Electrical Engineering
Week 4

Charles A. DiMarzio
EECE–2210
Northeastern University

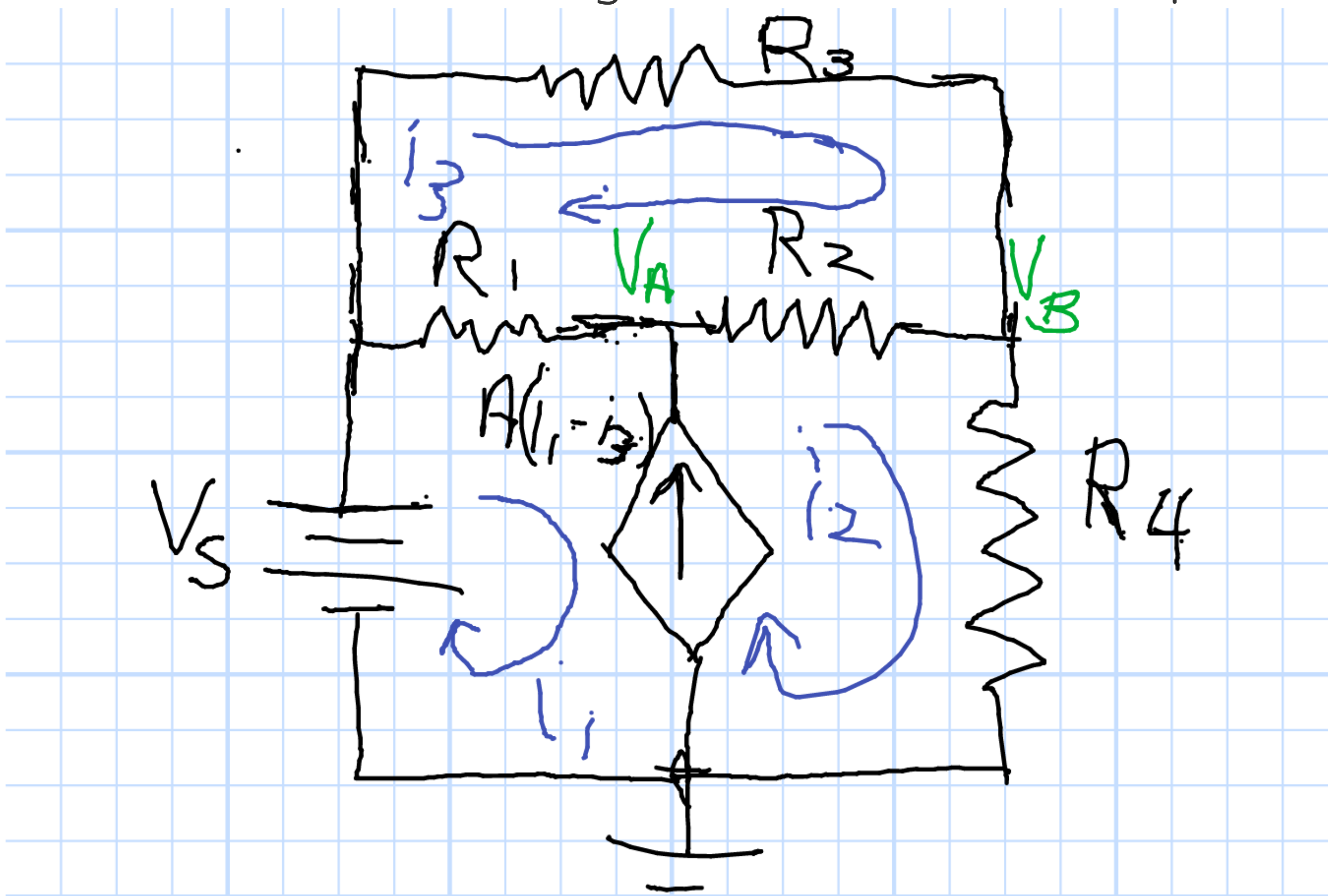
Sep 2022

Week 4 Agenda

- Mesh Analysis
- Thévenin Equivalent Circuits
- Norton Equivalent Circuits
- Solutions Using Superposition
- Wheatstone Bridge

Mesh Analysis

Solve Circuits Using KVL Around All Loops

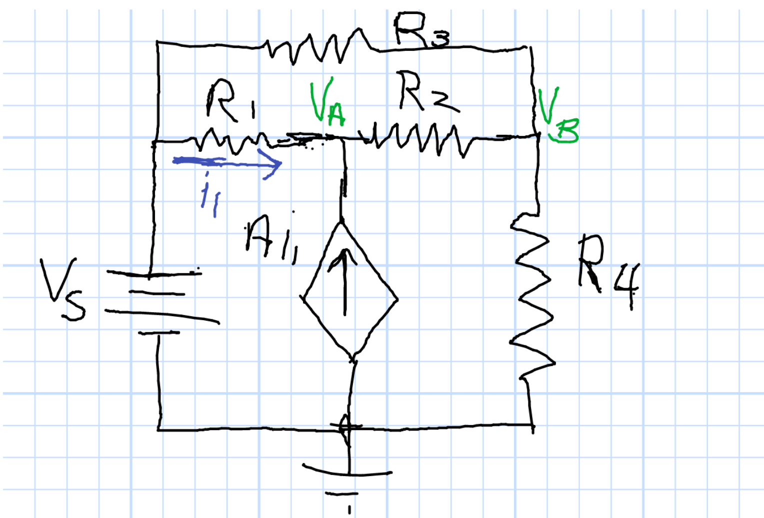


Remember Node Analysis

$V_s = 12\text{V} \qquad A = 3$

$R_1 = R_2 = 1k\Omega$

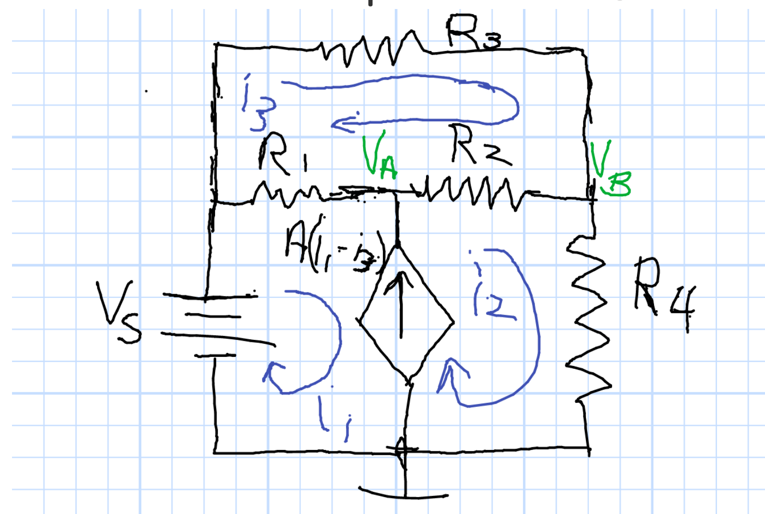
$R_3 = 5k\Omega \qquad R_4 = 200\Omega$



$$\mathbf{x} = \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 10 \\ 2 \end{pmatrix} \text{Volts}$$

Mesh Analysis

Problem of Dependent Source



Superloop 1,2

$$v_s - i_1R_1 + i_3R_1 - i_2R_2 + i_3R_2 - i_2R_4$$

$$= 0$$

Loop 3

$$i_3R_3 + (i_3 - i_2)\,R_2 + (i_3 - i_1)\,R_1 = 0$$

Example of Mesh Analysis

Dependent Source

$$A\,(i_1 - i_3) = i_2 - i_1$$

$$(A+1)\,i_1 - i_2 - Ai_3 = 0$$

Previous Page

$$v_s - i_1R_1 + i_3R_1 - i_2R_2 + i_3R_2 - i_2R_4 = 0$$

$$i_3R_3 + (i_3 - i_2)\,R_2 + (i_3 - i_1)\,R_1 = 0$$

Reorder

$$(A+1)\,i_1 - i_2 - Ai_3 = 0$$

$$-R_1i_1 - (R_2 + R_4)\,i_2 + (R_1 + R_2)\,i_3 = -v_s$$

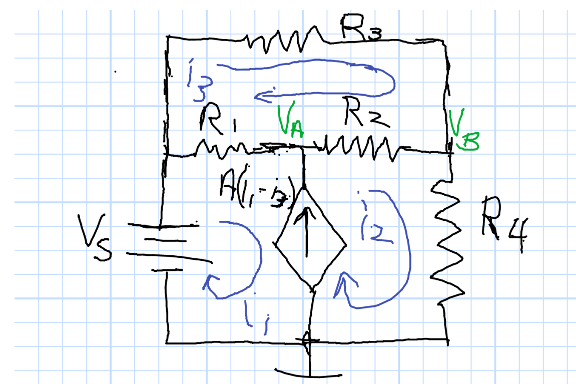
$$-R_1i_1 - R_2i_2 + (R_1 + R_2 + R_3)\,i_3 = 0$$

