Electrical Engineering
Week 3

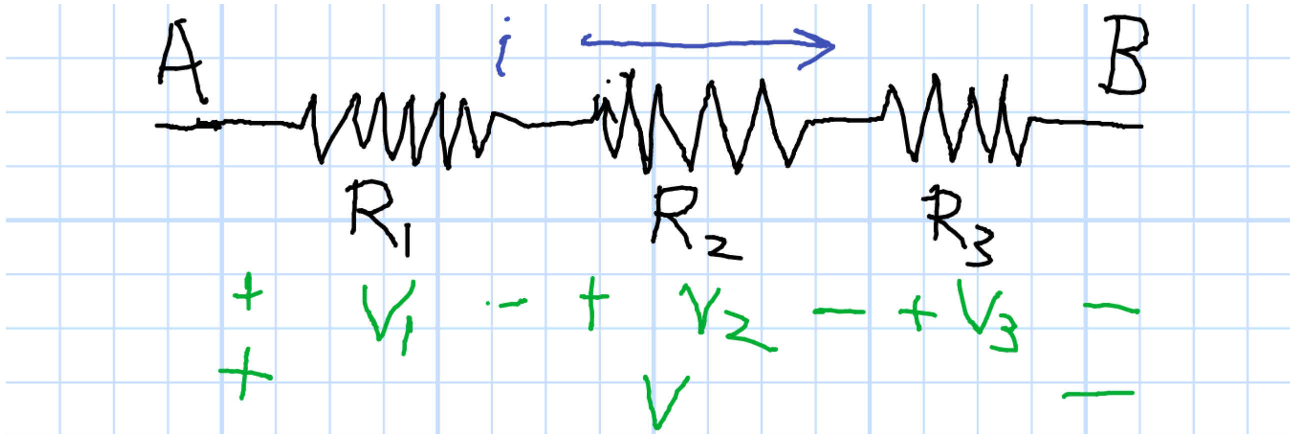
Charles A. DiMarzio
EECE–2210
Northeastern University

Aug 2020

Week 3 Agenda

- First Lab, Tuesday, Thursday, or Friday
- Resistors in Series
- Resistors in Parallel
- Equivalent Resistors
- Power in Resistor Combinations
- Voltage Dividers
- Current Dividers
- Node Analysis

Resistors in Series (1)

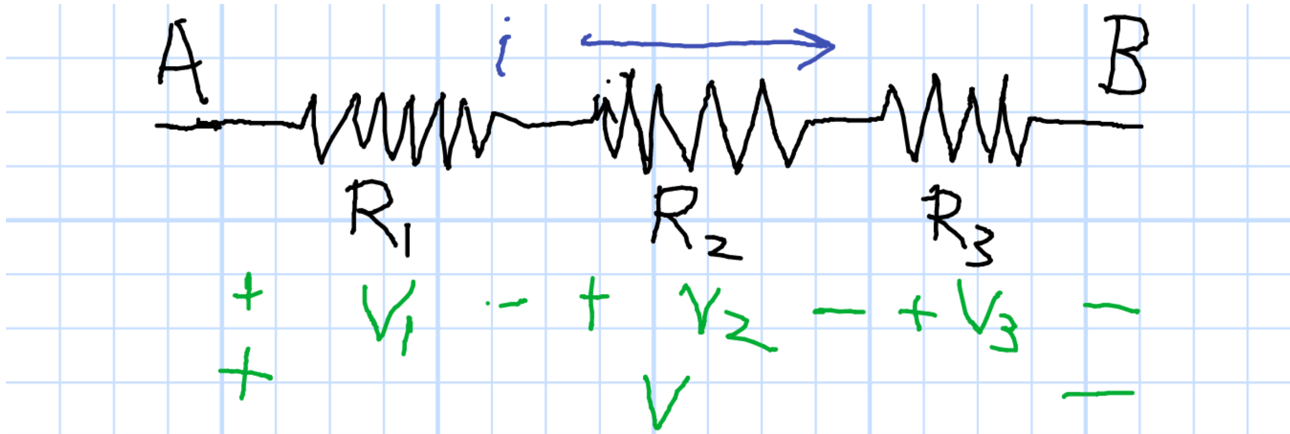


$$V = V_1 + V_2 + V_3 \qquad \text{KVL}$$

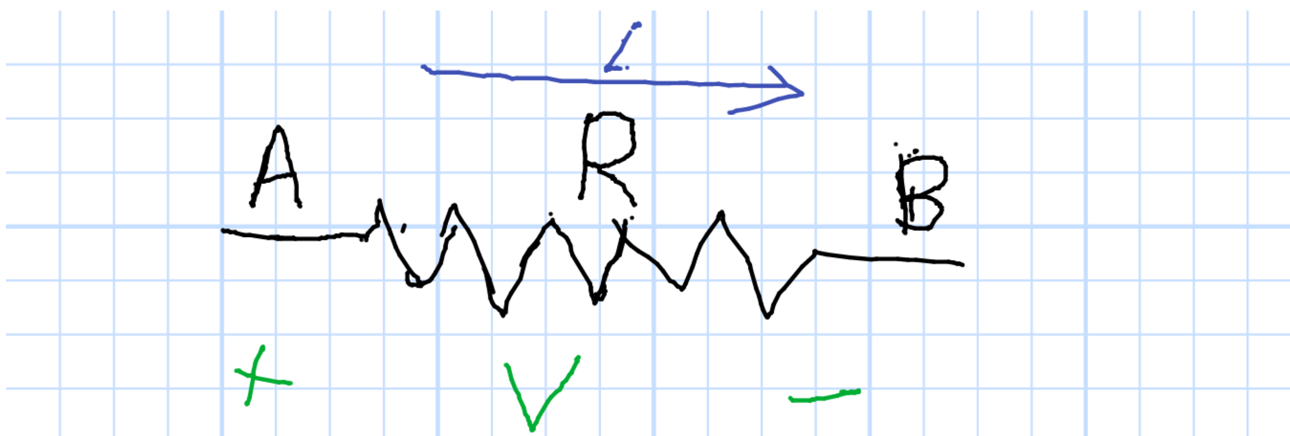
$$\frac{V}{i} = \frac{V_1}{i} + \frac{V_2}{i} + \frac{V_3}{i}$$

$$R = R_1 + R_2 + R_3$$

Resistors in Series (2)



$$R = R_1 + R_2 + R_3$$



Resistors in Series: Examples

Two 1kΩ Resistors in Series

$$R = R_1 + R_2$$

$$R = 2R_1$$

$$R = 2\text{k}\Omega$$

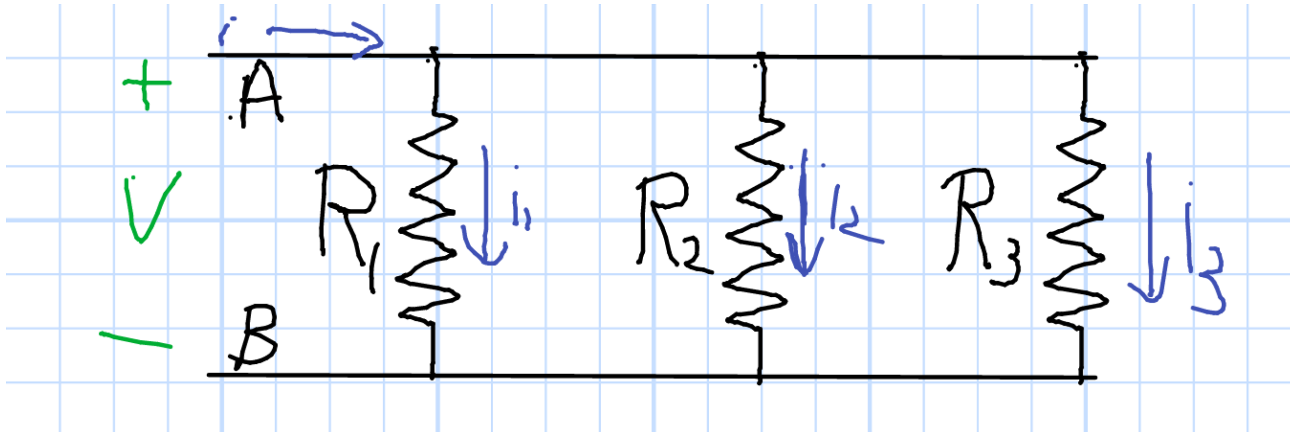
One Large Resistor and One Much Smaller

$$R = R_1 + R_2$$

$$R \approx Max\left(R_n\right)$$

For Example $R_2 = R_1/10$ $\qquad R = 0.91R_1$ $\qquad$ (10% error)

Resistors in Parallel (1)

