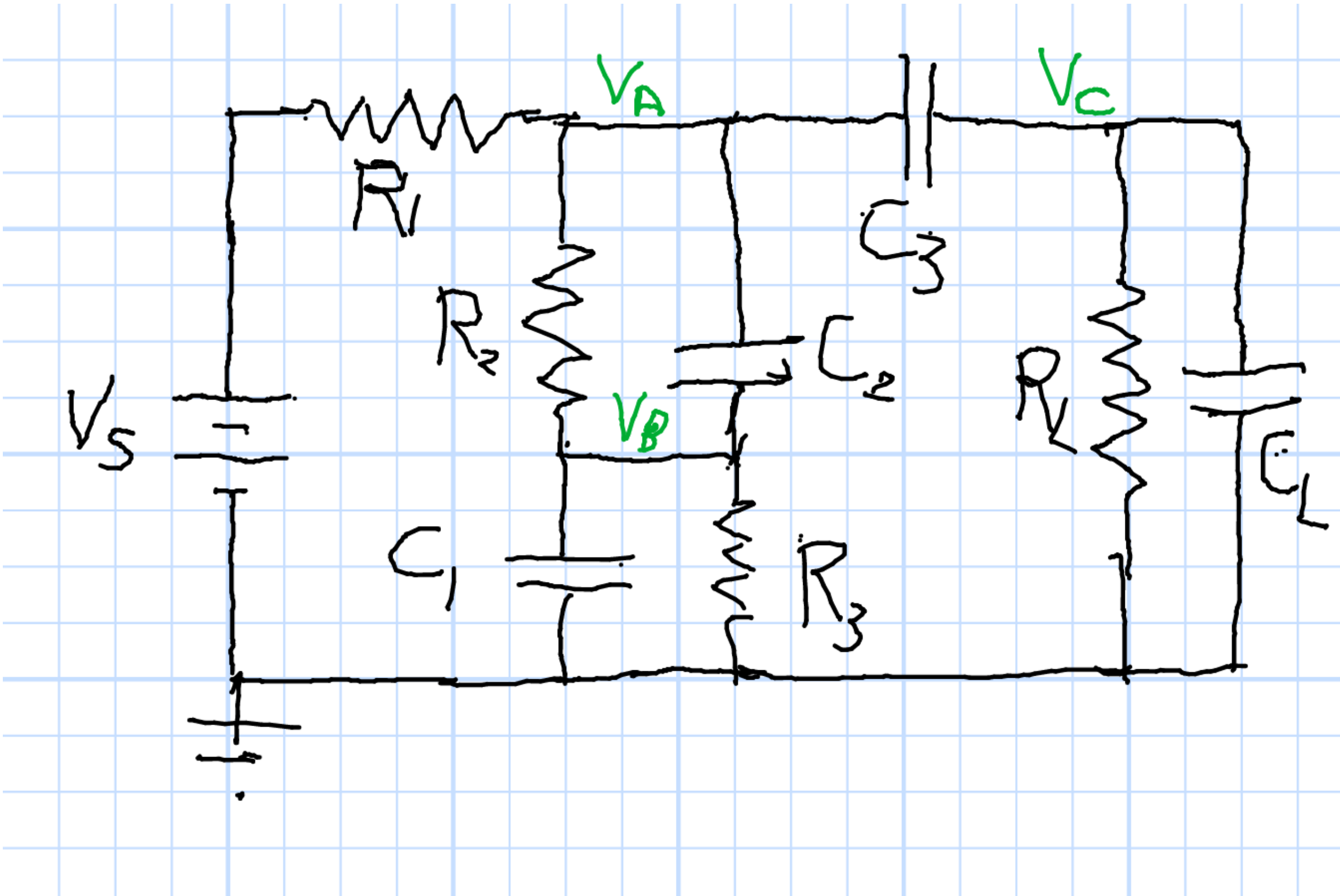


$C_{1:6} = 1\mu\text{F} \qquad C_{AB} = ?$

Steady State

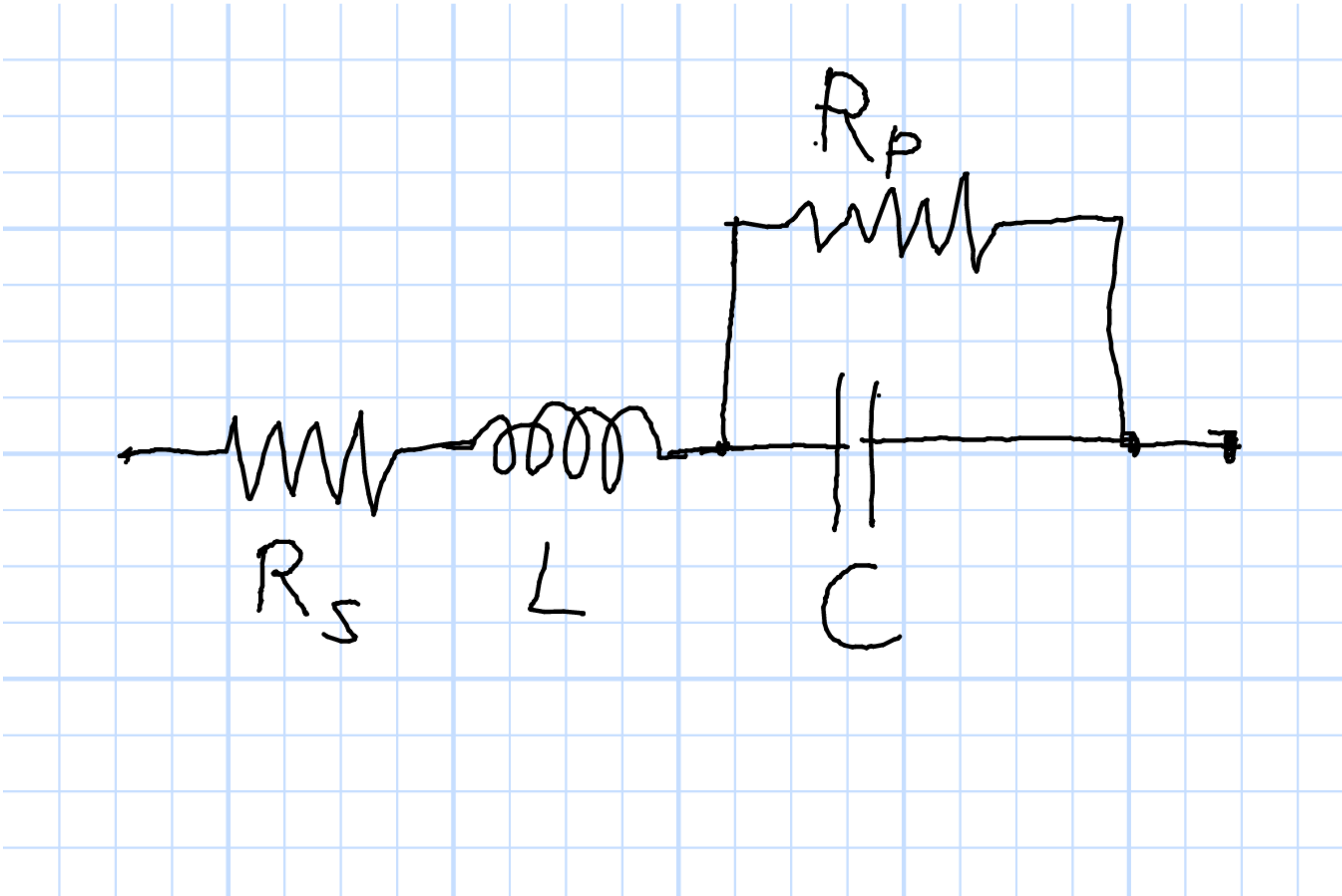
- DC Sources, Resistors, Capacitors (and later Inductors)
- $t \rightarrow \infty$
- $\frac{d\text{anything}}{dt} \rightarrow 0$
- Specifically $\frac{dv_{capacitor}}{dt} \rightarrow 0$
- Therefore $i_{capacitor} = 0$
- Treat Capacitors as Open and Solve

