Circuits and Signals: Biomedical Applications
Week 3

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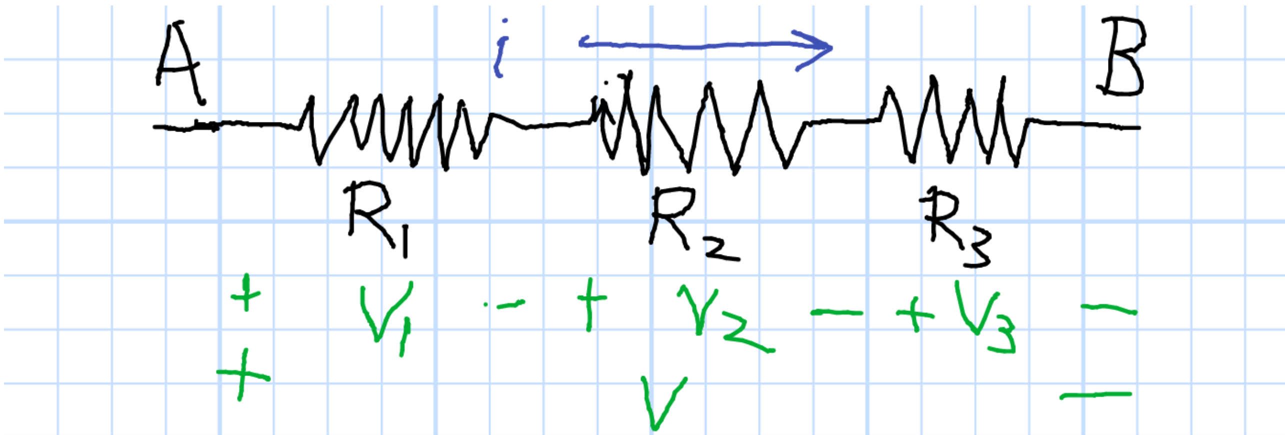
Sep 2023

Week 3 Agenda

- Voltage Dividers
- Current Dividers
- Node Analysis
- Mesh Analysis
- Using the Oscilloscope

Voltage Divider

New Concept: Voltage Dividers

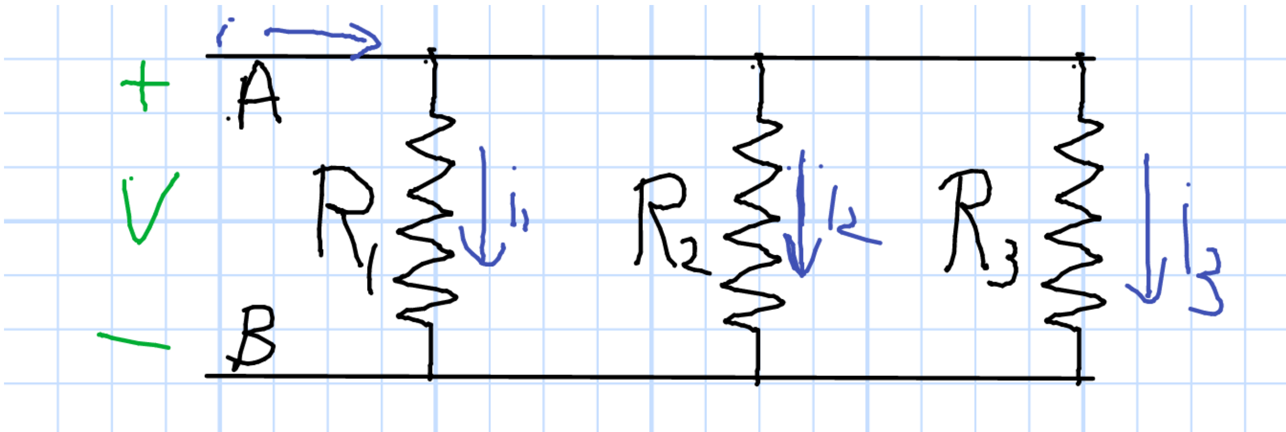


$$v_1 = iR_1 \qquad v = iR = i\left(R_1 + R_2 + R_3\right)$$

$$v_1 = v\frac{R_1}{R_1 + R_2 + R_3}$$

- Using a Voltmeter
- Understanding Battery Failure

Current Divider

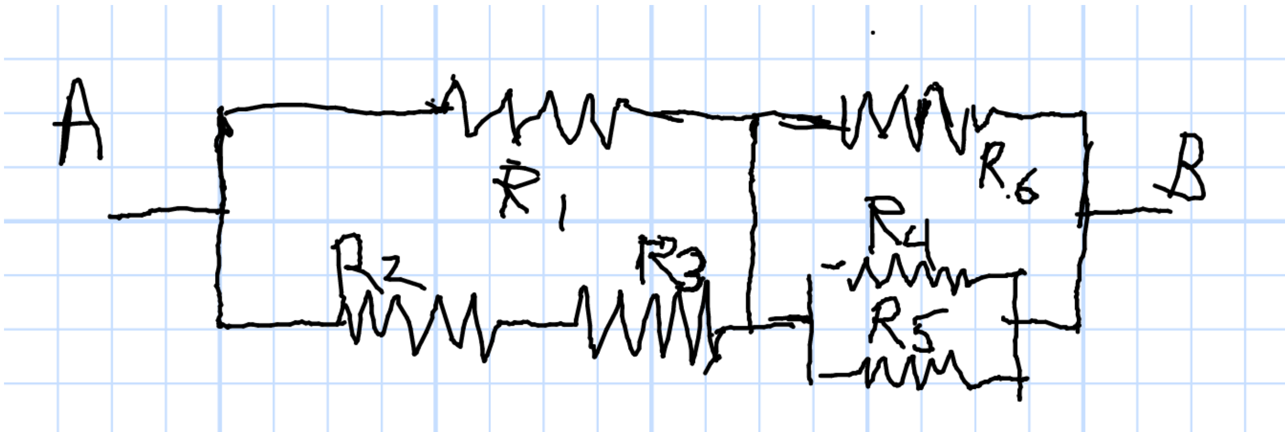


$$v = i_1 R_1 \qquad v = iR = i\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