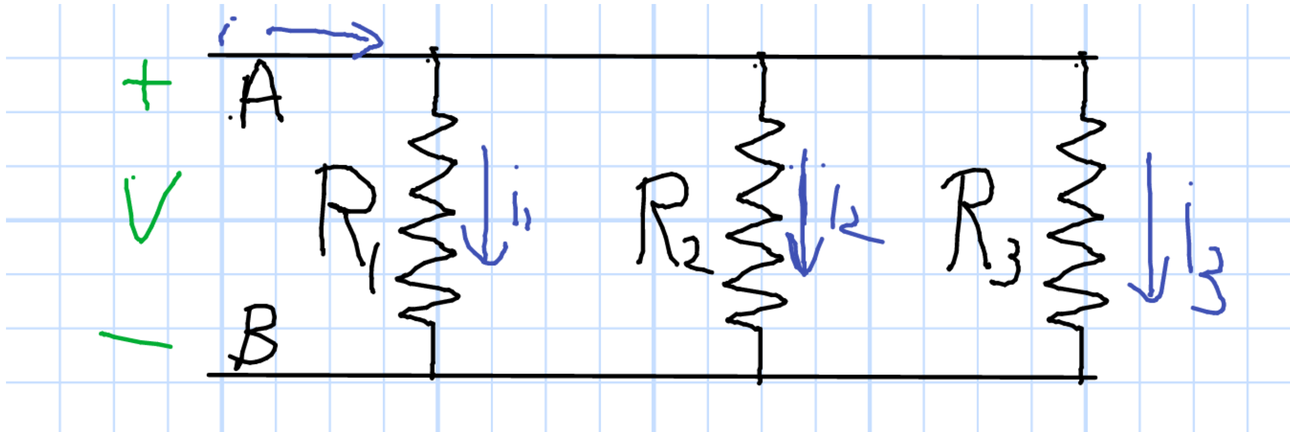


$$i = i_1 + i_2 + i_3 \qquad \text{KCL}$$

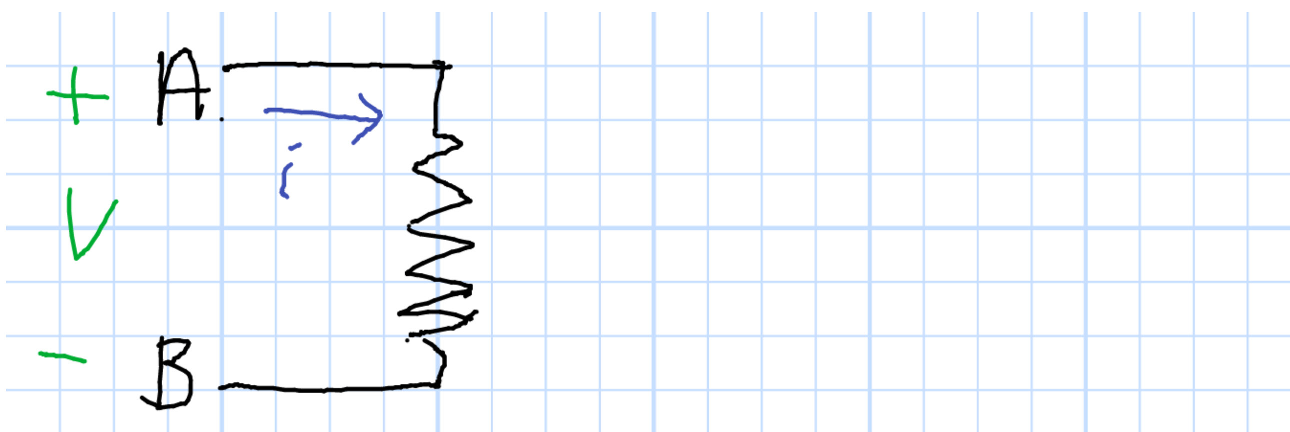
$$\frac{i}{V} = \frac{i_1}{V} + \frac{i_2}{V} + \frac{i_3}{V}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Resistors in Parallel (2)



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



Parallel–Resistor Equations

$$R = R_1 \parallel R_2$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

Conductances Add

$$G = G_1 + G_2$$

Resistors in Parallel: Example

Two 1kΩ Resistors in Parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R = \frac{R_1}{2}$$

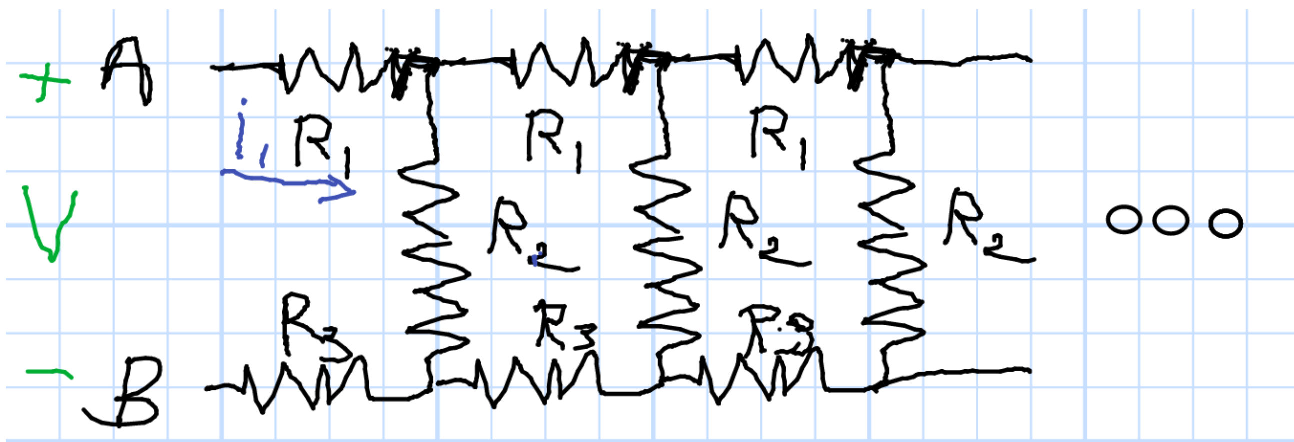
$$R = 500\Omega$$

One Large Resistor and One Much Smaller

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R \approx Min\left(R_n\right)$$

Example: Ladder Network



Infinite Network: Equivalent Resistor? ($R_1 = R_2 = R_3 = 50\Omega$)
Assume the Answer is R
Add One More Link