Steady–State Example



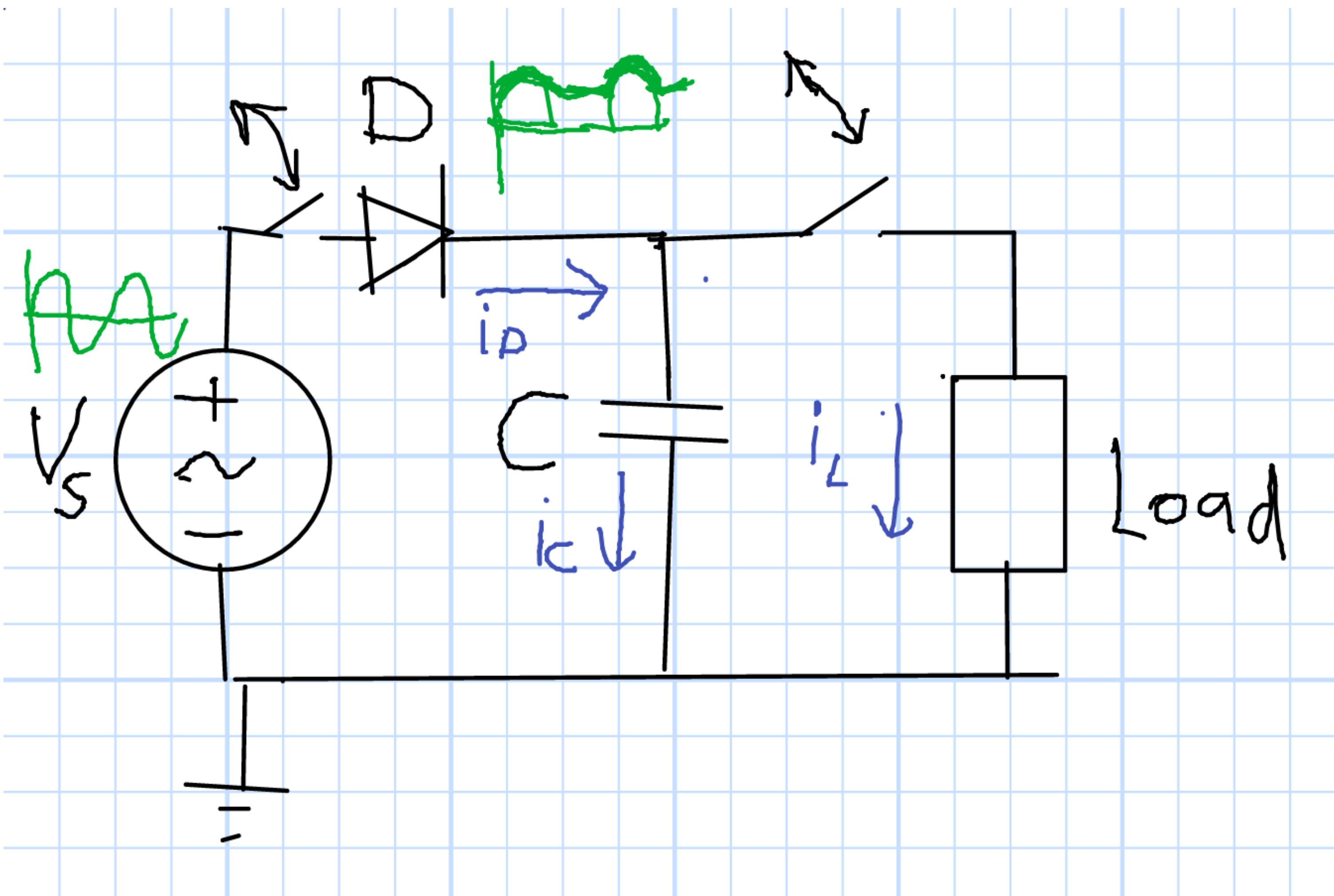
Turn on V_s at $t = 0$. $v_A = ?$ $v_B = ?$ $v_C = ?$ $i_L = ?$

Real Capacitors



R_s, L Low. R_p High.

High Voltage Power Supply



Don't Try this at Home, Kids!

High–Voltage Capacitor:

$v = 10\text{kV} \qquad C = 10,000\mu\text{F} \qquad R_p = 10\text{G}\Omega$

Discharge Time at Constant Current:

$$q = Cv = 10\text{Coulombs}$$

$$i(0) = \frac{v}{R_P} = 1\mu\text{A}$$

$$t = \frac{q}{i} = 10^7\text{sec} = 116\ \text{Days}$$

Exponential decay will be slower.

“Shorting Bar”