$$i_1 = i\frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

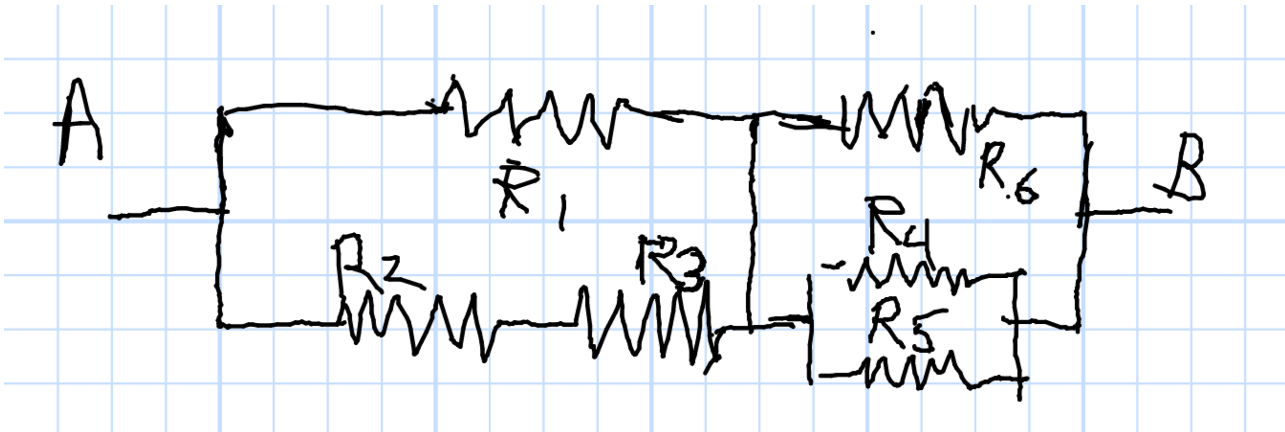
$$i_1 = \frac{G_1}{G_1+G_2+G_3}$$

Solve the Circuit



Solve With $v_A = 12\text{V}$ $v_B = 0$ All $R_n = 50\Omega$

Solve the Circuit



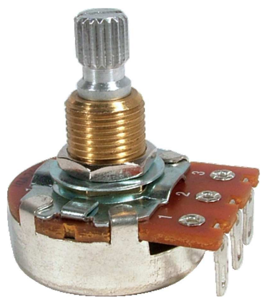
Solve With $v_A = 12\text{V}$ $v_B = 0$ All $R_n = 50\Omega$

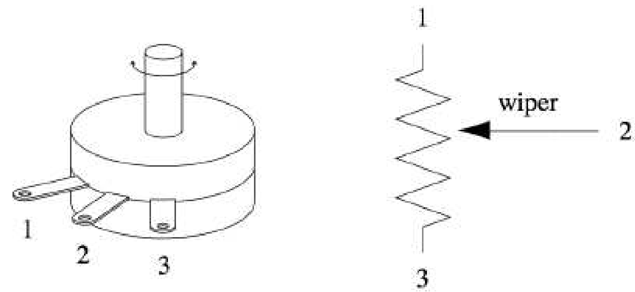
$$R_1 \parallel (R_2 + R_3) = 33.3\Omega \qquad R_6 \parallel R_4 \parallel R_5 = 16.7 \qquad \rightarrow \qquad R = 50\Omega$$

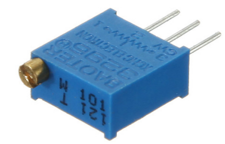
$$i = \frac{12\text{V}}{50\Omega} = 240\text{mA} \qquad i_1 = 160\text{mA} \qquad i_2 = 80\text{mA}$$

$$i_{4,5,6} = 80\text{mA} \qquad v_{crossbar} = 80\text{mA} \times 50\Omega = 4\text{V}$$

Volume Control



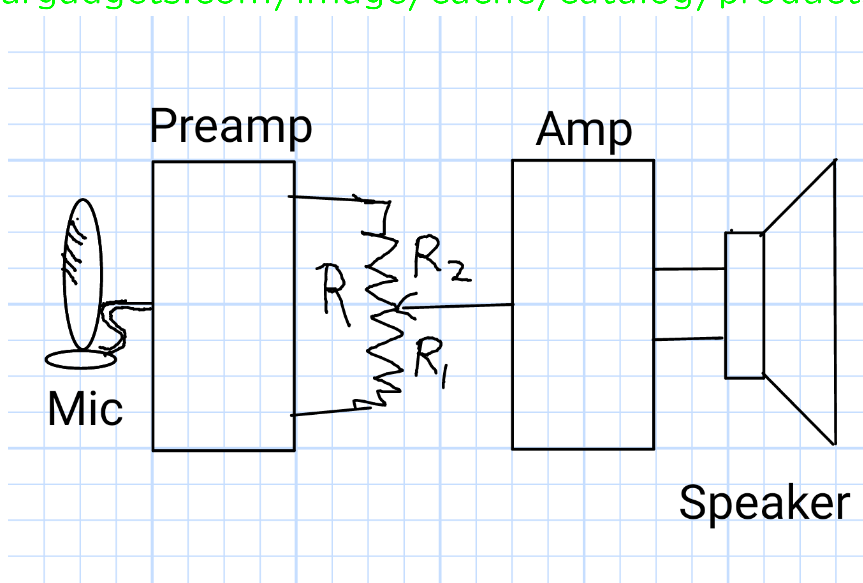




1:

