Electrical Engineering
Week 12

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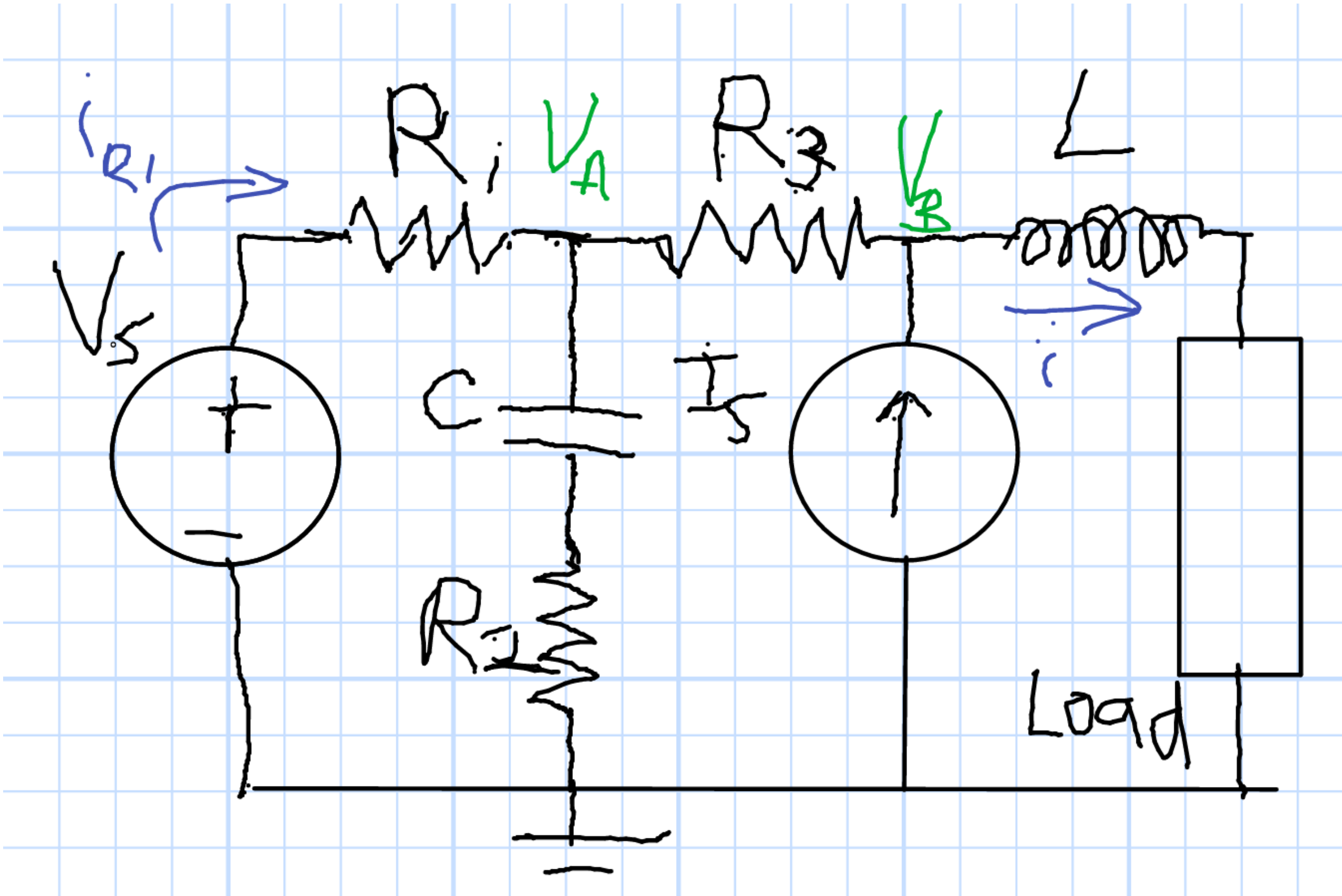
EECE–2210
Northeastern University