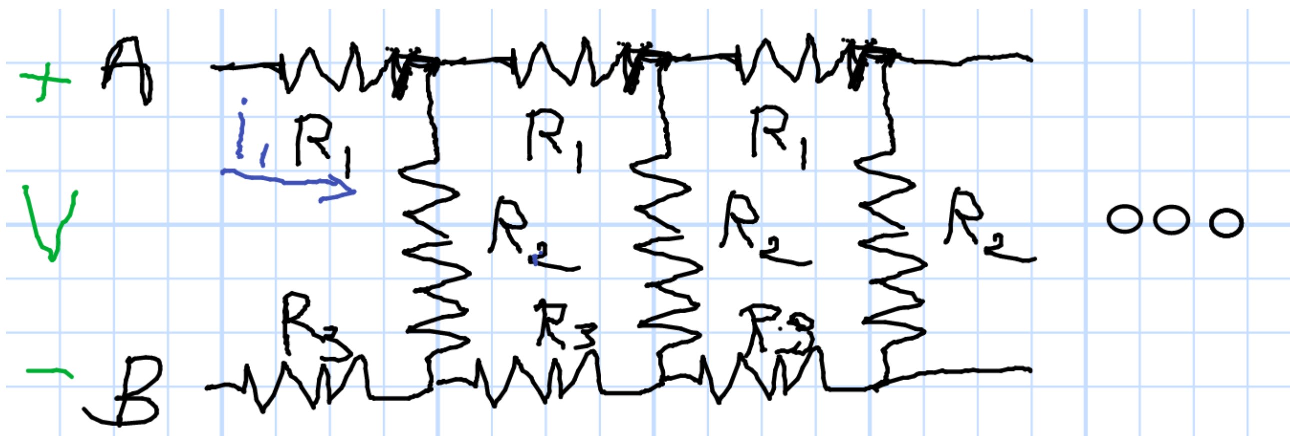
$$R = R_1 + (R_2 \parallel R) + R_3$$

$$R = R_1 + \frac{R_2 R}{R_2 + R} + R_3$$

$$R(R_2 + R) = R_1(R_2 + R) + R_2 R + R_3(R_2 + R)$$

$$R(R_2 + R - R_1 - R_2 - R_3) = R_1 R_2 + R_3 R_2$$

Ladder Solution



Previous Page

$$R\left(R_2 + R - R_1 - R_2 - R_3\right) = R_1 R_2 + R_3 R_2$$

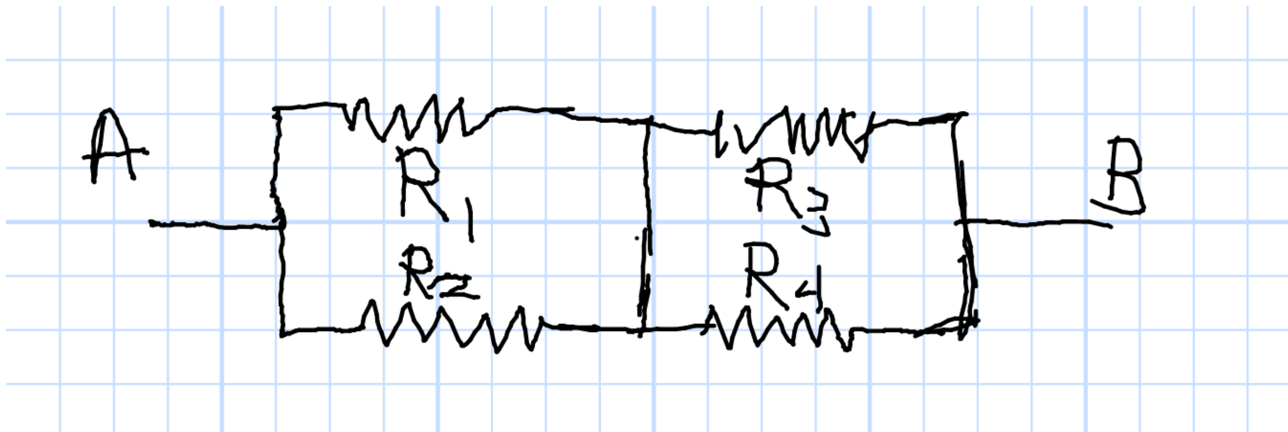
$$R^2 - R\left(R_1 + R_3\right) - R_1 R_2 + R_3 R_2 = 0$$

Possible Solutions

$$R = \frac{\left(R_1 + R_3\right) \pm \sqrt{\left(R_1 + R_3\right)^2 + 4 R_2 \left(R_1 + R_3\right)}}{2}$$

For All 50Ω Resistors, $R = 137\Omega$ (Failed Solution, $R = -37\Omega$)

Power Issues



What Resistors to Use?

$R = 1000$, All Resistors Equal

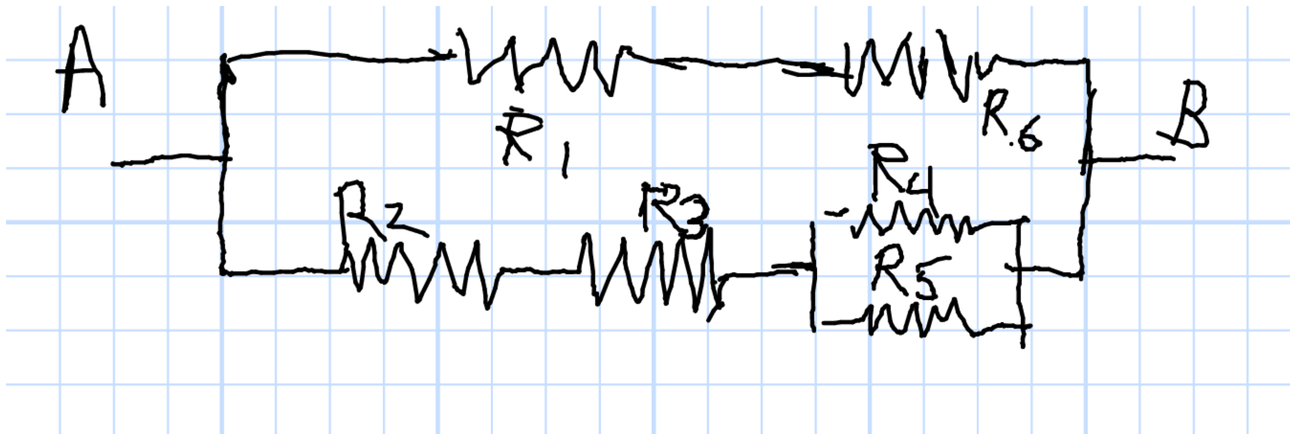
$$R_{1:4} = ?$$

What is the Power in Each Resistor as a Fraction of the Total?

$$\frac{P_n}{P_{total}} = ?$$

What if I leave out the vertical wire in the middle?

Equivalent Resistance (1)



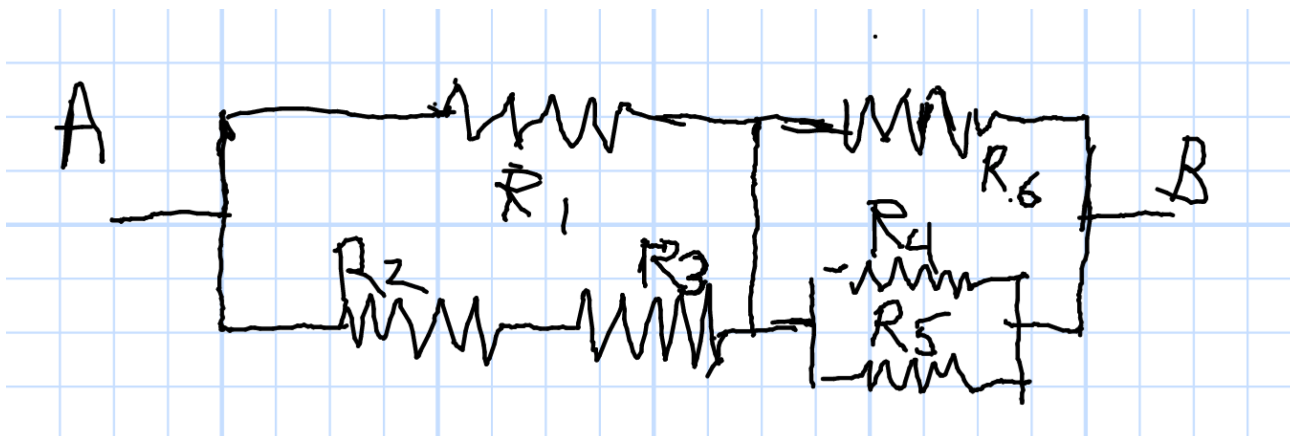
$$[R_1 + R_6] \parallel [R_2 + R_3 + (R_4 \parallel R_5)]$$

$$R_n = 50\Omega \qquad \text{All } n$$

$$R = [50 + 50] \parallel [50 + 50 + 25]$$

$$R = 55.6\Omega$$

Equivalent Resistance (2)