1 2 3

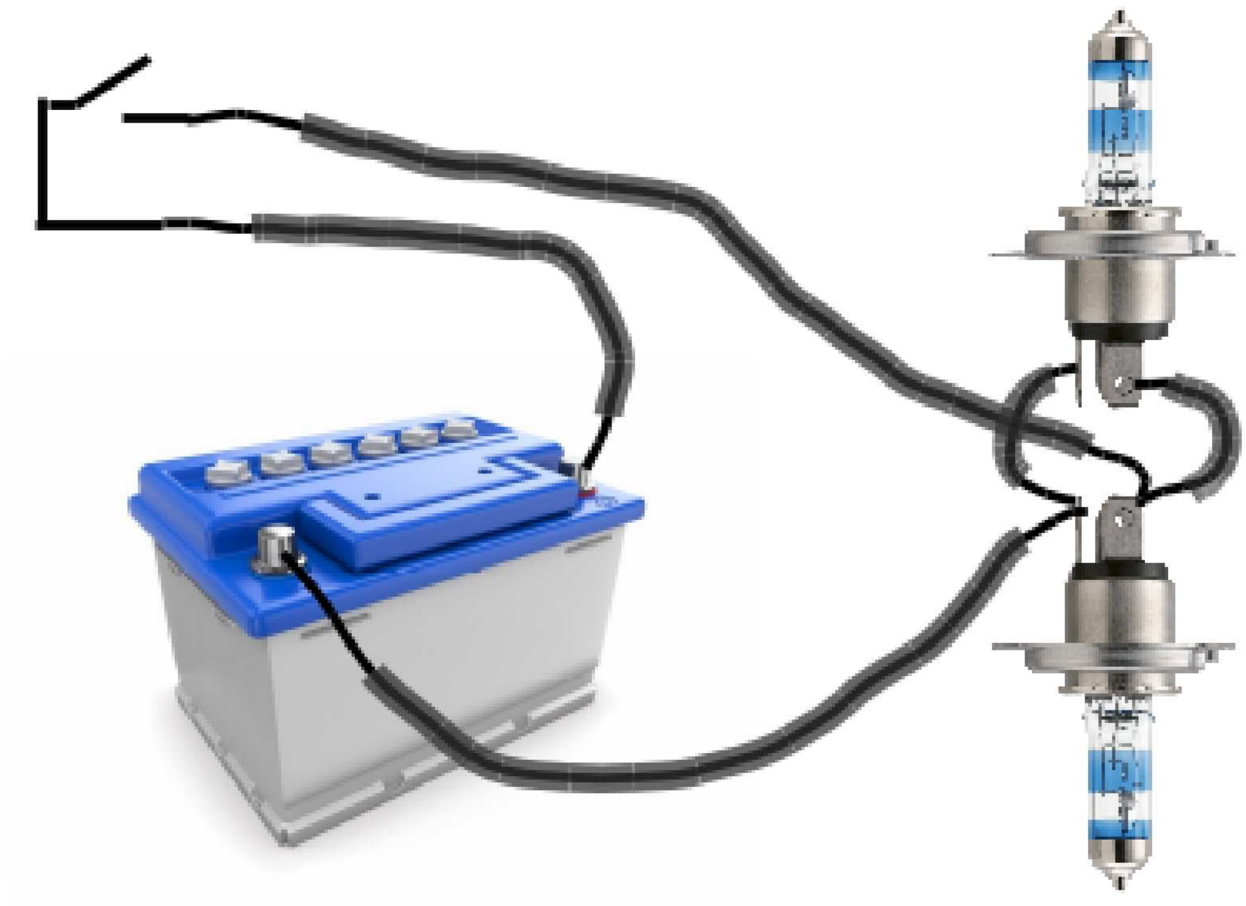
https://www.tubesandmore.com/sites/default/files/uc_products/
2: http://hades.mech.northwestern.edu/images/3/3e/Sensor-potentiometer.png
3: https://www.bazaargadgets.com/image/cache/catalog/products/electronics/arduino/



Would this make a good light dimmer switch?