$$V_s = 12\text{V} \qquad A = 3$$

$$R_1 = R_2 = 1k\Omega$$

$$R_3 = 5k\Omega$$

$$R_4 = 200\Omega$$



Mesh Analysis Solution

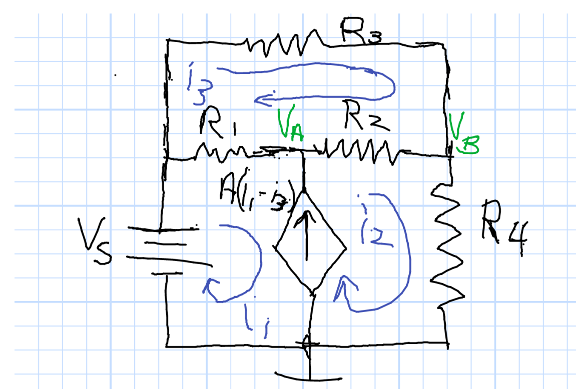
Previous Page

$$V_s = 12\text{V} \qquad A = 3$$
$$R_1 = R_2 = 1k\Omega$$
$$R_3 = 5k\Omega$$
$$R_4 = 200\Omega$$

$$(A+1)\,i_1 - i_2 - Ai_3 = 0$$

$$-R_1 i_1 - (R_2 + R_4)\,i_2 + (R_1 + R_2)\,i_3 = -v_s$$

$$-R_1 i_1 - R_2 i_2 + (R_1 + R_2 + R_3)\,i_3 = 0$$



$$\mathcal{M}\begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -v_s \\ 0 \end{pmatrix}$$

Matlab Mesh Results

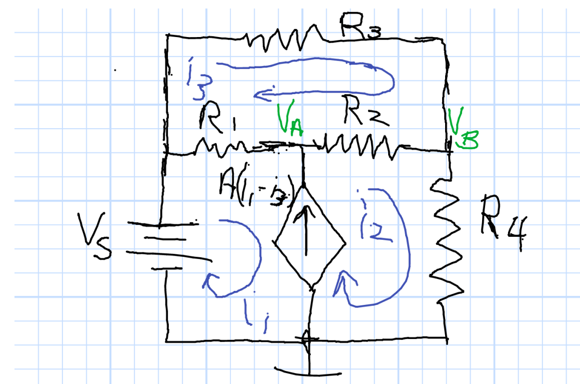
```
>> vs=12;A=3;R1=1000;R2=R1;...
R3=5000;R4=200;y=[0;-vs;0];
>> M=[A+1,-1,-A;...
-R1,-(R2+R4),R1+R2;...
-R1,-R2,R1+R2+R3]
M =
      4      -1      -3
  -1000   -1200    2000
  -1000   -1000    7000
>> x=inv(M)*y
x =
    0.0040
    0.0100
    0.0020
>> vBcheck=x(2)*R4
vBcheck =   2
```

$V_s = 12\text{V} \qquad A = 3$

$R_1 = R_2 = 1k\Omega$

$R_3 = 5k\Omega$

$R_4 = 200\Omega$

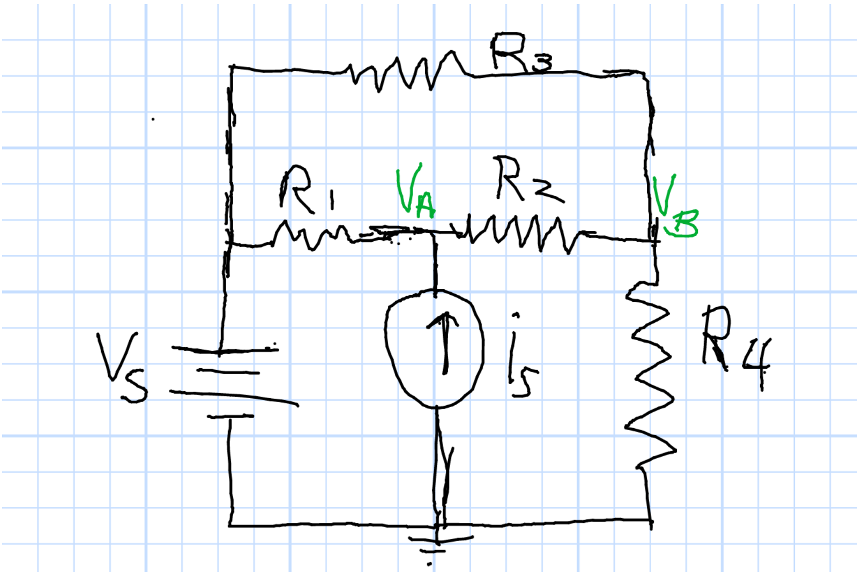


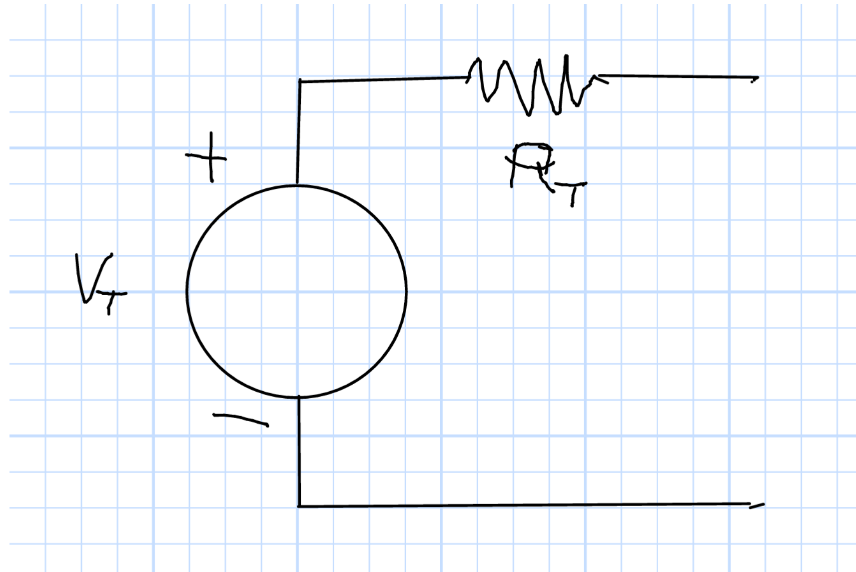
Node Solution

$$\begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 10 \\ 2 \end{pmatrix} \text{Volts}$$

```
>> vAcheck=vBcheck+x(2)*R2-x(3)*R2
vAcheck =   10
```

Thévenin Equivalent Circuit





R_4 is the Load: Draw a circuit for everything else.

Linear Circuits: i *vs.* v is a straight line.

We need two points to determine the line.

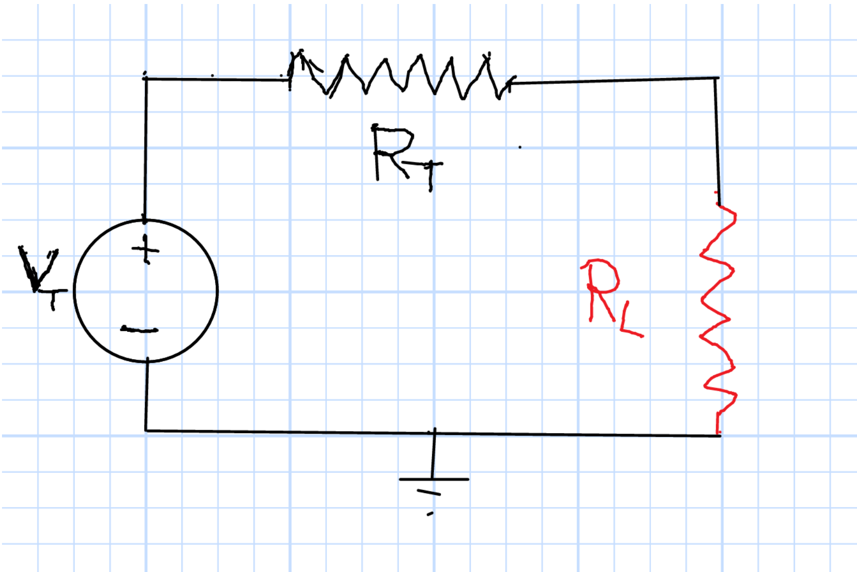
$i = 0$: Open Circuit Voltage. $v = 0$: Short–Circuit Current