$$[R_1 \parallel (R_2 + R_3)] + [R_4 \parallel R_5 \parallel R_6]$$

$$R_n = 50\Omega \qquad \text{All } n$$

$$R = [50 \parallel (100)] + [50/3]$$

$$R = 50\Omega$$

Series and Parallel

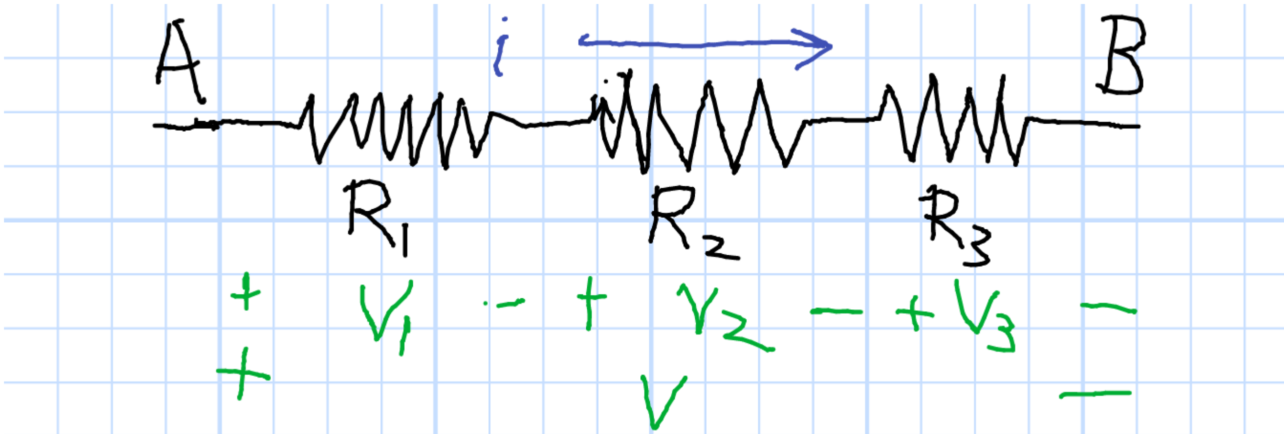
Series

- Voltage Sources Add
- Current Sources Fail
- Resistors Add

Parallel

- Voltage Sources Fail
- Current Sources Add
- Resistors Add Inverses

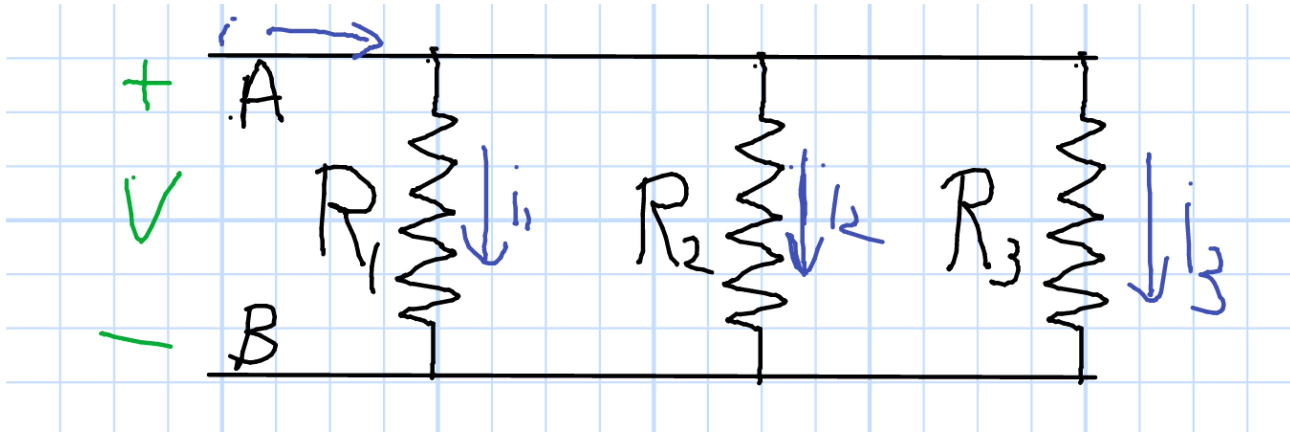
Voltage Divider



$$v_1 = iR_1 \qquad v = iR = i\left(R_1 + R_2 + R_3\right)$$

$$v_1 = v\frac{R_1}{R_1 + R_2 + R_3}$$

Current Divider



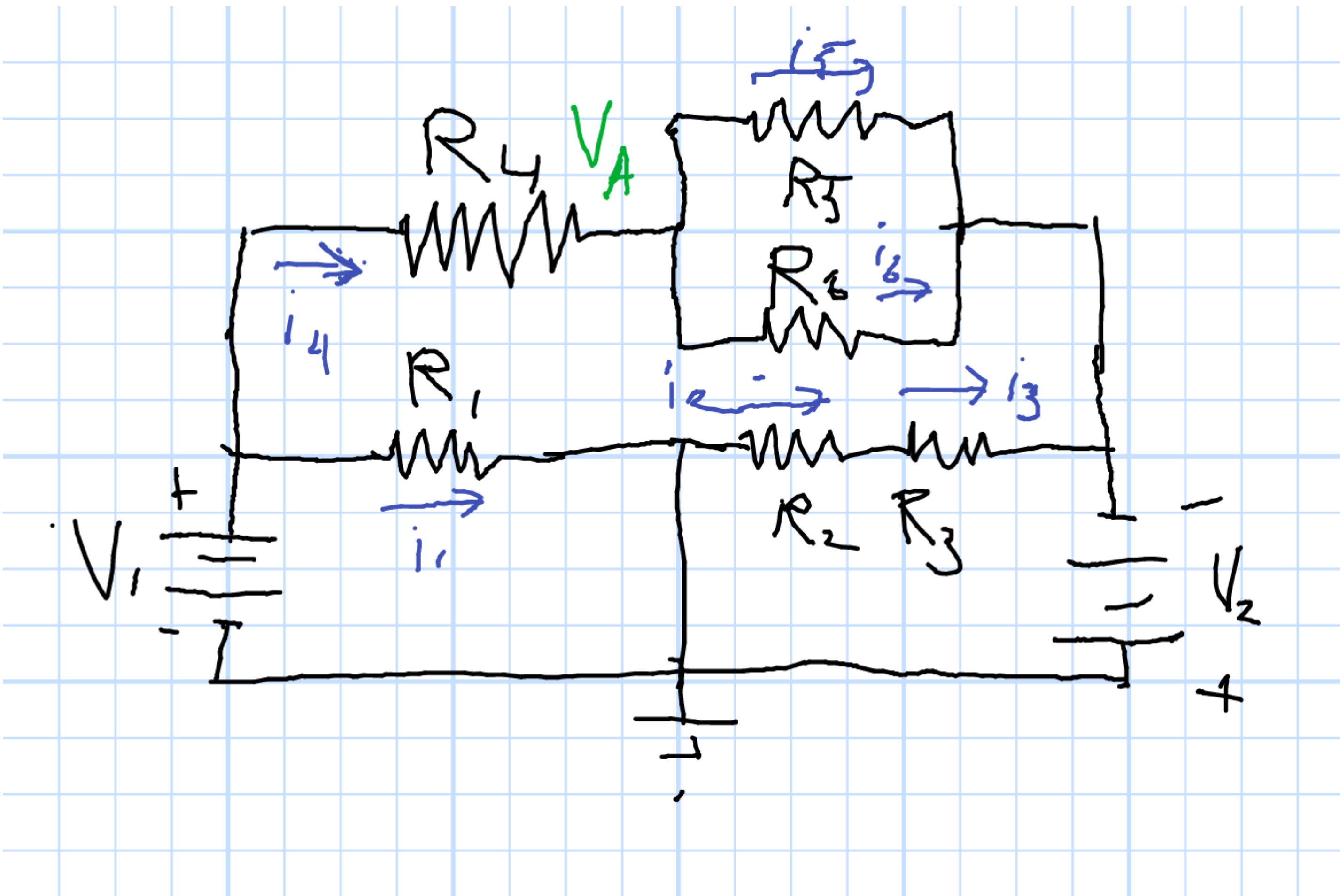
$$v = i_1 R_1 \qquad v = iR = i\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

$$i_1 = i\frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

$$i_1 = \frac{G_1}{G_1+G_2+G_3}$$

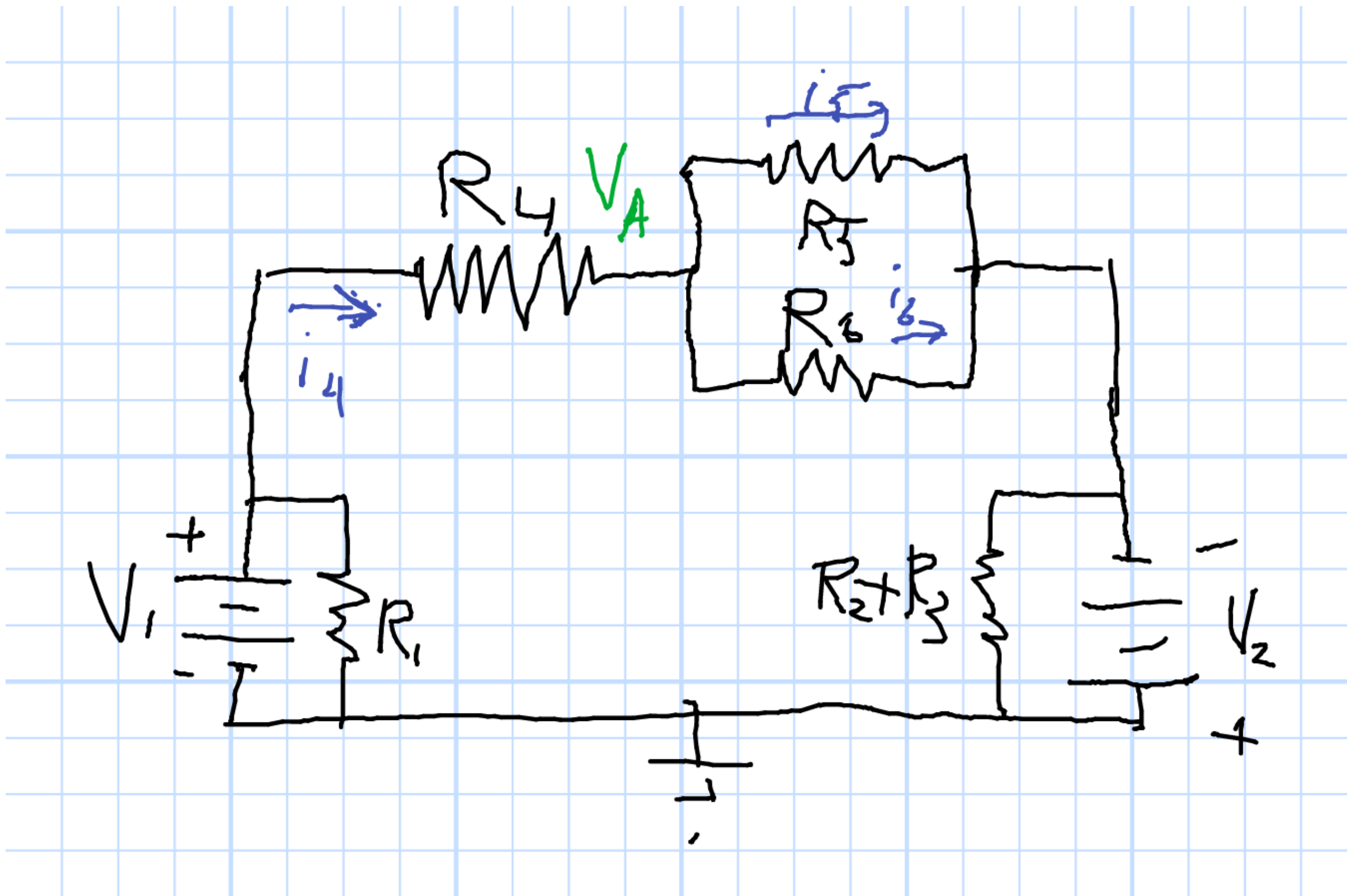
A Tricky One (1)