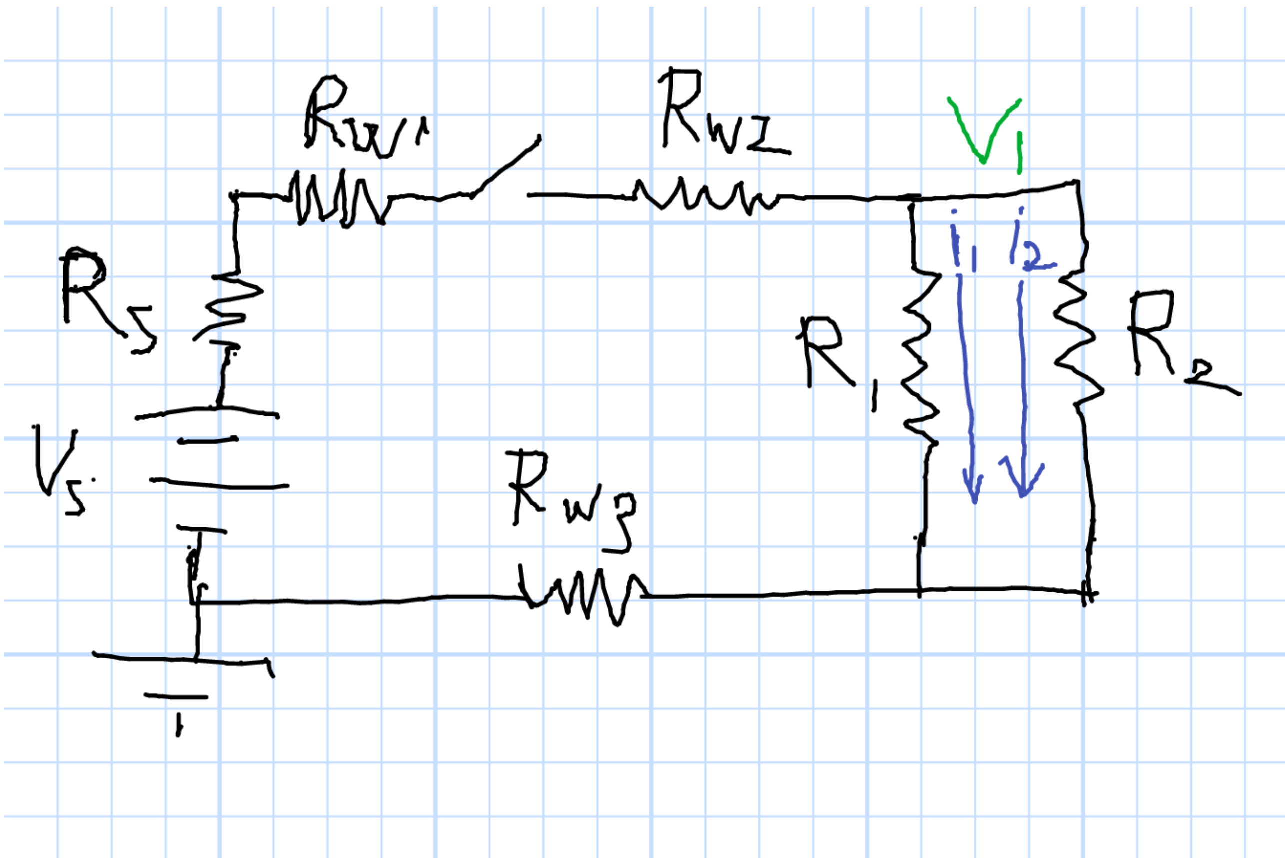
Current Divider

What Happens if One Burns Out



Current Divider

No Big Deal; $R_{1,2} >> \sum R_w + R_s$
Otherwise Remaining Light Brightens

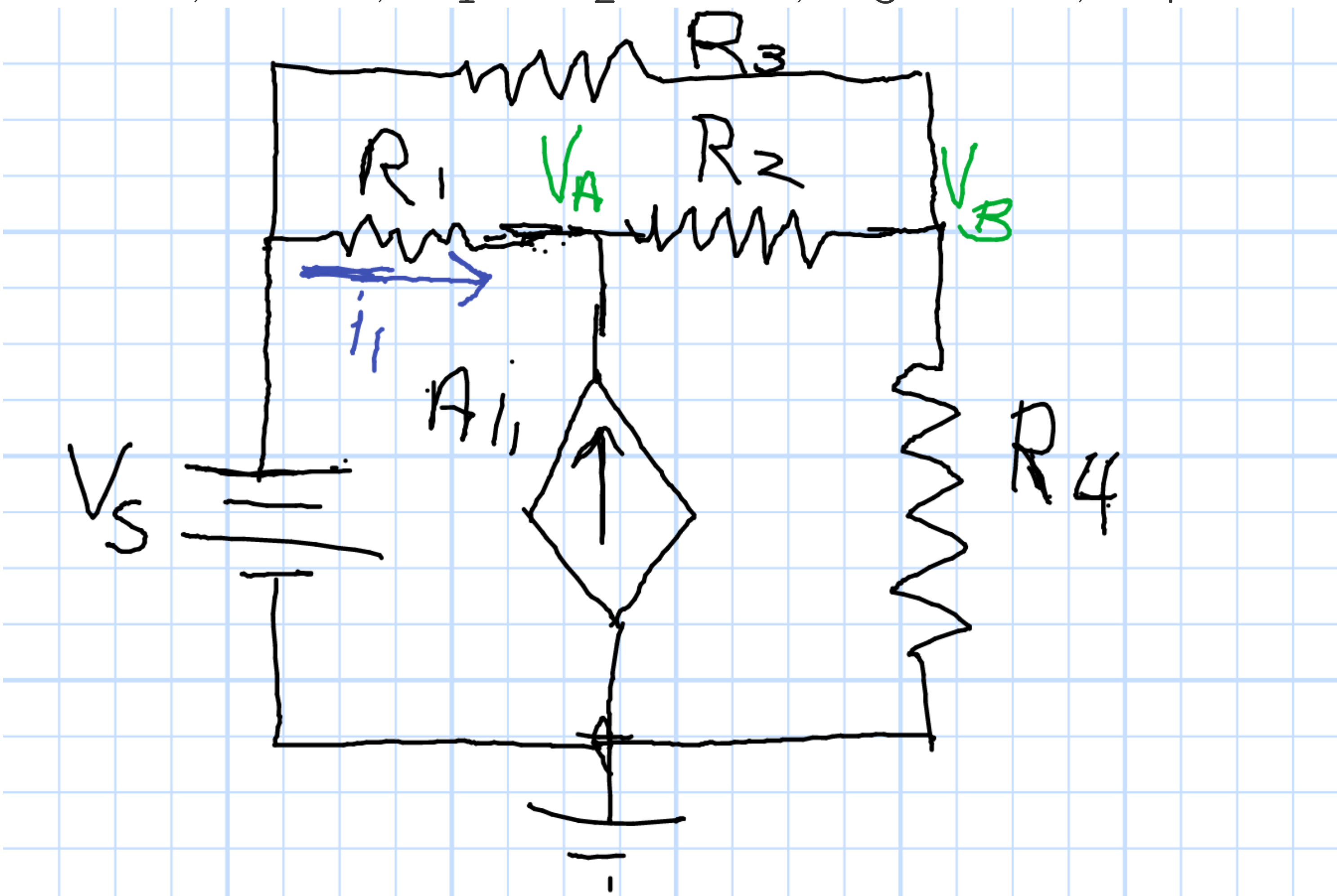


Node Analysis

- We've Learned a Bag of Tricks
  - Simple Circuits
  - Series and Parallel
  - Dividers
- What if None of them Works? Is there Something that Always Works?
  - Node Analysis (KCL and Ohm's Law)
  - Mesh Analysis (KVL and Ohm's Law)

Solve This Circuit

$V_s = 12\text{V},\ A = 3,\ R_1 = R_2 = 1k\Omega,\ R_3 = 5k\Omega,\ R_4 = 200\Omega$



Approach to Solution

Matrix Equation with KCL at Each Node

$$\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\mathcal{M}\mathbf{x} = \mathbf{y}$$

$$\text{Circuit Parameters} \times \text{Unknowns} = \text{Knowns}$$

Solution

$$\mathbf{x} = \mathcal{M}^{-1}\mathbf{y}$$

Do you remember how to find the inverse of a matrix?