Nov 2021

Week 12 Agenda: Equivalent Circuits Again

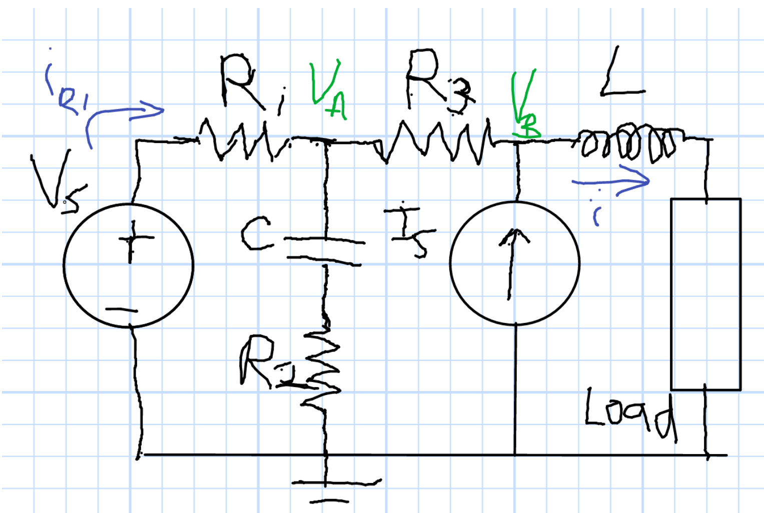
- Thévenin Equivalent
- Norton Equivalent
- Power Transfer

Some AC Circuit



$R_1 = 1\text{k}\Omega$, $R_2 = 3\text{k}\Omega$, $R_3 = 2\text{k}\Omega$, $C = 10\text{nF}$, $L = 80\text{mH}$,
$f = 10\text{kHz}$ $V_S = 5\text{V}$, $I_S = 10\text{mA}$

Open–Circuit Voltage (1)



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- Open Circuit

$$Z_{load} \rightarrow \infty$$

- No Current in L
- Inbound Current at One Node

$$\frac{V_S - V_A}{R_1} + I_S - \frac{V_A}{R_2 + \frac{1}{j\omega C}} = 0$$

$$V_A\left(\frac{1}{R_2 + \frac{1}{j\omega C}} + \frac{1}{R_1}\right) = \frac{V_S}{R_1} + I_S$$

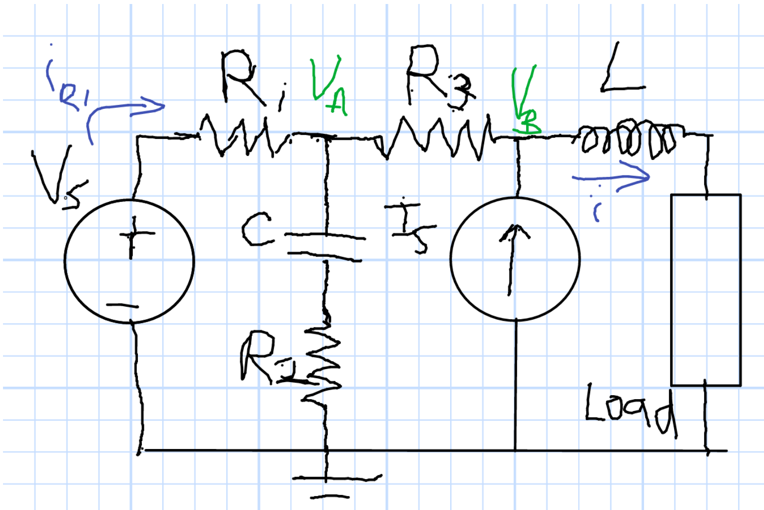
- Result

$$V_A = 11.7625 + 1.2881j$$

- Check Inbound Current

$$\approx -2 \times 10^{-18} + 2 \times 10^{-18}j$$

Open–Circuit Voltage (2)