Reduce Using Series & Parallel Combinations: $v_{1,2} = 12\text{V}$, $R_n = 100\Omega$



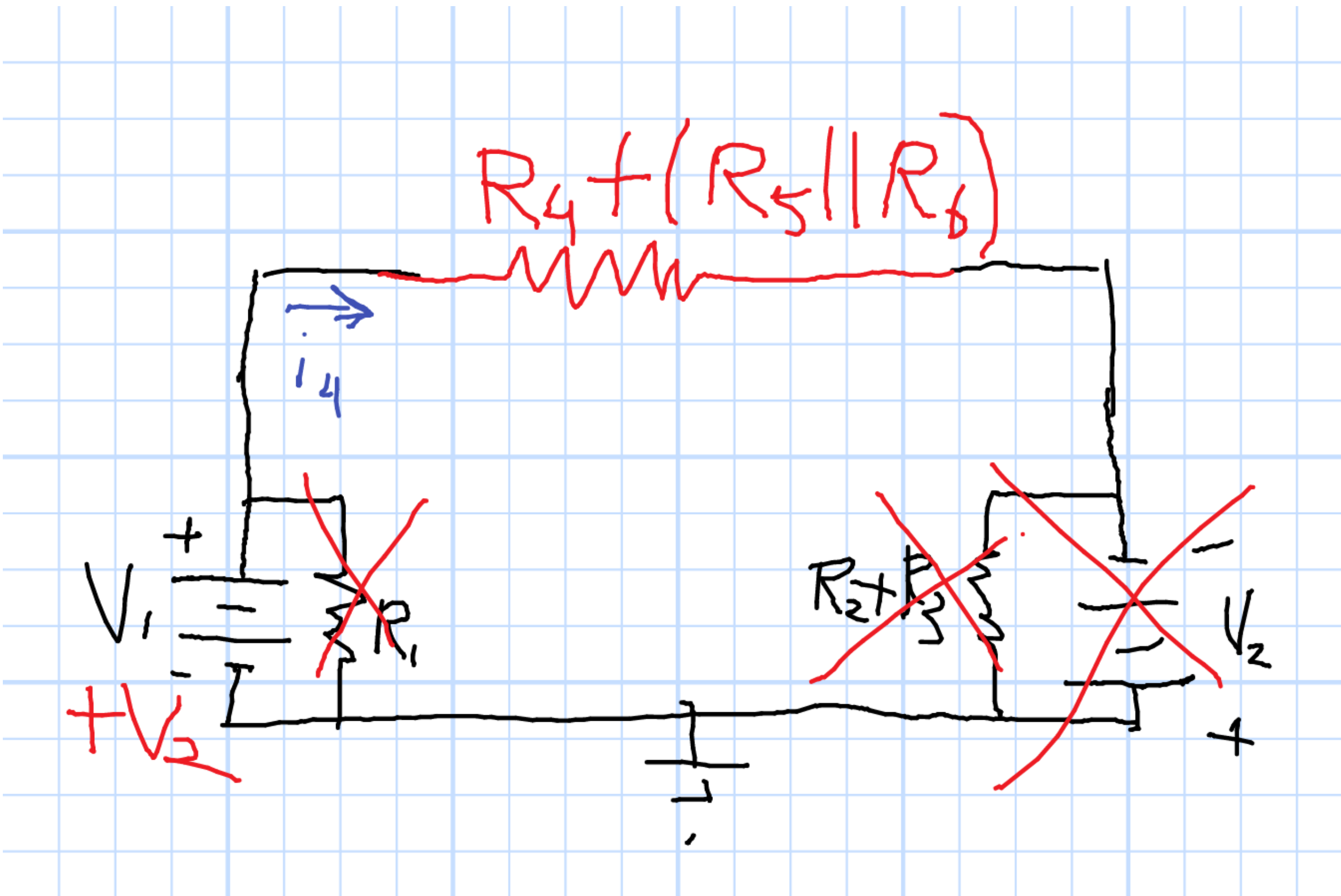
A Tricky One (2)

Reduce Using Series & Parallel Combinations: $v_{1,2} = 12\text{V}$, $R_n = 100\Omega$



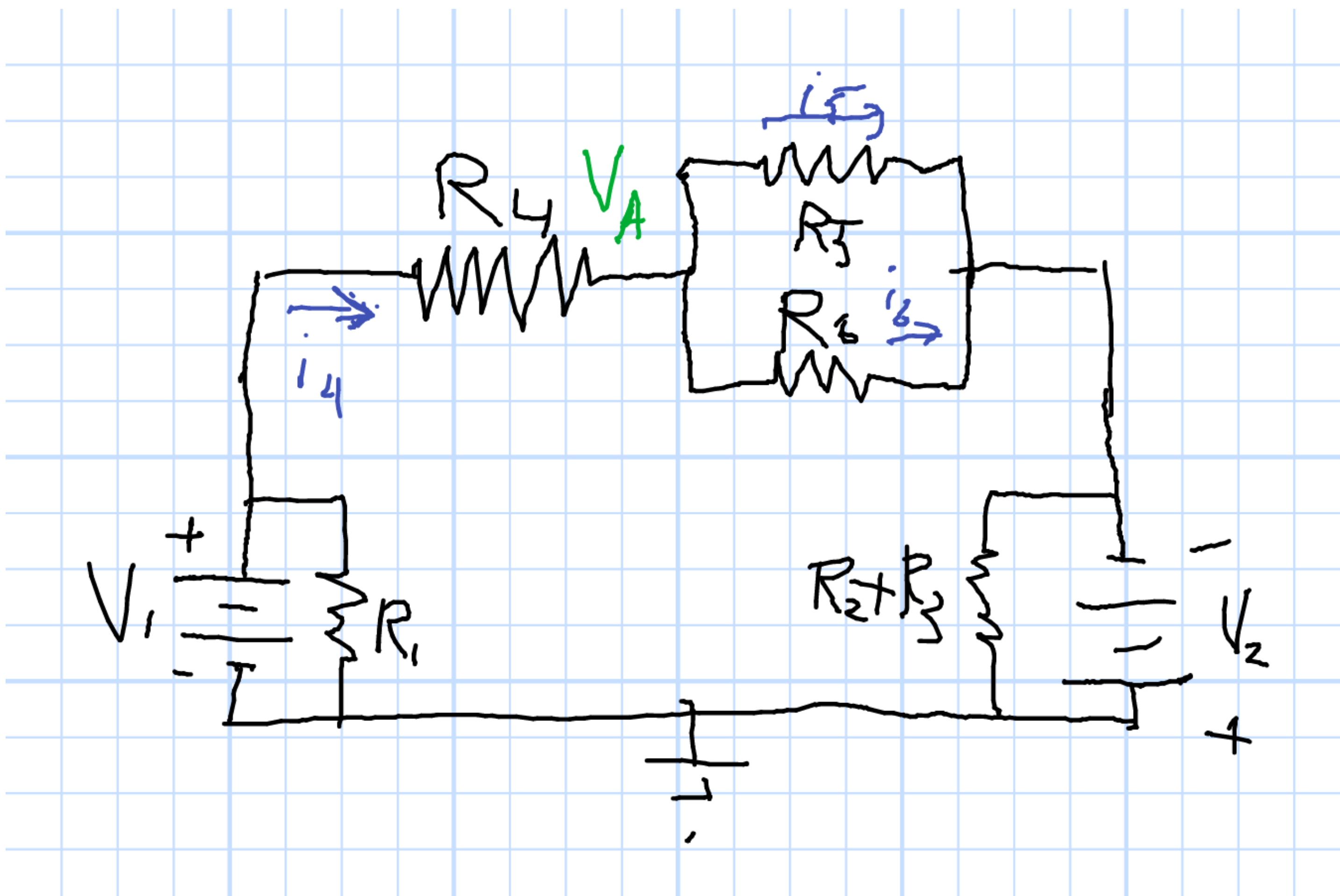
A Tricky One (3)

$i_4 = 24\text{V}/150\Omega = 160\text{mA}$



A Tricky One (4)

Voltage Divider: $v_A = -12\text{V} + 24\text{V} \times 50/150 = -4\text{V}$; i_n are easy.