Approach to Solution

Matrix Equation with KCL at Each Node

$$\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\mathcal{M}\mathbf{x} = \mathbf{y}$$

Circuit Parameters $\times$ Unknowns $=$ Knowns

Solution

$$\mathbf{x} = \mathcal{M}^{-1}\mathbf{y}$$

Do you remember how to find the inverse of a matrix?

Use Matlab: `x = inv(M)*y`

KCL at Node A

Inbound Currents at A:

$$\frac{v_s - v_A}{R_1} + \frac{v_B - v_A}{R_2} + Ai_1 = 0$$

$$\frac{v_s - v_A}{R_1} + \frac{v_B - v_A}{R_2} + A\frac{v_s - v_A}{R_1} = 0$$

$$\frac{v_s}{R_1} - \frac{v_A}{R_1} + \frac{v_B}{R_2} - \frac{v_A}{R_2} + A\frac{v_s}{R_1} - A\frac{v_A}{R_1} = 0$$

Constants on the Right

$$-\frac{v_A}{R_1} - \frac{v_A}{R_2} - A\frac{v_A}{R_1} + \frac{v_B}{R_2} = -\frac{v_s}{R_1} - A\frac{v_s}{R_1}$$

$$\left[-\frac{1+A}{R_1} - \frac{1}{R_2}\right] v_A + \frac{1}{R_2} v_B = -\frac{1+A}{R_1} v_s$$

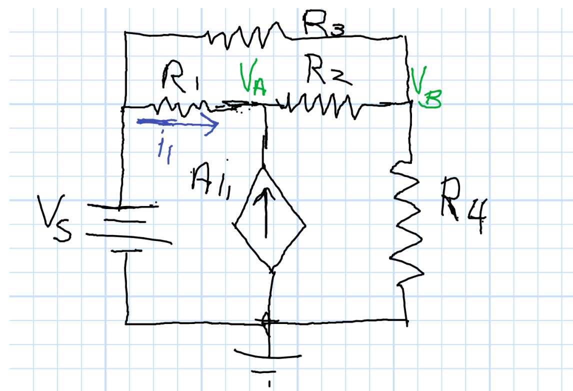
$V_s = 12\text{V}$

$A = 3$

$R_1 = R_2 = 1k\Omega$

$R_3 = 5k\Omega$

$R_4 = 200\Omega$



KCL at Node B

Inbound Currents at B:

$$\frac{v_s - v_B}{R_3} + \frac{v_A - v_B}{R_2} + \frac{0 - v_B}{R_4} = 0$$

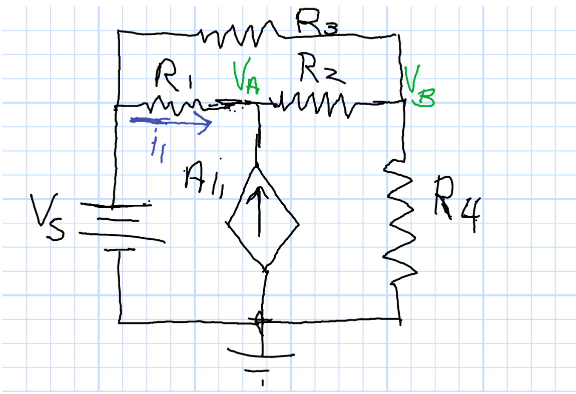
$$\frac{v_s}{R_3} - \frac{v_B}{R_3} + \frac{v_A}{R_2} - \frac{v_B}{R_2} - \frac{v_B}{R_4} = 0$$

Constants on the Right

$$-\frac{v_B}{R_3} + \frac{v_A}{R_2} - \frac{v_B}{R_2} - \frac{v_B}{R_4} = -\frac{v_s}{R_3}$$

$$\frac{1}{R_2}v_A - \left[\frac{1}{R_3} - \frac{1}{R_2} - \frac{1}{R_4}\right] v_B = -\frac{v_s}{R_3}$$

$V_s = 12\text{V}$
$A = 3$
$R_1 = R_2 = 1k\Omega$
$R_3 = 5k\Omega$
$R_4 = 200\Omega$



Solve

Inbound Currents at A:

$$\left[-\frac{1+A}{R_1}-\frac{1}{R_2}\right]v_A+\frac{1}{R_2}v_B=-\frac{1+A}{R_1}v_s$$

Inbound Currents at B:

$$\frac{1}{R_2}v_A-\left[\frac{1}{R_3}+\frac{1}{R_2}+\frac{1}{R_4}\right]v_B=-\frac{v_s}{R_3}$$

Matrix Equation

$$\begin{pmatrix} \left[-\frac{1+A}{R_1}-\frac{1}{R_2}\right] & \frac{1}{R_2} \\ \frac{1}{R_2} & -\left[\frac{1}{R_3}+\frac{1}{R_2}+\frac{1}{R_4}\right] \end{pmatrix} \times \cdots$$

$$\cdots \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} -\frac{1+A}{R1}v_s \\ -\frac{v_s}{R_3} \end{pmatrix}$$

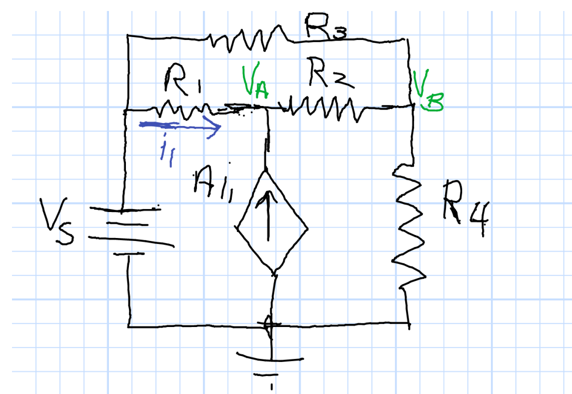
$V_s = 12\text{V}$

$A = 3$

$R_1 = R_2 = 1k\Omega$

$R_3 = 5k\Omega$

$R_4 = 200\Omega$



Result

$$\begin{pmatrix} \left[-\frac{1+A}{R_1} - \frac{1}{R_2}\right] & \frac{1}{R_2} \\ \frac{1}{R_2} & -\left[\frac{1}{R_3} + \frac{1}{R_2} + \frac{1}{R_4}\right] \end{pmatrix} \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} -\frac{1+A}{R_1} v_s \\ -\frac{v_s}{R_3} \end{pmatrix}$$