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- Open Circuit

$$Z_{load} \rightarrow \infty$$

- Previous Page

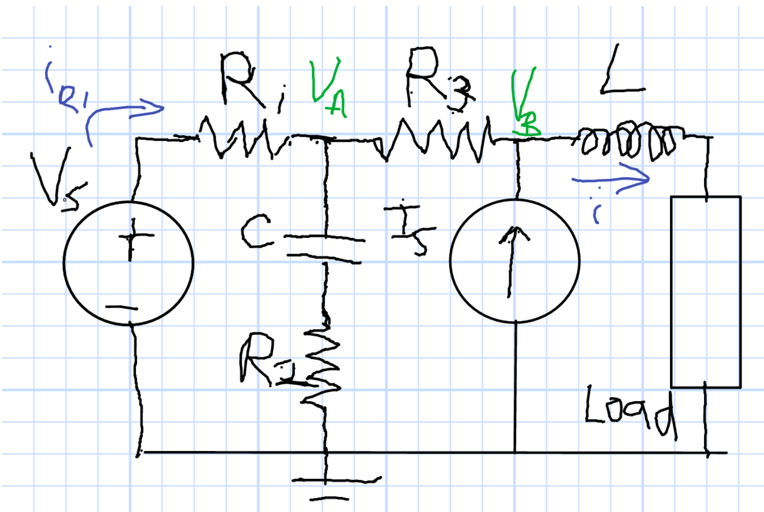
$$V_A = 11.7625 + 1.2881j$$

$$11.83\text{V}\angle{-6.25^\circ}$$

$$V_{OC} = V_A + I_s R_3 = 31.8 - 1.29j$$

$$= 31.8\text{V}\angle{-2.3^\circ}$$

Short–Circuit Current



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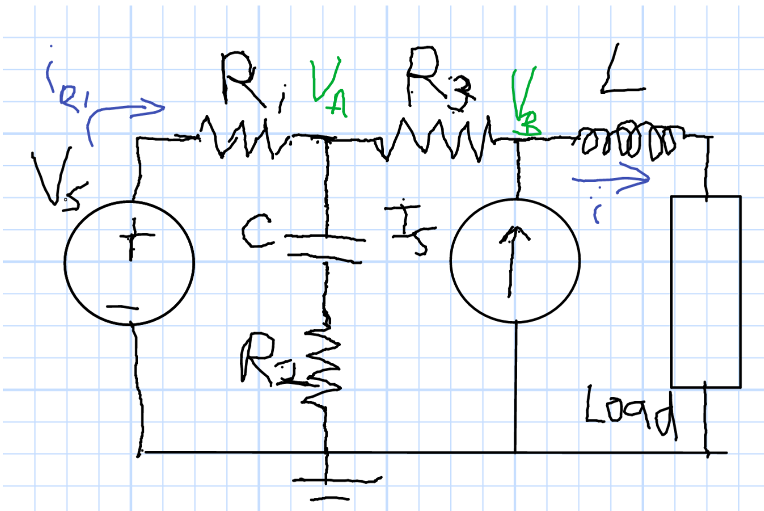
- Short Circuit

$$Z_{load} = 0$$

- Two Nodes (Oh No!)
- Try Superposition

Short the Voltage Source



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$V_S = 5\text{V}$, $I_S = 10\text{mA}$