Solve the Circuit

Solve With $v_A = 12\text{V}$ $v_B = 0$

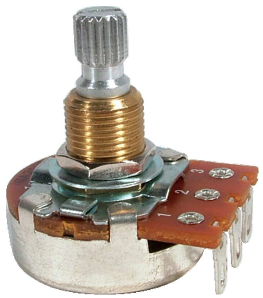
Remember $R_n = 50\Omega$ for All n $\rightarrow$ $R = 50\Omega$

Solution (Current Divider)

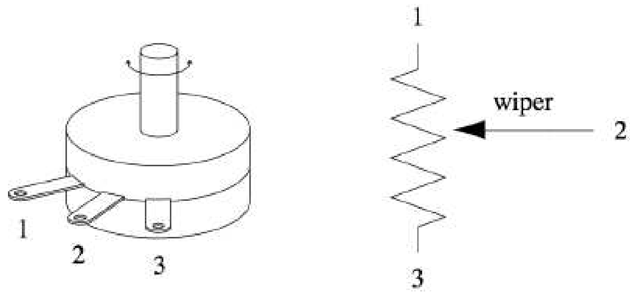
$$i = \frac{12\text{V}}{50\Omega} = 240\text{mA} \qquad i_1 = 160\text{mA} \qquad i_2 = 80\text{mA}$$

$$i_{4,5,6} = 80\text{mA} \qquad v_{crossbar} = 80\text{mA} \times 50\Omega = 4\text{V}$$

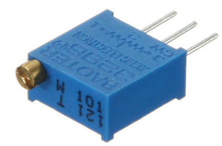
Volume Control



1

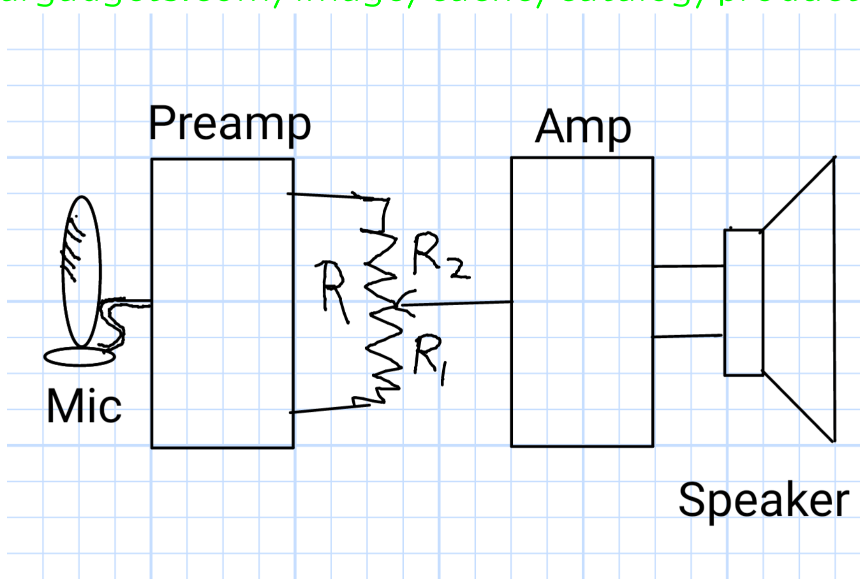


2



3

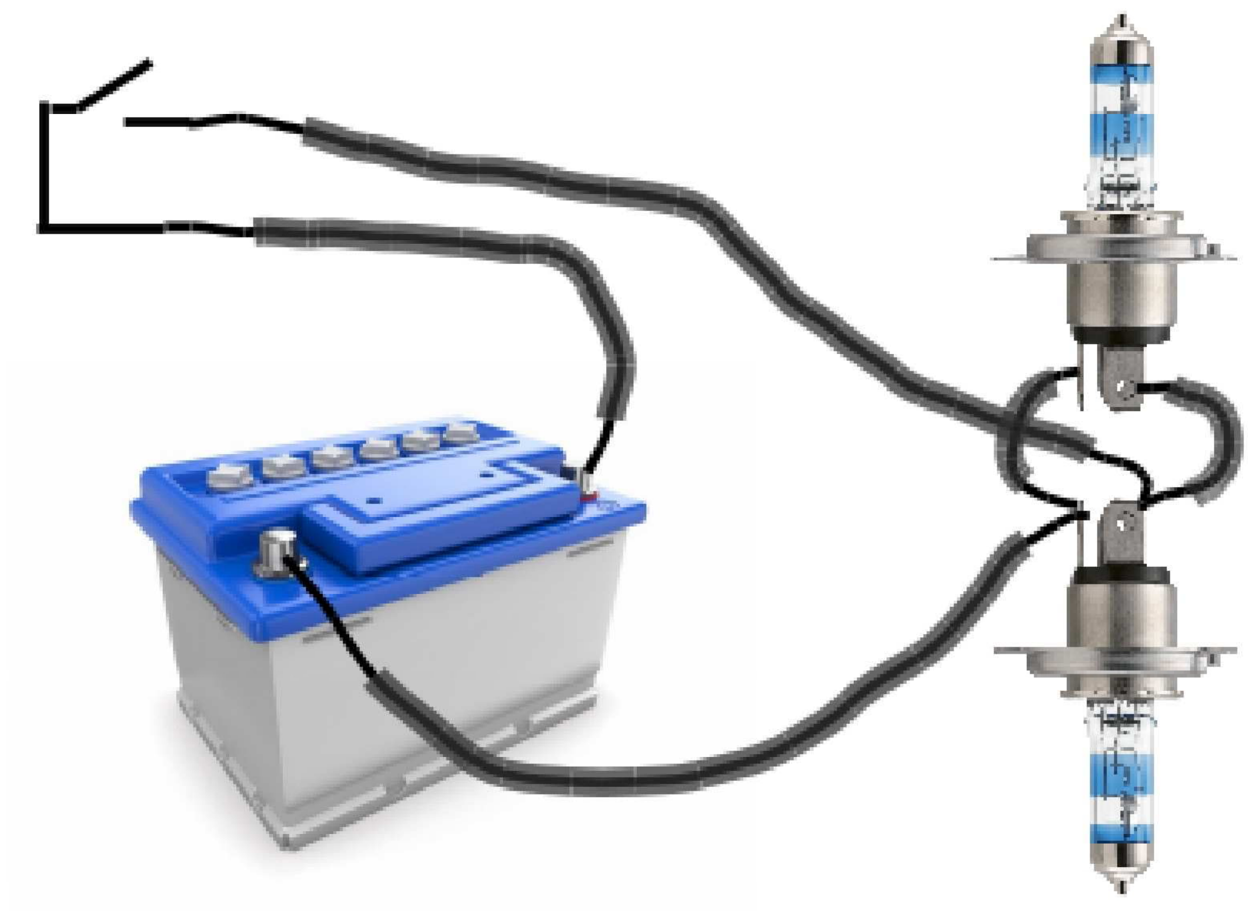
1:
https://www.tubesandmore.com/sites/default/files/uc_products/
2: http://hades.mech.northwestern.edu/images/3/3e/Sensor-potentiometer.png
3: https://www.bazaargadgets.com/image/cache/catalog/products/electronics/arduino/



Would this make a good light dimmer switch?