From Matlab

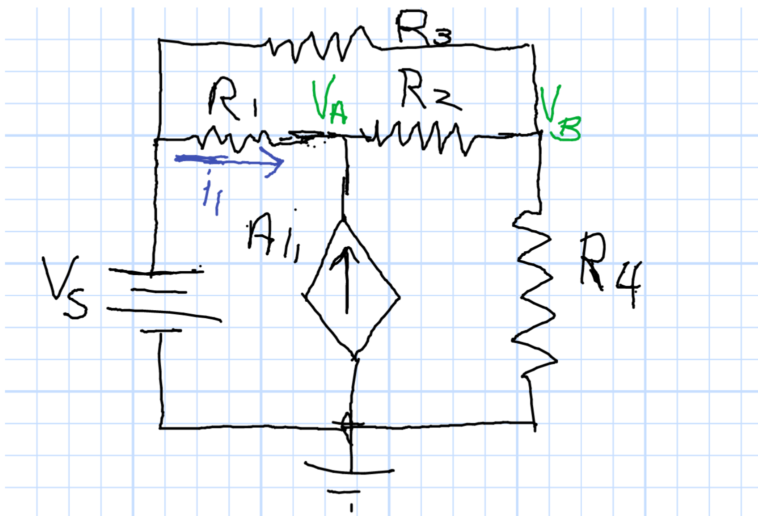
$V_s = 12\text{V} \qquad A = 3$

$R_1 = R_2 = 1k\Omega$

$R_3 = 5k\Omega \qquad R_4 = 200\Omega$

$$\begin{pmatrix} -0.0050 & 0.0010 \\ 0.0010 & -0.0062 \end{pmatrix} \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \ldots$$

$$\ldots \begin{pmatrix} -0.0480 \\ -0.0024 \end{pmatrix}$$



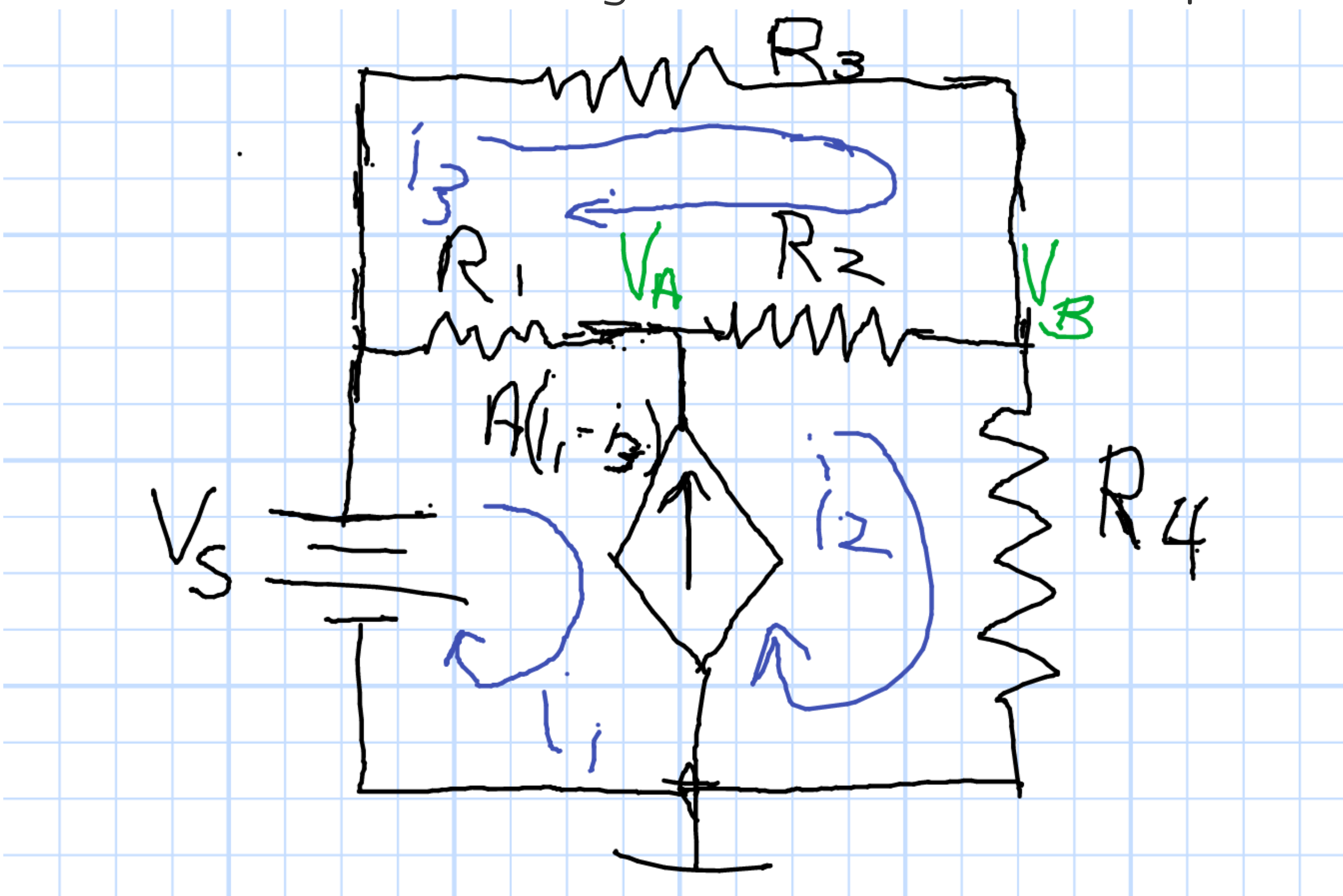
$$\mathbf{y} = \mathcal{M}\mathbf{x} \qquad \mathbf{x} = \mathcal{M}^{-1}\mathbf{y}$$

$$\mathbf{x} = \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 10 \\ 2 \end{pmatrix} \text{Volts}$$