- Short Circuit

$$V_S = 0$$

- Current Source Sees Z_1

$$Z_1 = L \parallel \{R_3 + [R_1 \parallel (R_2 + Z_C)]\}$$

- Voltage

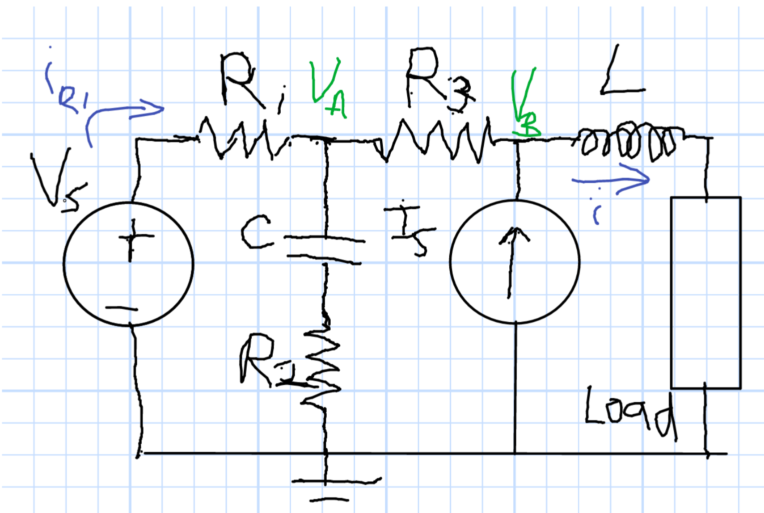
$$V_B = I_S Z_1$$

- Current Contribution

$$I_{1sc} = \frac{V_B}{Z_L}$$

$$I_{1sc} = 2.3\text{mA} - 4.4j\text{mA}$$

Open the Current Source



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- Open Circuit

$$I_S = 0$$

- Voltage Source Sees Z_2

$$Z_2 = R_1 + (Z_C + R_2) \parallel (R_3 + Z_L)$$

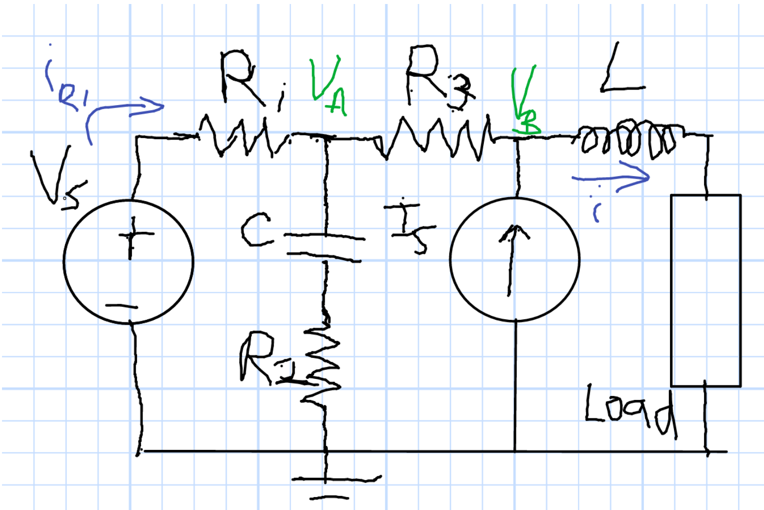
$$I_{R1} = \frac{V_S}{Z_2}$$

- Current Contribution (Divider)