Load Resistor: $i = \frac{v}{R_4}$

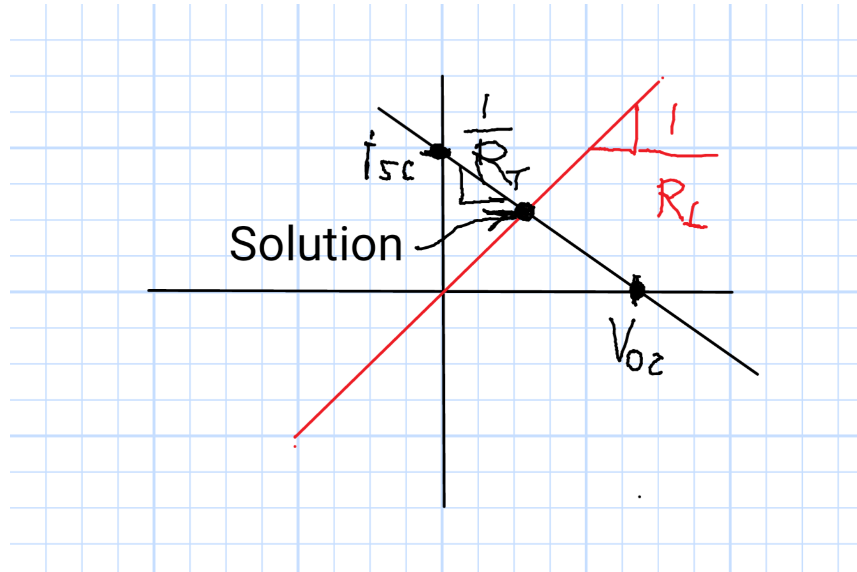
Solution: Voltage Divider

Everything Else: $i = \frac{v_T - v}{R_T}$

Thévenin Concept



Circuit Equation



Load Equation

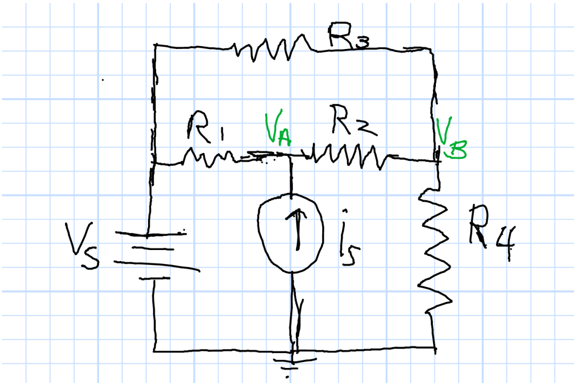
$$i = \frac{v_T - v}{R_T}$$

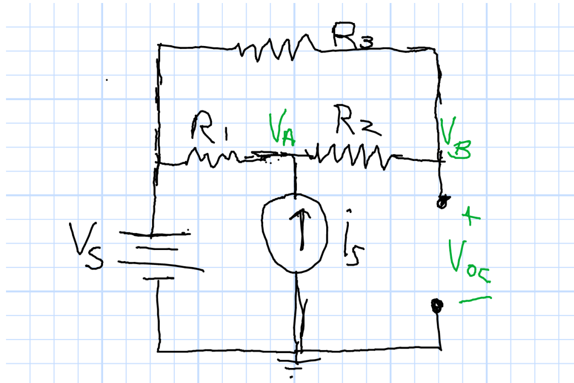
Load Line

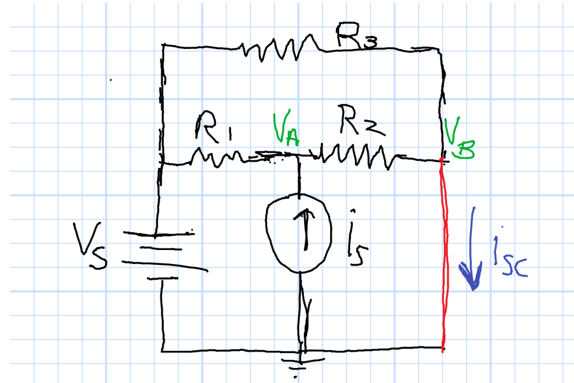
$$i = \frac{v}{R}$$

Solution is at the interesection.

Thévenin Calculation





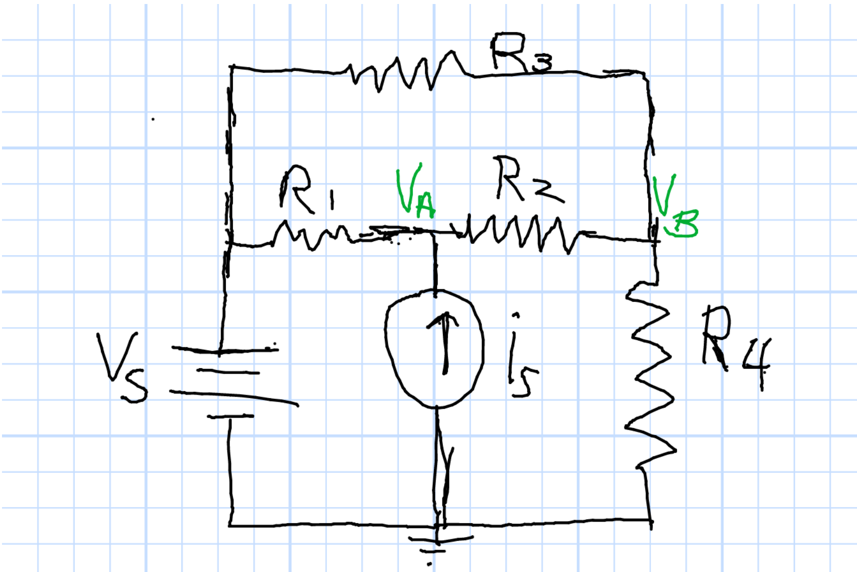


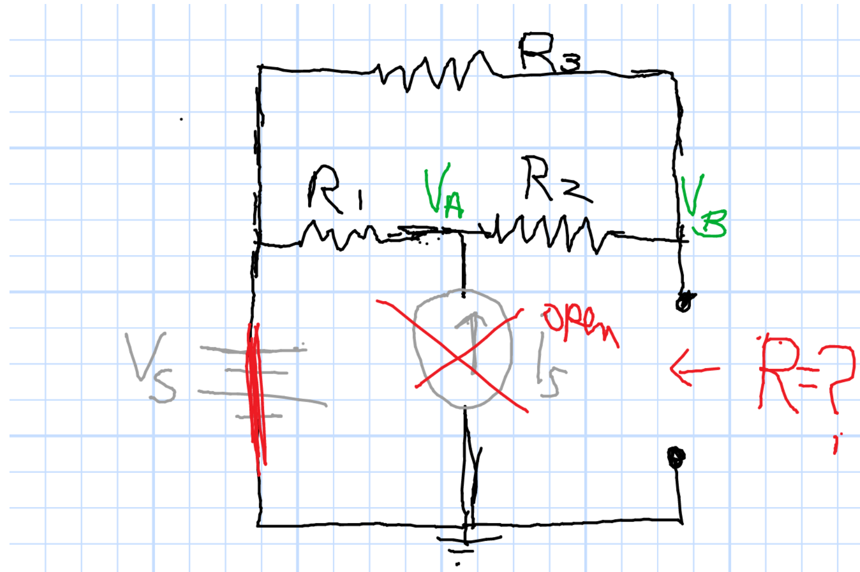
- Calculate Open–Circuit Voltage, v_{oc}.

- Calculate Short–Circuit Current, i_{sc}.

- Find Equivalent Circuit, $v_T = v_{oc}$ and $R_T = \frac{v_{oc}}{i_{sc}}$.

Zeroing All Sources





Alternative to Compute R_T

Voltage Sources Shorted ($v = 0$)