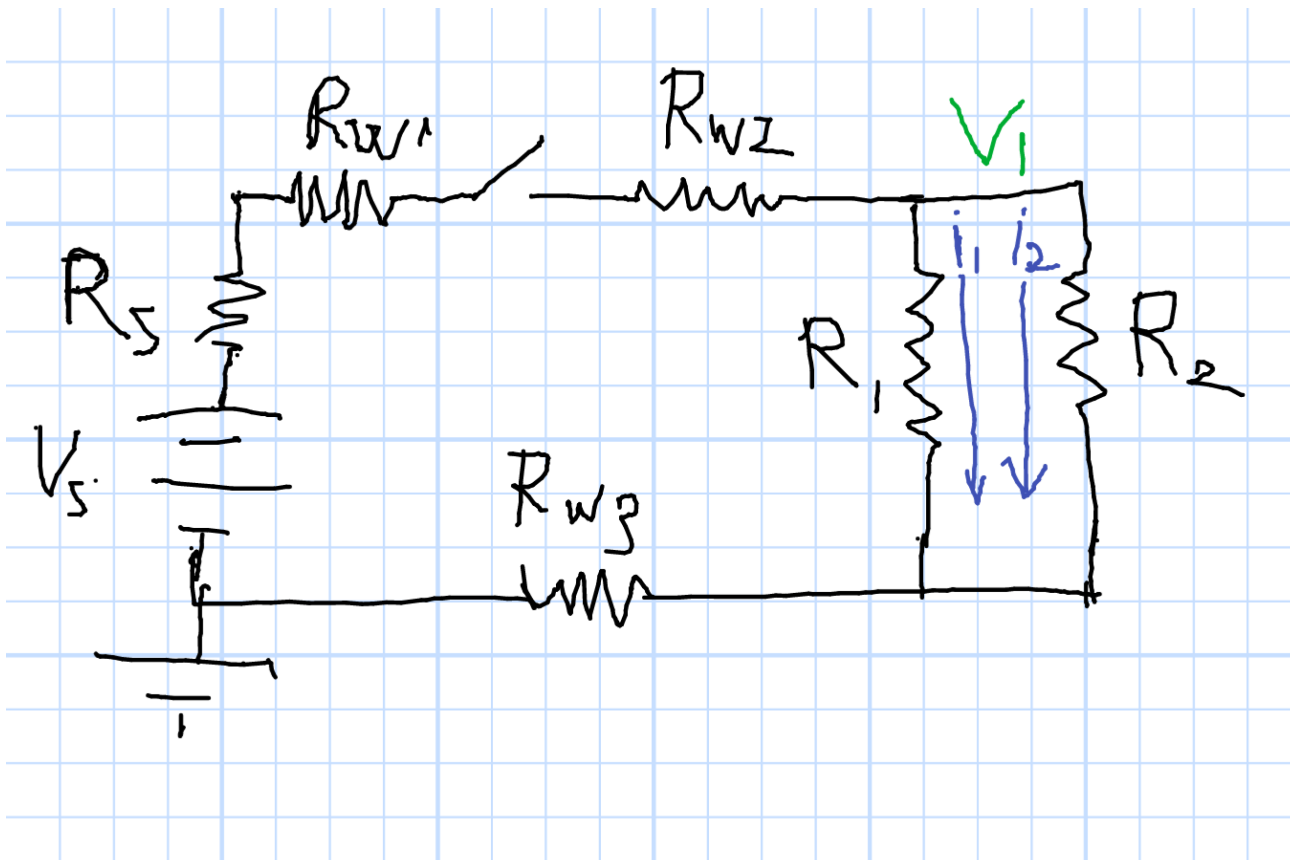
Current Divider

What Happens if One Burns Out

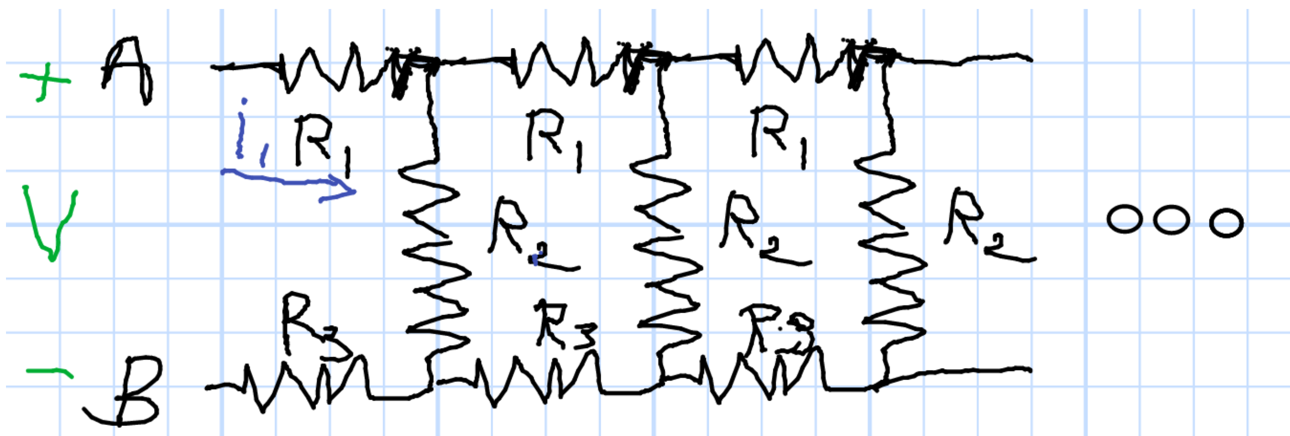


Current Divider

No Big Deal; $R_{1,2} >> \sum R_w + R_s$
Otherwise Remaining Light Brightens



Power in Ladder



Use $v = 13.7$V: Remember For All 50Ω Resistors, $R = 137\Omega$

$$i = \frac{v}{R} = 100\text{mA} \qquad p_1 = p_3 = (100\text{mA})^2 \times 50\Omega = 500\text{mW}$$

Current Divider

$$i_2 = 100\text{mA}\frac{137\Omega}{137\Omega + 50\Omega} \qquad p_2 = i_2^2 \times 50\Omega = 268\text{mW}$$

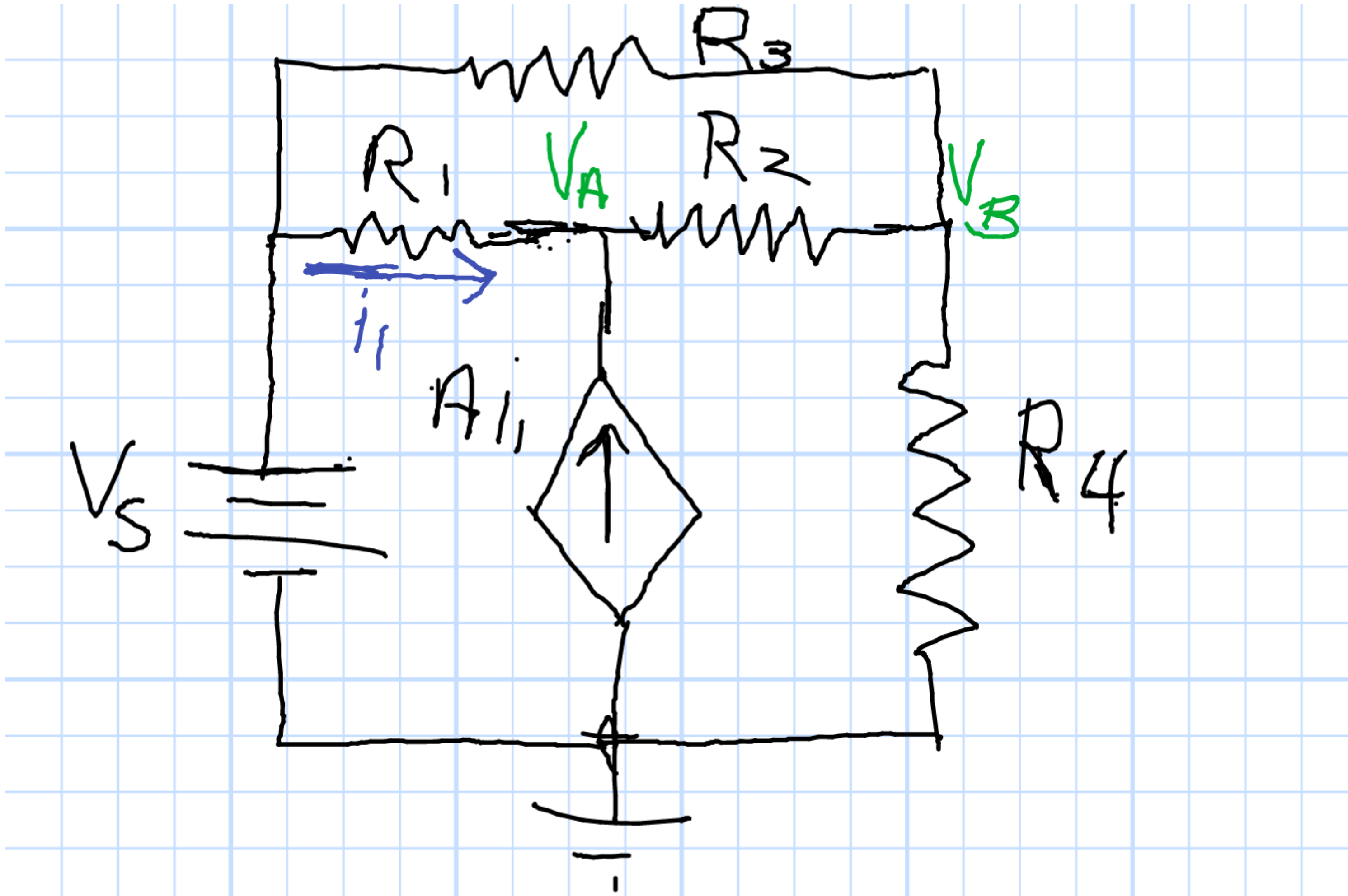
$$i_{next-stage} = 100\text{mA}\frac{50\Omega}{137\Omega + 50\Omega} = 27\text{mA}\ldots \textit{etc.}$$

Node Analysis

- We've Learned a Bag of Tricks
  - Simple Circuits
  - Series and Parallel
  - Dividers
- What if None of them Works? Is there Something that Always Works?
  - Node Analysis (KCL and Ohm's Law)
  - Mesh Analysis (KVL and Ohm's Law)

Solve This Circuit

$V_s = 12\text{V}, \ A = 3, \ R_1 = R_2 = 1k\Omega, \ R_3 = 5k\Omega, \ R_4 = 200\Omega$



Approach to Solution

Matrix Equation with KCL at Each Node

$$\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\mathcal{M}\mathbf{x} = \mathbf{y}$$

$$\text{Circuit Parameters} \times \text{Unknowns} = \text{Knowns}$$

Solution

$$\mathbf{x} = \mathcal{M}^{-1}\mathbf{y}$$

Do you remember how to find the inverse of a matrix?