$$I_{2sc} = I_{R1} \frac{Z_C + R_2}{(Z_C + R_2)(R_3 + Z_L)}$$

$$I_{2sc} = 270\mu\text{A} + 640j\mu\text{A}$$

Superposition Result for I_{sc}



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- Current Source Alone

$$I_{1sc} = 2.3\text{mA} - 4.4j\text{mA}$$

- Voltage Source Alone

$$I_{2sc} = 270\mu\text{A} + 640j\mu\text{A}$$

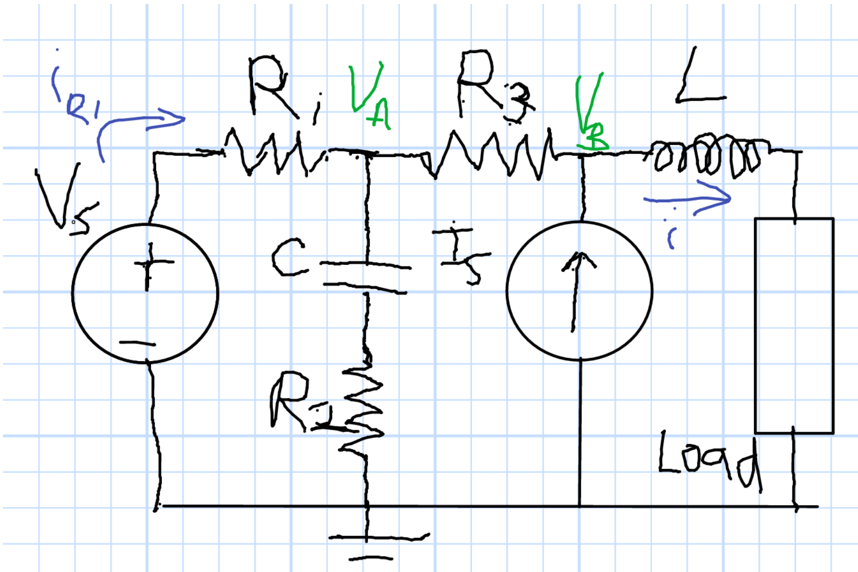
- Superposition

$$I_{sc} = I_{1sc} + I_{2sc}$$

$$I_{sc} = 2.6\text{mA} - 5.0j\text{mA}$$

$$I_{sc} = 5.6\text{mA}\angle{-63^\circ}$$

Impedance



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- $V_S = 0$ (Short)
- $I_S = 0$ (Open)
- Load Sees $Z_T = Z_N$

$$Z_T = Z_L + R_3 + [R_1 \parallel (Z_C + R_2)]$$

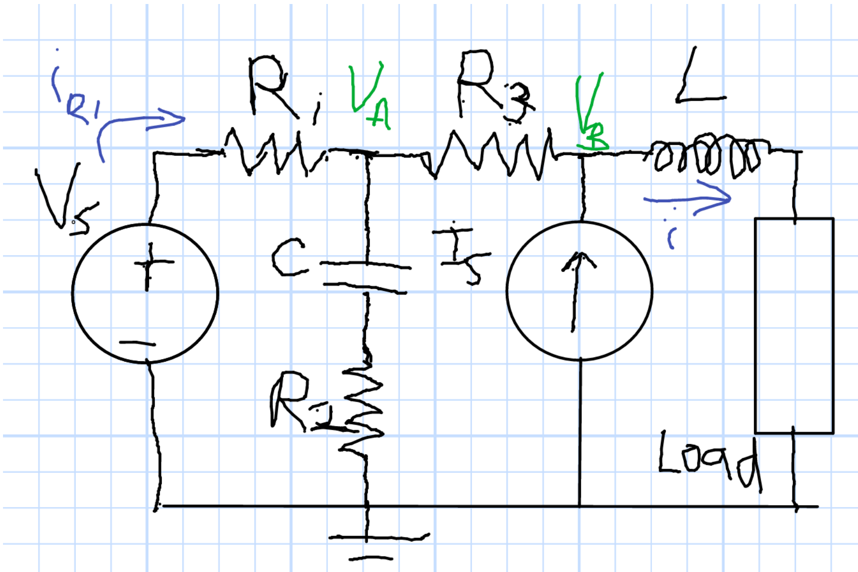
$$Z_T = 2.8\text{k}\Omega + 4.9j\text{k}\Omega$$

$$= 5.7\text{k}\Omega\angle -61^\circ$$

- Trust but Verify

```
>> check=ISC*Z_T-VOC
check =
  -7.1054e-15 + 1.3323e-15i
```