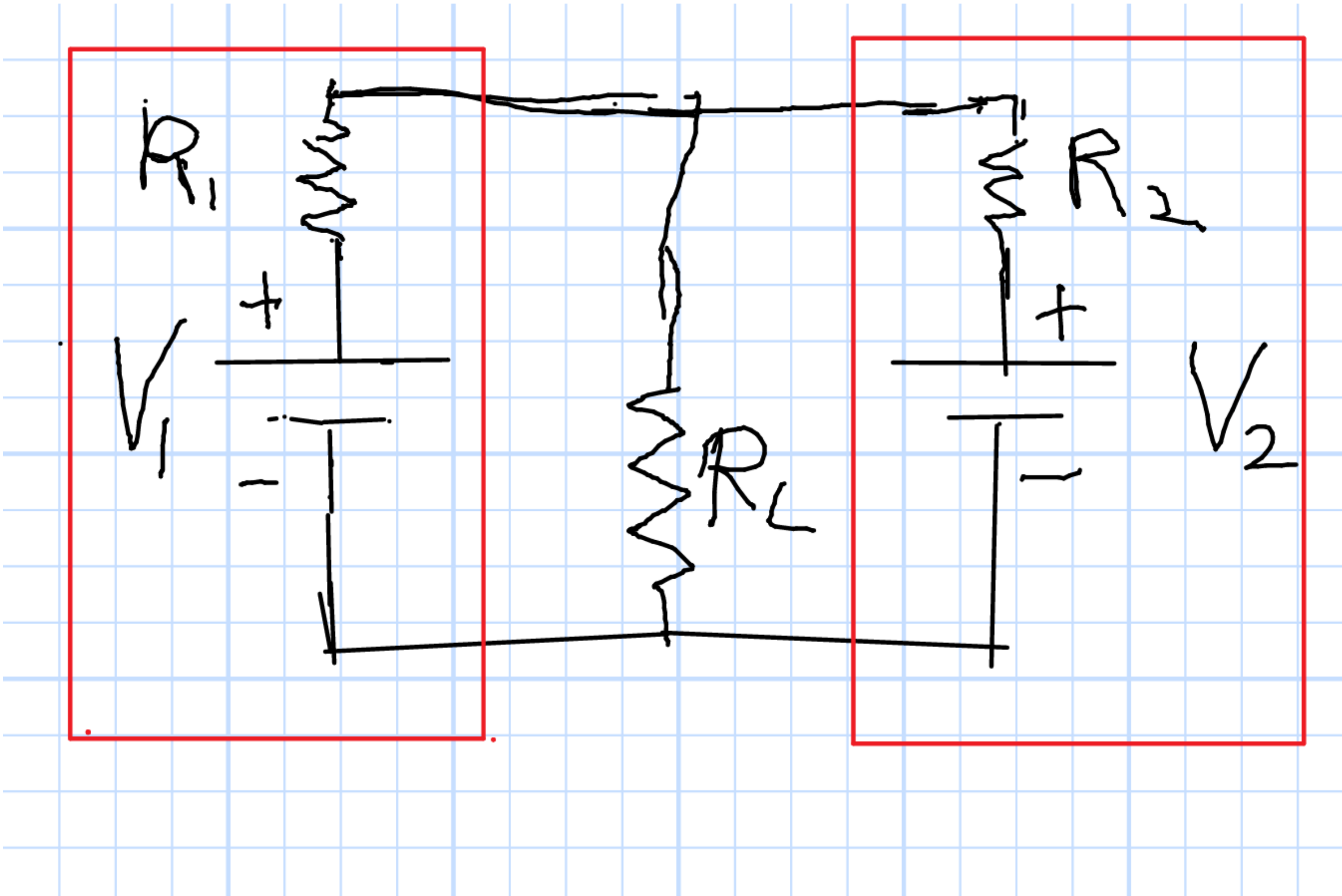
Current Sources Opened ($i = 0$)

Use Series and Parallel Combinations

Do This if v_{oc} or i_{sc} Looks Messy.

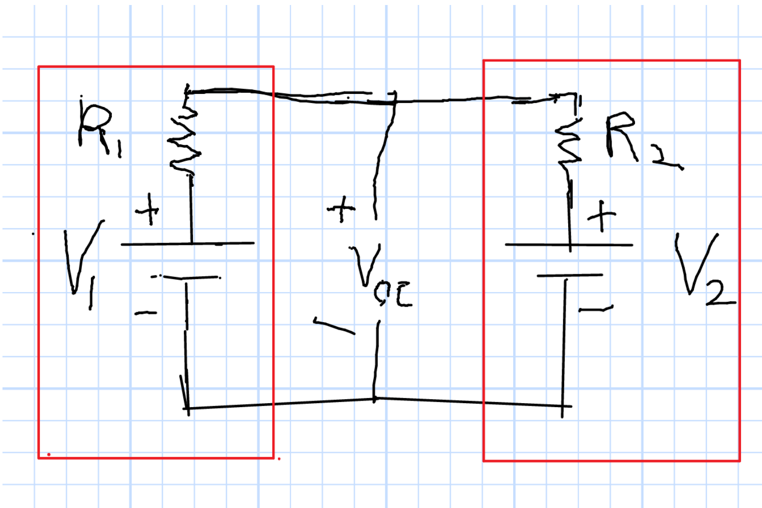
In This Example, $R_T = (R_1 + R_2) \parallel R_3 = 1430$Ohms

Thévenin Example: Parallel Batteries



Parallel Batteries Solution

$V_1 = 12\text{V},\ R_1 = 1\Omega,\ V_2 = 11.9\text{V},\ R_2 = 3\Omega$



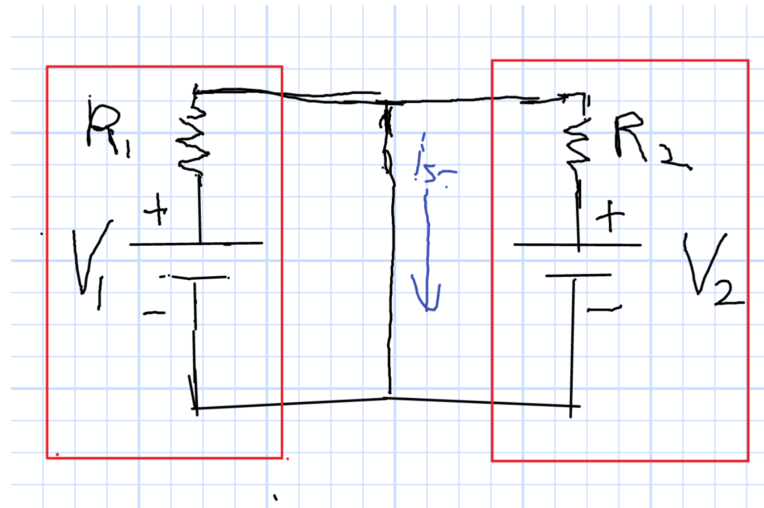
Voltage Divider

$$v_{oc} = v_1 - (v_1 - v_2)\frac{R_1}{R_1+R_2}$$

$$v_1\frac{R_2}{R_1+R_2} + v_2\frac{R_1}{R_1+R_2}$$

$$= \frac{v_1R_2+V_2R_1}{R_1+R_2}$$

$$= 11.97\text{V}$$



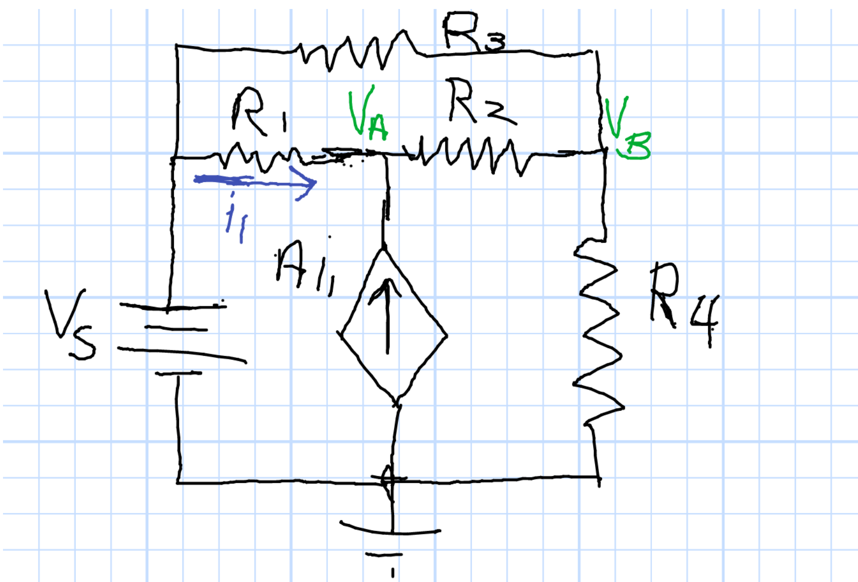
KCL

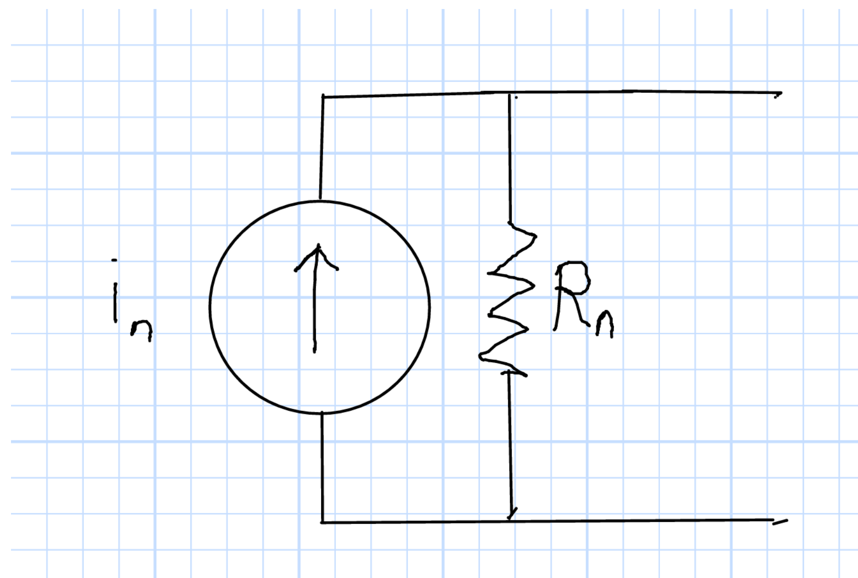
$$i_{sc} = \frac{v_1}{R_1} + \frac{v_2}{R_2} = \frac{v_1R_2+v_2R_1}{R_1R_2}$$

Thévenin Resistor

$$R_T = \frac{v_{oc}}{i_{sc}} = 0.75\Omega$$

Norton Equivalent Circuit





R_4 is the Load: Draw a circuit for everything else.