Check Units

Mesh Analysis

Solve Circuits Using KVL Around All Loops

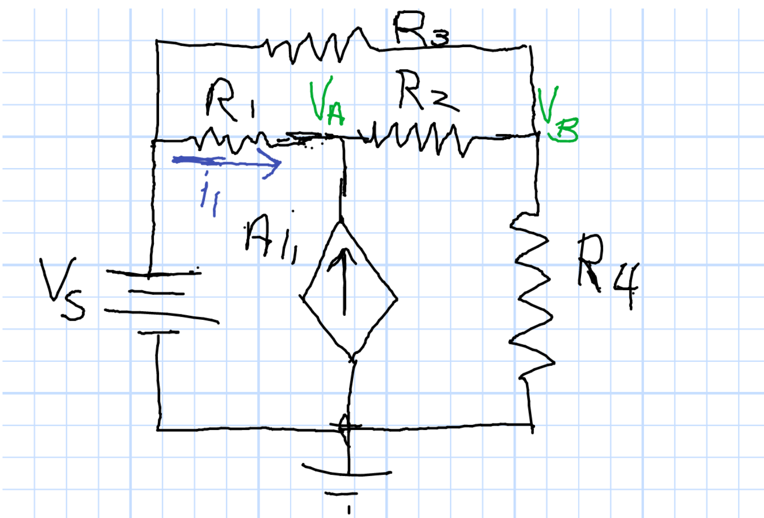


Remember Node Analysis

$V_s = 12\text{V} \qquad A = 3$

$R_1 = R_2 = 1k\Omega$

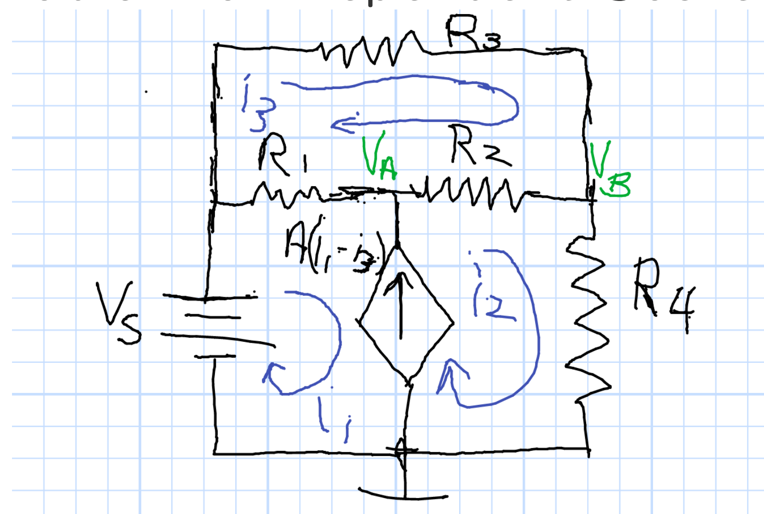
$R_3 = 5k\Omega \qquad R_4 = 200\Omega$



$$\mathbf{x} = \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 10 \\ 2 \end{pmatrix} \text{Volts}$$

Mesh Analysis

Problem of Dependent Source



Superloop 1,2

$$v_s - i_1R_1 + i_3R_1 - i_2R_2 + i_3R_2 - i_2R_4$$

$$= 0$$

Loop 3

$$i_3R_3 + (i_3 - i_2)\,R_2 + (i_3 - i_1)\,R_1 = 0$$

Example of Mesh Analysis

Dependent Source

$$A\left(i_1 - i_3\right) = i_2 - i_1$$

$$\left(A+1\right)i_1 - i_2 - Ai_3 = 0$$

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$$v_s - i_1R_1 + i_3R_1 - i_2R_2 + i_3R_2 - i_2R_4 = 0$$

$$i_3R_3 + \left(i_3 - i_2\right)R_2 + \left(i_3 - i_1\right)R_1 = 0$$

Reorder

$$\left(A+1\right)i_1 - i_2 - Ai_3 = 0$$

$$-R_1i_1 - \left(R_2 + R_4\right)i_2 + \left(R_1 + R_2\right)i_3 = -v_s$$

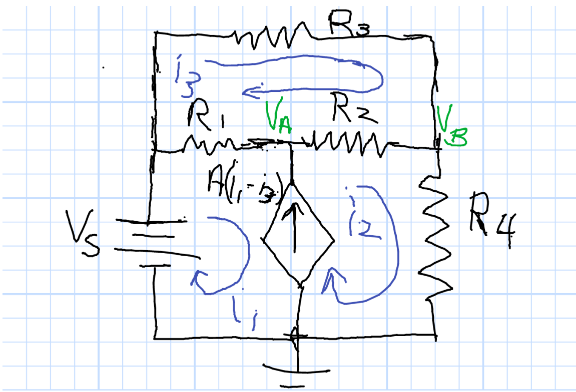
$$-R_1i_1 - R_2i_2 + \left(R_1 + R_2 + R_3\right)i_3 = 0$$

$$V_s = 12\text{V} \qquad A = 3$$

$$R_1 = R_2 = 1k\Omega$$

$$R_3 = 5k\Omega$$

$$R_4 = 200\Omega$$



Mesh Analysis Solution

Previous Page

$V_s = 12\text{V} \qquad A = 3$

$R_1 = R_2 = 1k\Omega$

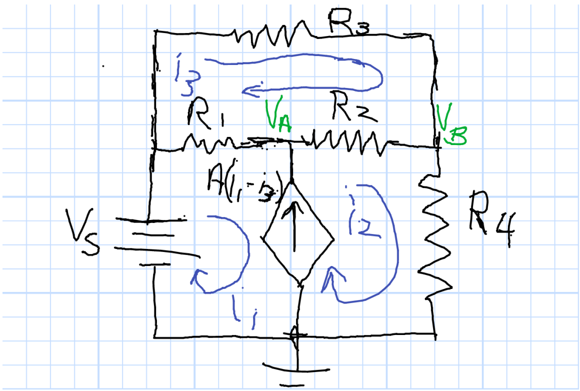
$R_3 = 5k\Omega$

$R_4 = 200\Omega$

$$(A+1)\,i_1 - i_2 - Ai_3 = 0$$

$$-R_1 i_1 - (R_2 + R_4)\,i_2 + (R_1 + R_2)\,i_3 = -v_s$$

$$-R_1 i_1 - R_2 i_2 + (R_1 + R_2 + R_3)\,i_3 = 0$$



$$\mathcal{M}\begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -v_s \\ 0 \end{pmatrix}$$