Approach to Solution

Matrix Equation with KCL at Each Node

$$\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\mathcal{M}\mathbf{x} = \mathbf{y}$$

Circuit Parameters $\times$ Unknowns $=$ Knowns

Solution

$$\mathbf{x} = \mathcal{M}^{-1}\mathbf{y}$$

Do you remember how to find the inverse of a matrix?

Use Matlab: `x = inv(M)*y`

KCL at Node A

Inbound Currents at A:

$$\frac{v_s - v_A}{R_1} + \frac{v_B - v_A}{R_2} + Ai_1 = 0$$

$$\frac{v_s - v_A}{R_1} + \frac{v_B - v_A}{R_2} + A\frac{v_s - v_A}{R_1} = 0$$

$$\frac{v_s}{R_1} - \frac{v_A}{R_1} + \frac{v_B}{R_2} - \frac{v_A}{R_2} + A\frac{v_s}{R_1} - A\frac{v_A}{R_1} = 0$$

Constants on the Right

$$-\frac{v_A}{R_1} - \frac{v_A}{R_2} - A\frac{v_A}{R_1} + \frac{v_B}{R_2} = -\frac{v_s}{R_1} - A\frac{v_s}{R_1}$$

$$\left[-\frac{1+A}{R_1} - \frac{1}{R_2}\right] v_A + \frac{1}{R_2} v_B = -\frac{1+A}{R_1} v_s$$

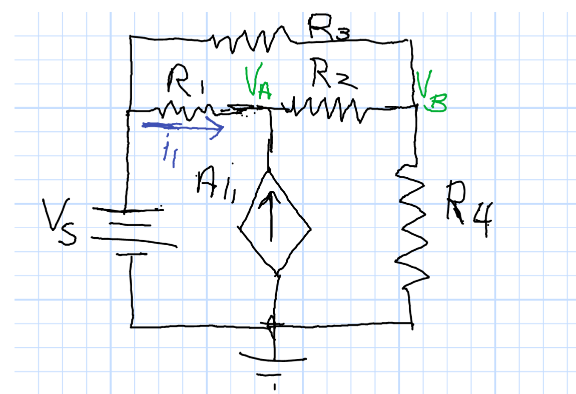
$V_s = 12\text{V}$

$A = 3$

$R_1 = R_2 = 1k\Omega$

$R_3 = 5k\Omega$

$R_4 = 200\Omega$



KCL at Node B

Inbound Currents at B:

$$\frac{v_s - v_B}{R_3} + \frac{v_A - v_B}{R_2} + \frac{0 - v_B}{R_4} = 0$$

$$\frac{v_s}{R_3} - \frac{v_B}{R_3} + \frac{v_A}{R_2} - \frac{v_B}{R_2} - \frac{v_B}{R_4} = 0$$

Constants on the Right

$$-\frac{v_B}{R_3} + \frac{v_A}{R_2} - \frac{v_B}{R_2} - \frac{v_B}{R_4} = -\frac{v_s}{R_3}$$

$$\frac{1}{R_2}v_A - \left[\frac{1}{R_3} - \frac{1}{R_2} - \frac{1}{R_4}\right] v_B = -\frac{v_s}{R_3}$$

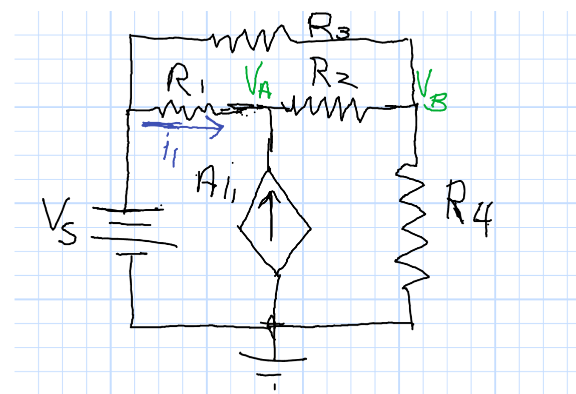
$V_s = 12\text{V}$

$A = 3$

$R_1 = R_2 = 1k\Omega$

$R_3 = 5k\Omega$

$R_4 = 200\Omega$



Solve

Inbound Currents at A:

$$\left[-\frac{1+A}{R_1}-\frac{1}{R_2}\right]v_A+\frac{1}{R_2}v_B=-\frac{1+A}{R_1}v_s$$

Inbound Currents at B:

$$\frac{1}{R_2}v_A-\left[\frac{1}{R_3}+\frac{1}{R_2}+\frac{1}{R_4}\right]v_B=-\frac{v_s}{R_3}$$

Matrix Equation

$$\begin{pmatrix}\left[-\frac{1+A}{R_1}-\frac{1}{R_2}\right] & \frac{1}{R_2}\\ \frac{1}{R_2} & -\left[\frac{1}{R_3}+\frac{1}{R_2}+\frac{1}{R_4}\right]\end{pmatrix}\times\cdots$$

$$\cdots\begin{pmatrix}v_A\\ v_B\end{pmatrix}=\begin{pmatrix}-\frac{1+A}{R1}v_s\\ -\frac{v_s}{R_3}\end{pmatrix}$$

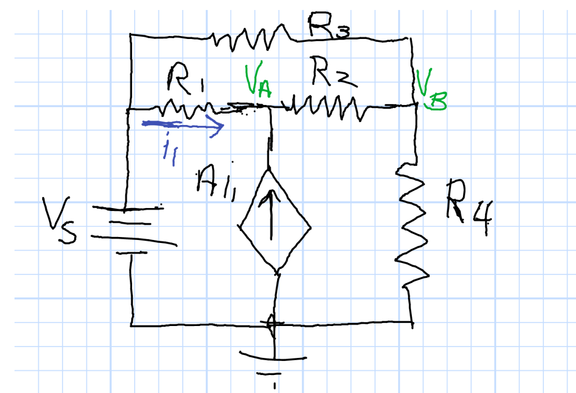
$V_s = 12$V

$A = 3$

$R_1 = R_2 = 1k\Omega$

$R_3 = 5k\Omega$

$R_4 = 200\Omega$



Result

$$\begin{pmatrix} \left[-\frac{1+A}{R_1} - \frac{1}{R_2}\right] & \frac{1}{R_2} \\ \frac{1}{R_2} & -\left[\frac{1}{R_3} + \frac{1}{R_2} + \frac{1}{R_4}\right] \end{pmatrix} \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} -\frac{1+A}{R1} v_s \\ -\frac{v_s}{R_3} \end{pmatrix}$$

From Matlab

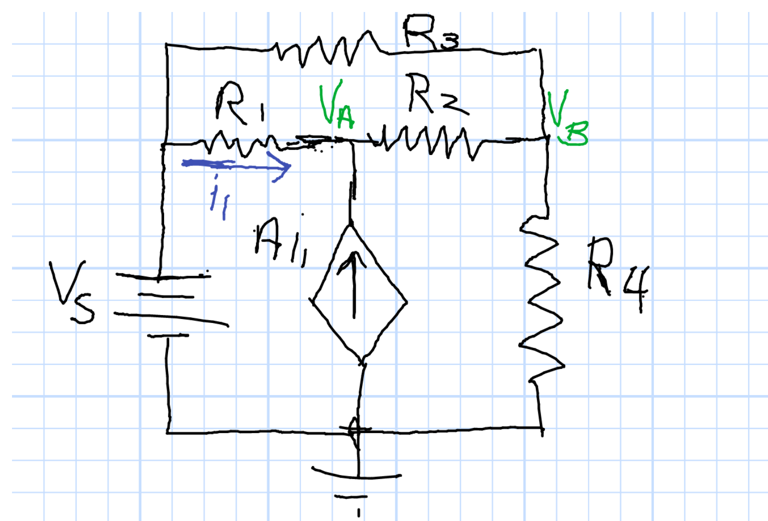
$V_s = 12\text{V} \qquad A = 3$

$R_1 = R_2 = 1k\Omega$

$R_3 = 5k\Omega \qquad R_4 = 200\Omega$

$$\begin{pmatrix} -0.0050 & 0.0010 \\ 0.0010 & -0.0062 \end{pmatrix} \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \dots$$

$$\dots \begin{pmatrix} -0.0480 \\ -0.0024 \end{pmatrix}$$



$$\mathbf{y} = \mathcal{M}\mathbf{x} \qquad \mathbf{x} = \mathcal{M}^{-1}\mathbf{y}$$

$$\mathbf{x} = \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 10 \\ 2 \end{pmatrix} \text{Volts}$$

Check Units