Linear Circuits: i *vs.* v is a straight line.

We need two points to determine the line.

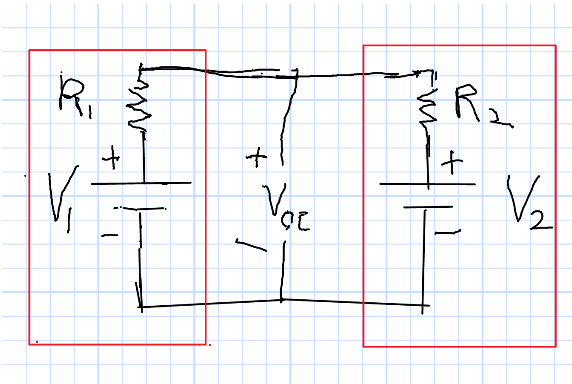
$i = 0$: Open Circuit Voltage. $v = 0$: Short–Circuit Current

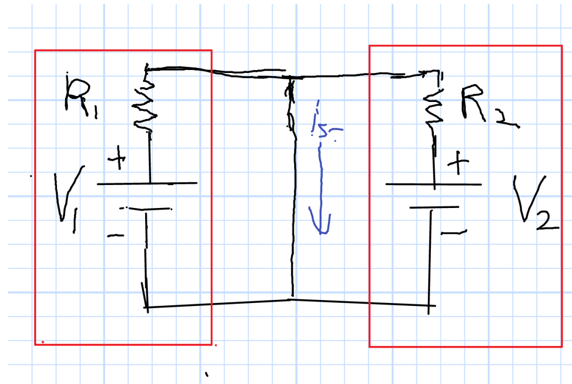
Load Resistor: $i = \frac{v}{R_4}$

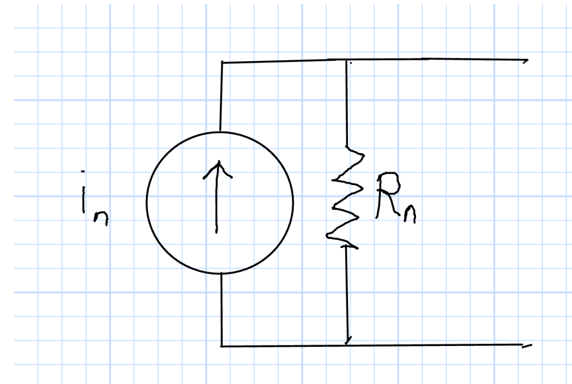
Solution: Current Divider

Everything Else: $i = i_N - \frac{v}{R_N}$

Norton Example







From Thévenin Equivalent

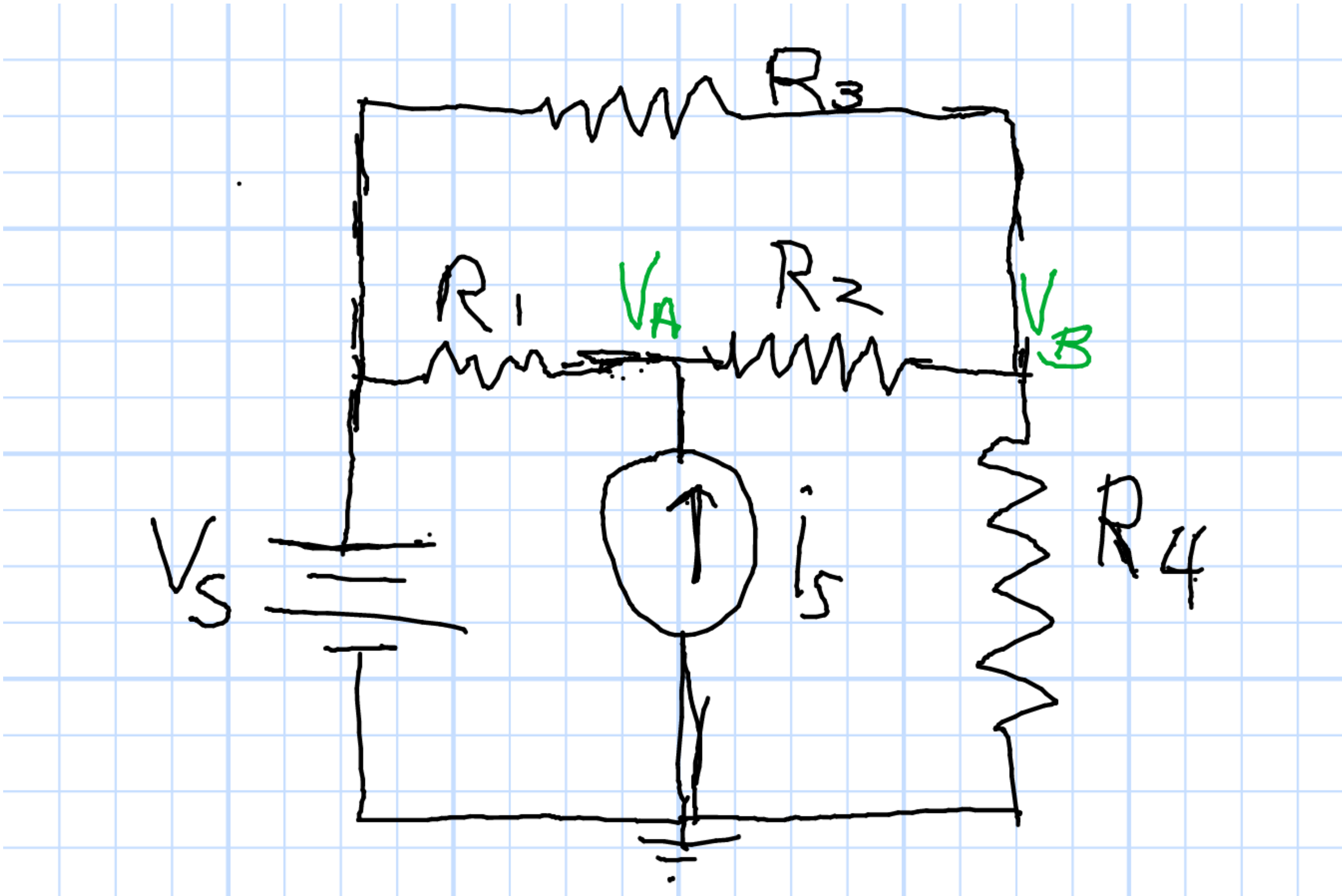
$$v_{oc} = 11.97\text{V} \qquad R_T = 0.75\Omega$$

$$i_{sc} = \frac{v_1}{R_1} + \frac{v_2}{R_2} = \frac{v_1 R_2 + v_2 R_1}{R_1 R_2} = 15.96\text{A}$$