Thévenin Equivalent



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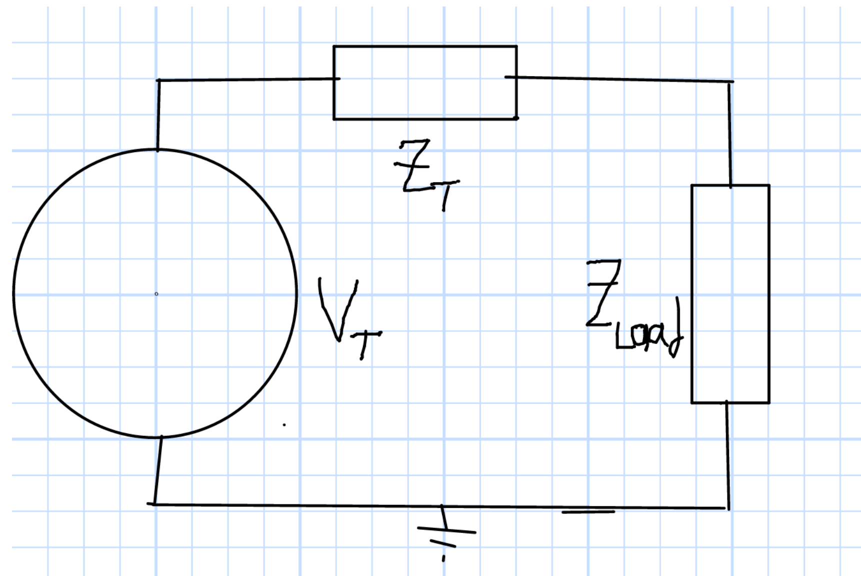
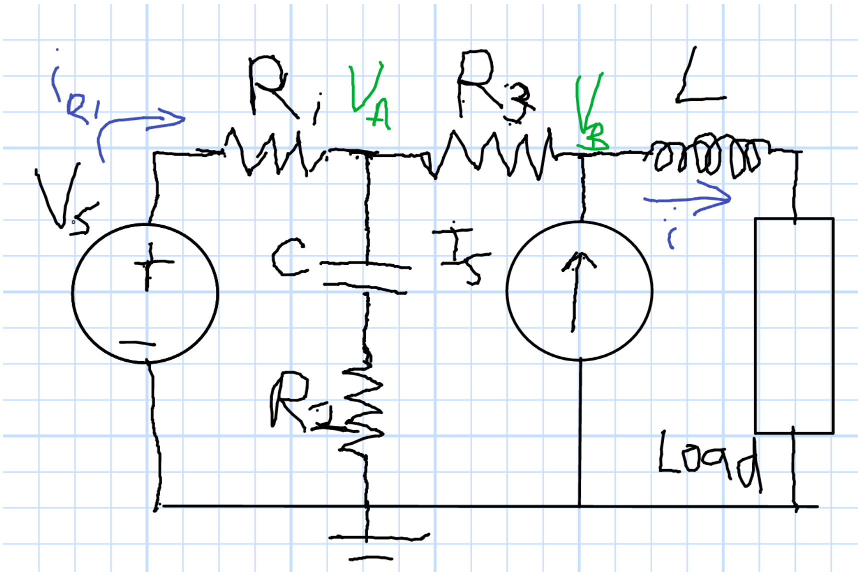


$V_T = V_{oc} = 31.8\text{V}\angle{-2.3^\circ}$

$Z_T = 5.7\text{k}\Omega\angle{-61^\circ}$

Norton Equivalent



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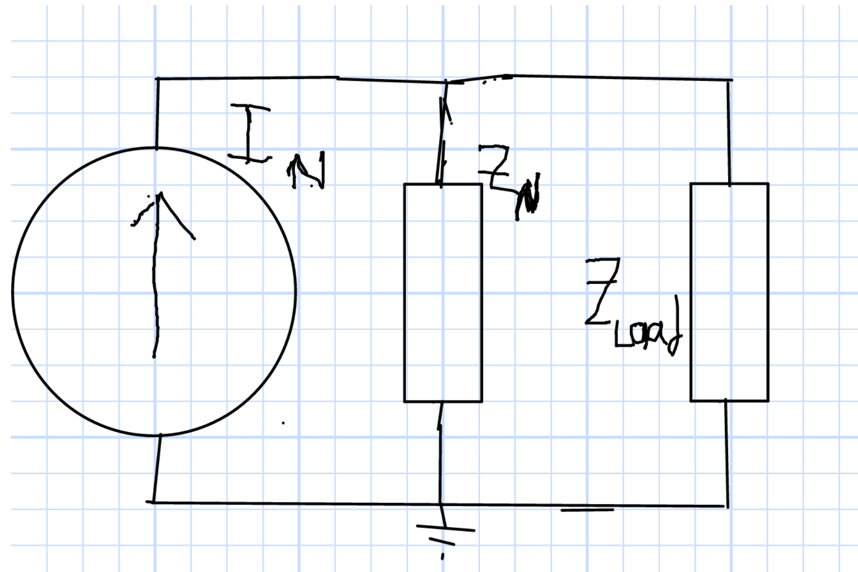

$I_N = I_{sc} = 5.6\text{mA}\angle{-63^\circ}$

$Z_N = Z_T = 5.7\text{k}\Omega\angle{-61^\circ}$

Happy Thanksgiving