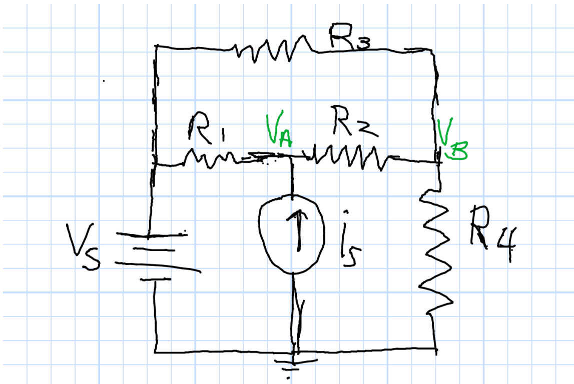
Norton Equivalent

$$i_N = i_{sc} = 15.96\text{A} \qquad R_N = R_T = 0.75\Omega$$

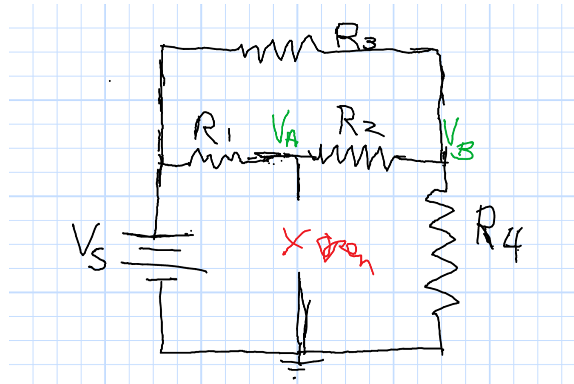
Superposition Example



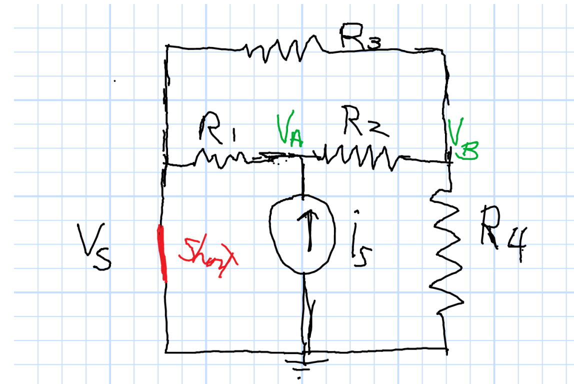
Zeroing Sources



Original



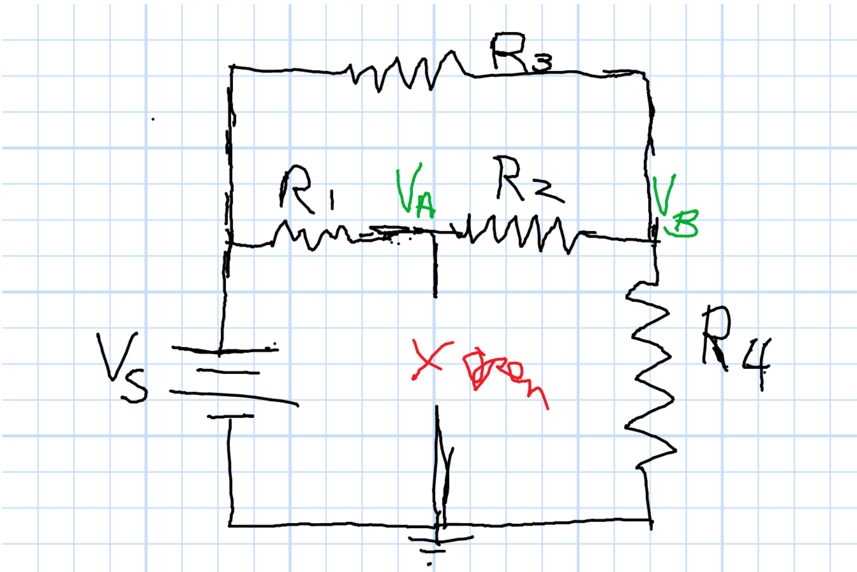
Zero Current Source
Open



Zero Voltage Source
Short

- Zero the Sources One at a Time
- Solve the Circuit for All Unknowns in Each Case
- Sum the Results

Superposition Solution



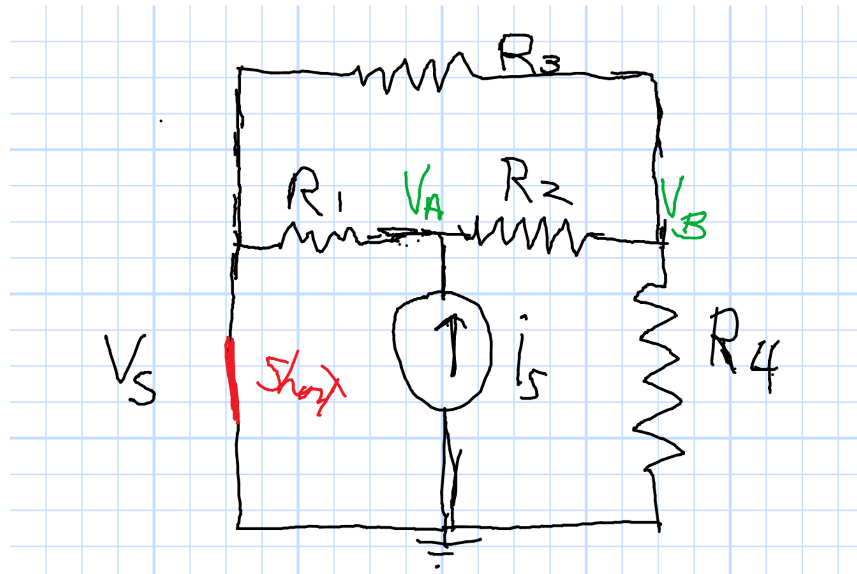
Series–Parallel Solution

$$i = v_s / \left[(R_1 + R_2) \parallel R_3 \right] + R_4$$

$$v_B = iR_4$$

Voltage Divider

$$v_A = v_s + (v_B - v_s)\,\frac{R_1}{R_1 + R_2}$$



Series–Parallel Solution

$$v_A = i_s \left\{ R_1 \parallel \left[R_2 + (R_3 \parallel R_4) \right] \right\}$$

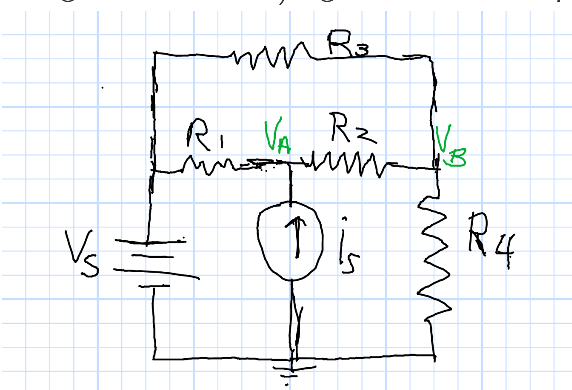
Current Divider (i_2 to the right)

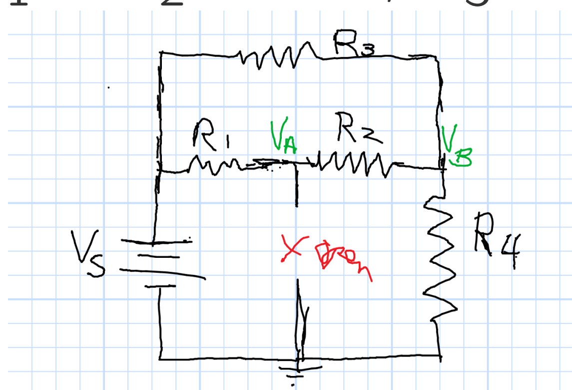
$$i_2 = i_s \frac{R_1}{R_1 + R_2 + (R_3 \parallel R_4)}$$

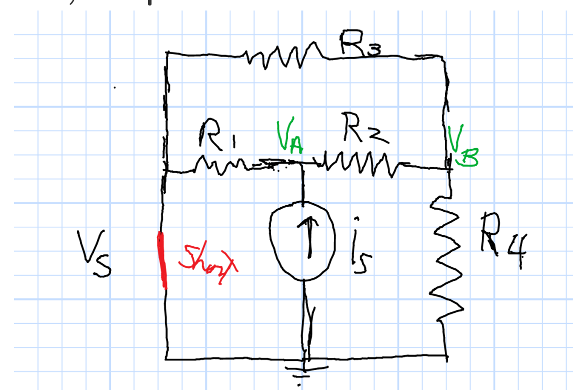
$$v_B = i_2\,(R_3 \parallel R_4)$$

Conclusion

$V_s = 12\text{V}, i_s = 6\text{mA},\ R_1 = R_2 = 1k\Omega,\ R_3 = 5k\Omega, R_4 = 200\Omega$







V_{total} $=$ v_1 $+$ v_2

v_1: $v_A = 6.74\text{V}$, $v_B = 1.47\text{V}$

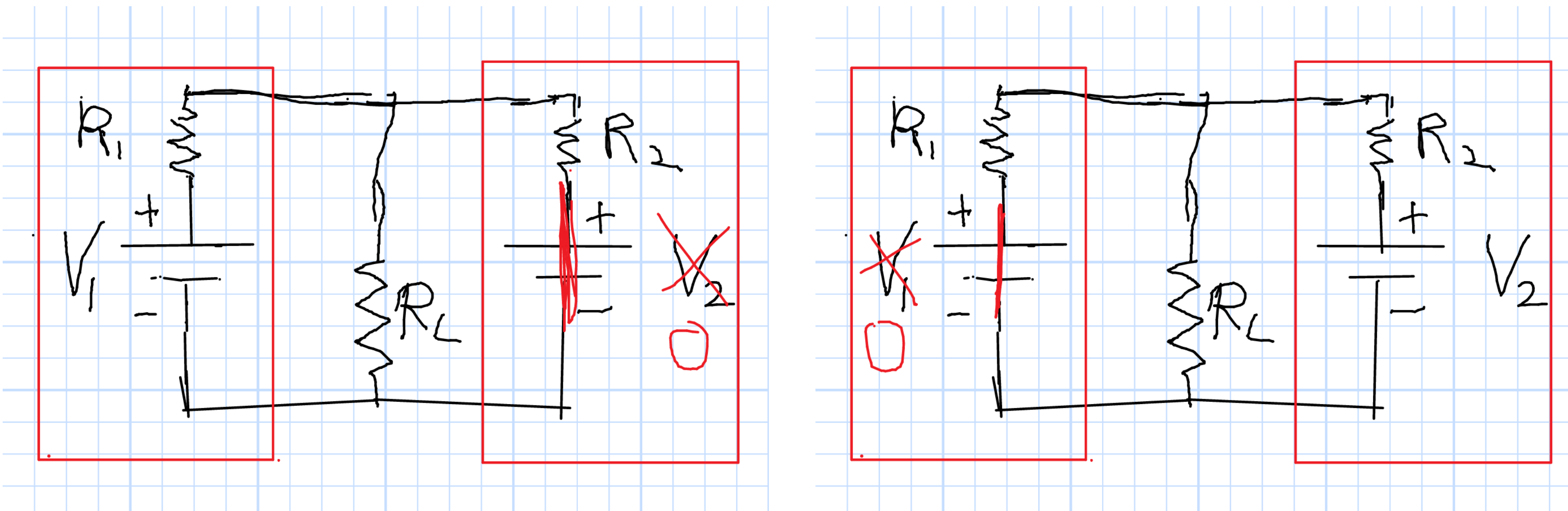
v_2: $v_A = 3.26\text{V}$, $v_B = 0.53\text{V}$

$V_A = 10.0\text{V} \qquad V_B = 2.0\text{V}$

Same Result We Had Before.

Parallel Batteries: Superposition

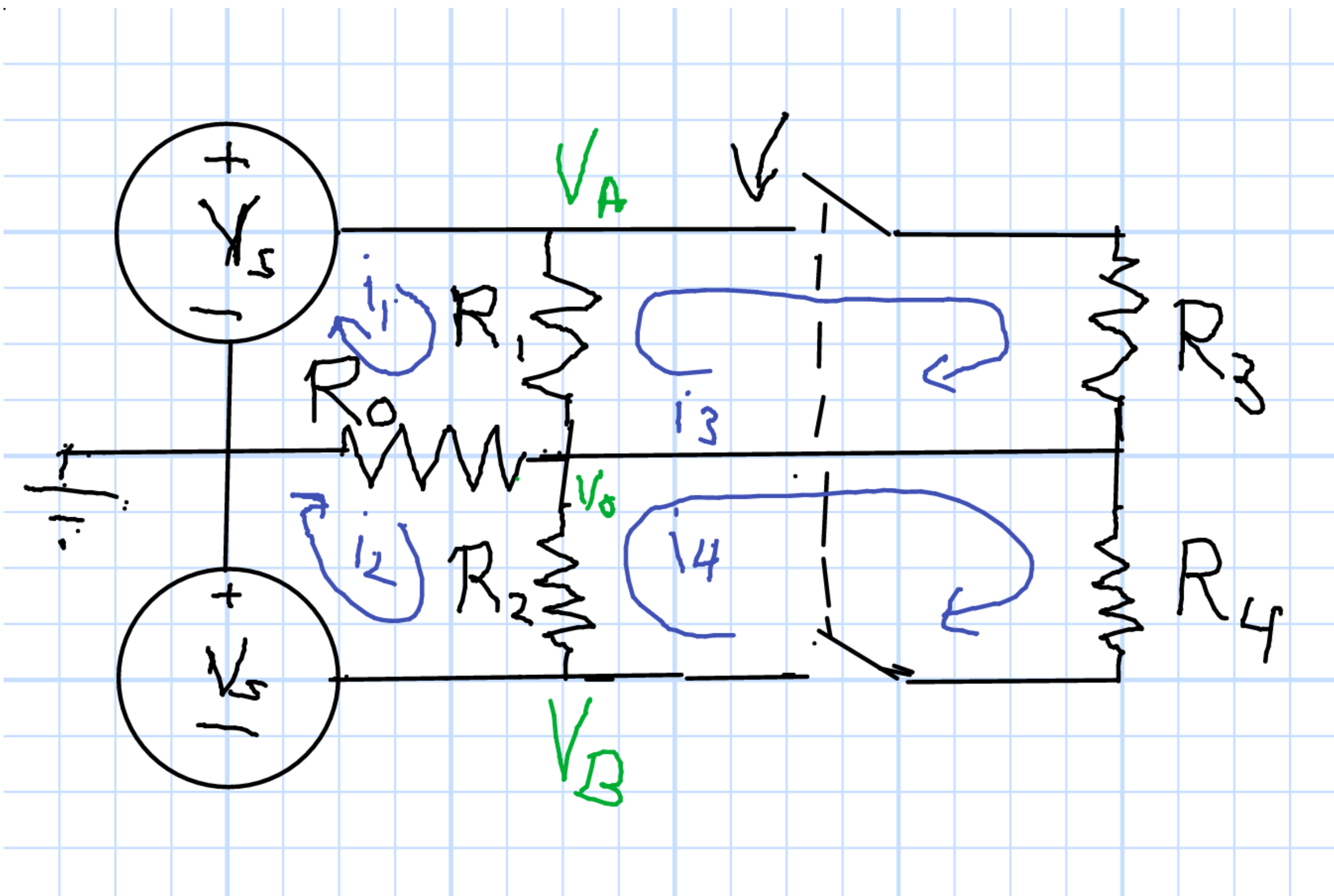
$V_1 = 12\text{V},\ R_1 = 1\Omega,\ V_2 = 11.9\text{V},\ R_2 = 3\Omega$



$$v_L = v_1 \frac{R_L \| R_2}{R_1 + (R_L \| R_2)} = 7.83\text{V} \qquad v_L = v_2 \frac{R_L \| R_1}{R_2 + (R_L \| R_1)} = 2.59\text{V}$$

$$v_L = 7.83\text{V} + 2.59\text{V} = 10.4\text{V}$$

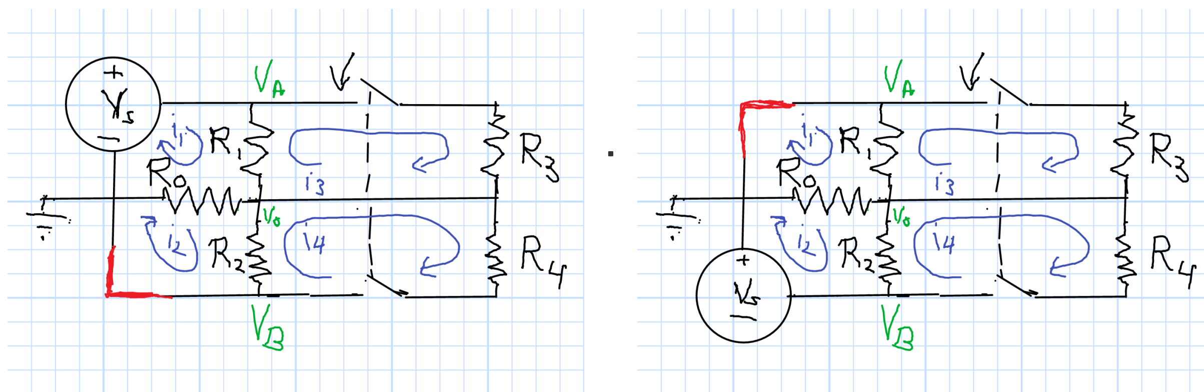
Balancing the Load



Source: $v_s = 120$V RMS. Bad Ground: $R_0 = 20\Omega$

Lights: $R_1 = 14.4\Omega$, $R_2 = 144\Omega$. Stove: $R_3 = R_4 = 7.2\Omega$.

Stove Off



Voltage Divider with Top Source: Zero Bottom so $R_2 \parallel R_0$

$$v_1 = v_s \frac{R_1}{R_1 + (R_0 \parallel R_2)} \qquad v_2 = v_s \frac{R_0 \parallel R_2}{R_1 + (R_0 \parallel R_2)}$$

Bottom Source: Zero Top so $R_1 \parallel R_0$

$$v_1 = -v_s \frac{R_1 \parallel R_0}{R_2 + (R_0 \parallel R_1)} \qquad v_2 = -v_s \frac{R_2}{R_2 + (R_0 \parallel R_1)}$$

Total

$$v_1 = 60.7\text{V} \qquad v_2 = 179.3\text{V} \qquad \text{Check } v_1 + v_2 = 240\text{V}$$

$$v_A = 120\text{V} \qquad v_B = -120\text{V} \qquad v_0 = v_A - v_1 = 59.3\text{V}$$

Stove On: Balance Load

Same as Previous Page, but with $R_1 \parallel R_3$ and $R_2 \parallel R_4$

Top Source

$$v_1 = v_s \frac{R_1 \| R_3}{(R_1 \| R_3) + (R_0 \| R_2 \| R_4)}$$

$$v_2 = v_s \frac{R_0 \| R_2 \| R_4}{(R_1 \| R_3) + (R_0 \| R_2 \| R_4)}$$

Bottom Source

$$v_1 = -v_s \frac{R_1 \| R_3 \| R_0}{(R_2 \| R_4) + (R_0 \| R_1 \| R_3)}$$

$$v_2 = -v_s \frac{R_2 \| R_4}{(R_2 \| R_4) + (R_0 \| R_1 \| R_3)}$$

Total

$v_1 = 101.4\text{V}$ $\quad v_2 = 138.6\text{V}$ $\quad$ Check $v_1 + v_2 = 240\text{V}$

$v_A = 120\text{V}$ $\quad v_B = -120\text{V}$ $\quad v_0 = v_A - v_1 = 18.6\text{V}$

Comparison

Stove Off

$$v_1 = 60.7\text{V} \qquad v_2 = 179.3\text{V}$$

$$v_A = 120\text{V} \qquad v_B = -120\text{V} \qquad v_0 = v_A - v_1 = 59.3\text{V}$$

Lights Represented by R_1 Are Dim
Lights Represented by R_2 Are Too Bright

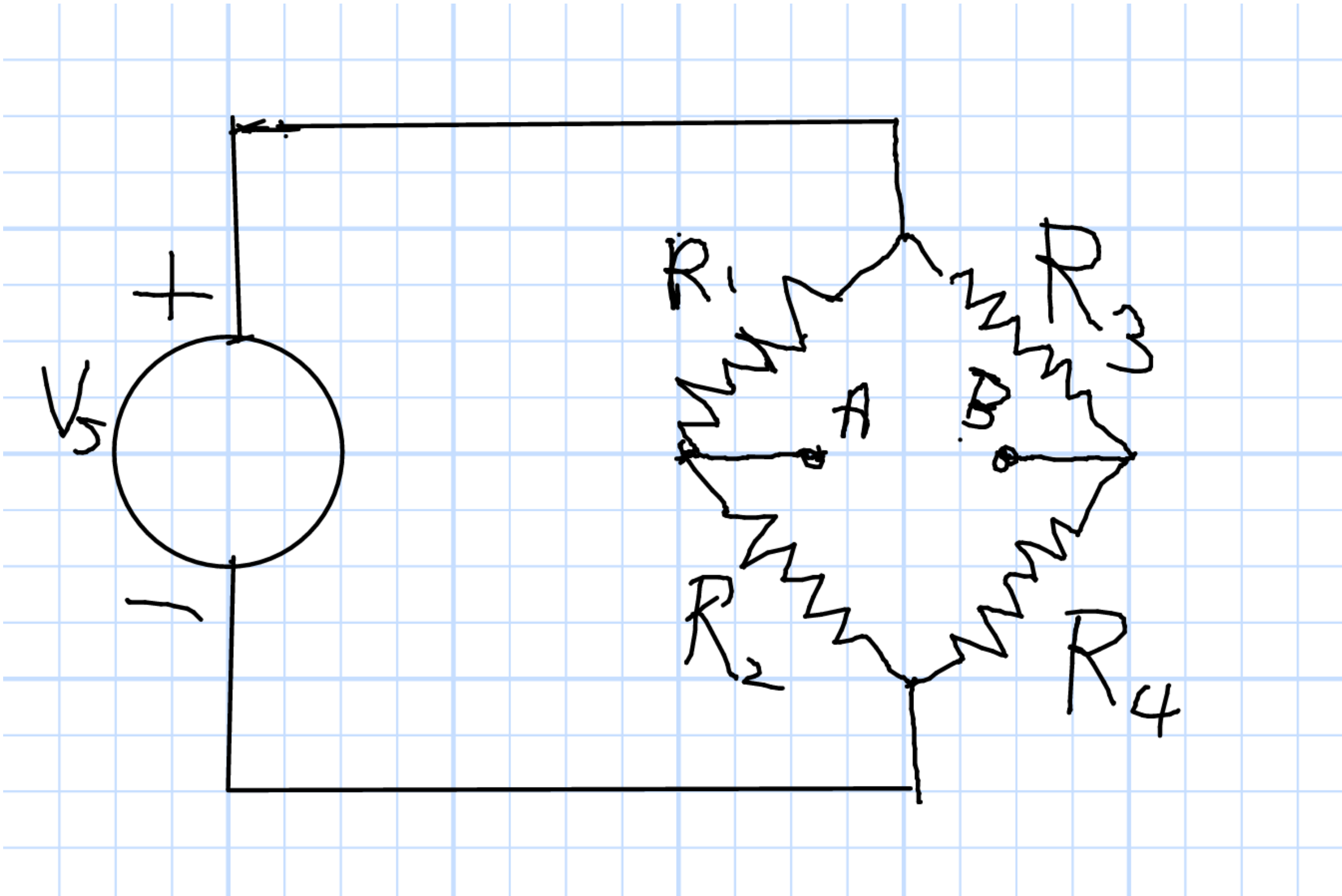
Stove On

$$v_1 = 101.4\text{V} \qquad v_2 = 138.6\text{V}$$

$$v_A = 120\text{V} \qquad v_B = -120\text{V} \qquad v_0 = v_A - v_1 = 18.6\text{V}$$

Lights Represented by R_1 Become Brighter
Lights Represented by R_2 Become Dimmer
Better Balance
"Ground" Voltage, v_0, Closer to Zero

Wheatstone Bridge

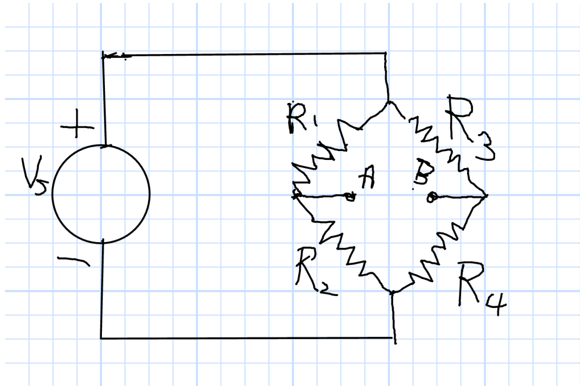


Balancing the Bridge

$$v_A = v_s \frac{R_2}{R_1 + R_2}$$

$$v_B = v_s \frac{R_3}{R_3 + R_4}$$

$$v_{BA} = v_s \left(\frac{R_2}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \right)$$



Application:

- R_4 is Unknown
- R_2 is Variable and Calibrated
- R_1 and R_3 are Precision Resistors
- Adjust for $V_{BA} = 0$
- Solve for R_4

Strain Gauge

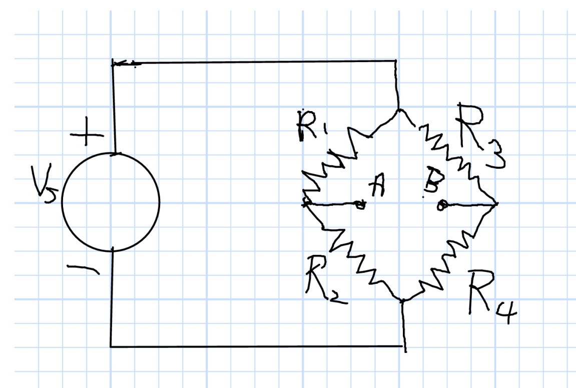
- R_4 is a Strain Guage
- $R_4 = R_0 + \Delta R$
- $R_{1:3} = R_0$ Precision Resistors
- Calculate v_{AB}

$$v_{BA} = v_s \left(\frac{R_0}{2R_0} - \frac{R_0 + \Delta R}{2R_0 + \Delta R} \right)$$

$$v_{BA} = v_s \left(\frac{1}{2} - \frac{R_0 + \Delta R}{2R_0 + \Delta R} \right)$$

$$v_{BA} \approx v_s \left(\frac{1}{2} - \frac{R_0 + \Delta R}{2R_0} \right)$$

$$v_{BA} \approx -v_s \frac{\Delta R}{2R_0}$$



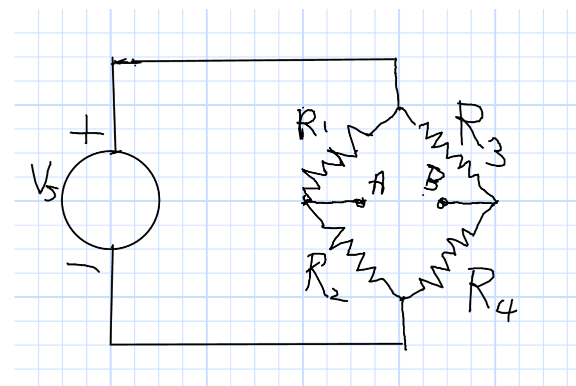
4 Balanced Strain Gauges

- $R_1 = R_4 = R_0 + \Delta R$ Are Strain Guage
- $R_2 = R_3 = R_0 + \Delta R$ Measure Opposite Strain
- Calculate v_{AB}

$$v_{BA} = v_s \left(\frac{R_0 + \Delta R}{2R_0} - \frac{R_0 - \Delta R}{2R_0} \right)$$

$$v_{BA} = v_s \frac{2\Delta R}{2R_0}$$

$$v_{BA} = v_s \frac{\Delta R}{R_0}$$



May Need a Variable to Get a Good Null