Matlab Mesh Results

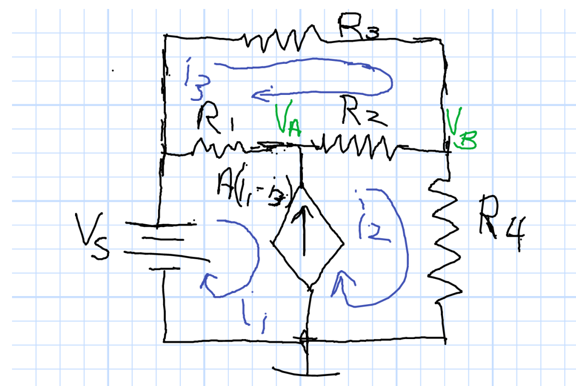
```
>> vs=12;A=3;R1=1000;R2=R1;...
R3=5000;R4=200;y=[0;-vs;0];
>> M=[A+1,-1,-A;...
-R1,-(R2+R4),R1+R2;...
-R1,-R2,R1+R2+R3]
M =
      4     -1     -3
  -1000  -1200   2000
  -1000  -1000   7000
>> x=inv(M)*y
x =
    0.0040
    0.0100
    0.0020
>> vBcheck=x(2)*R4
vBcheck =    2
```

$V_s = 12\text{V} \qquad A = 3$

$R_1 = R_2 = 1k\Omega$

$R_3 = 5k\Omega$

$R_4 = 200\Omega$

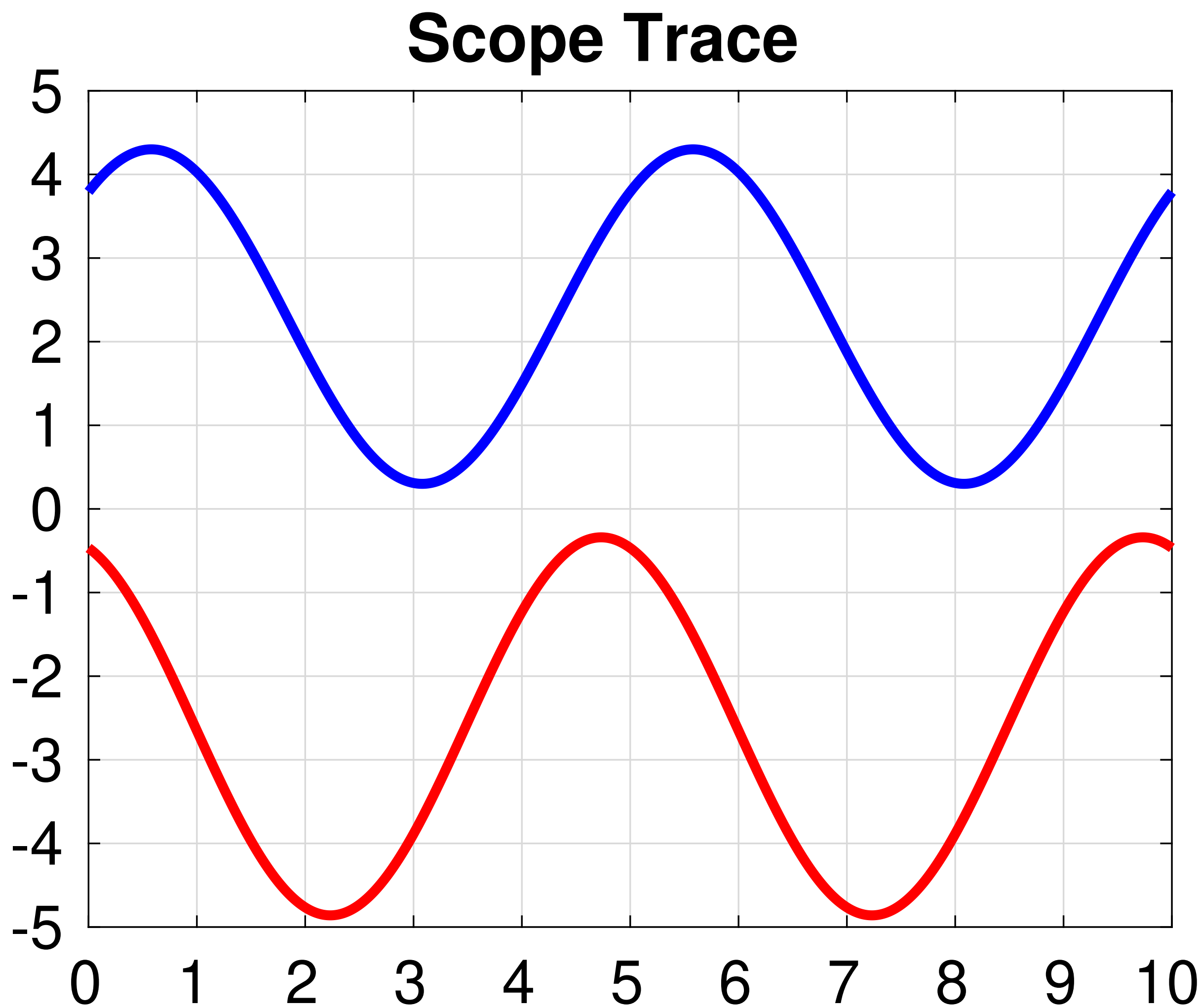


Node Solution

$$\begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 10 \\ 2 \end{pmatrix} \text{Volts}$$

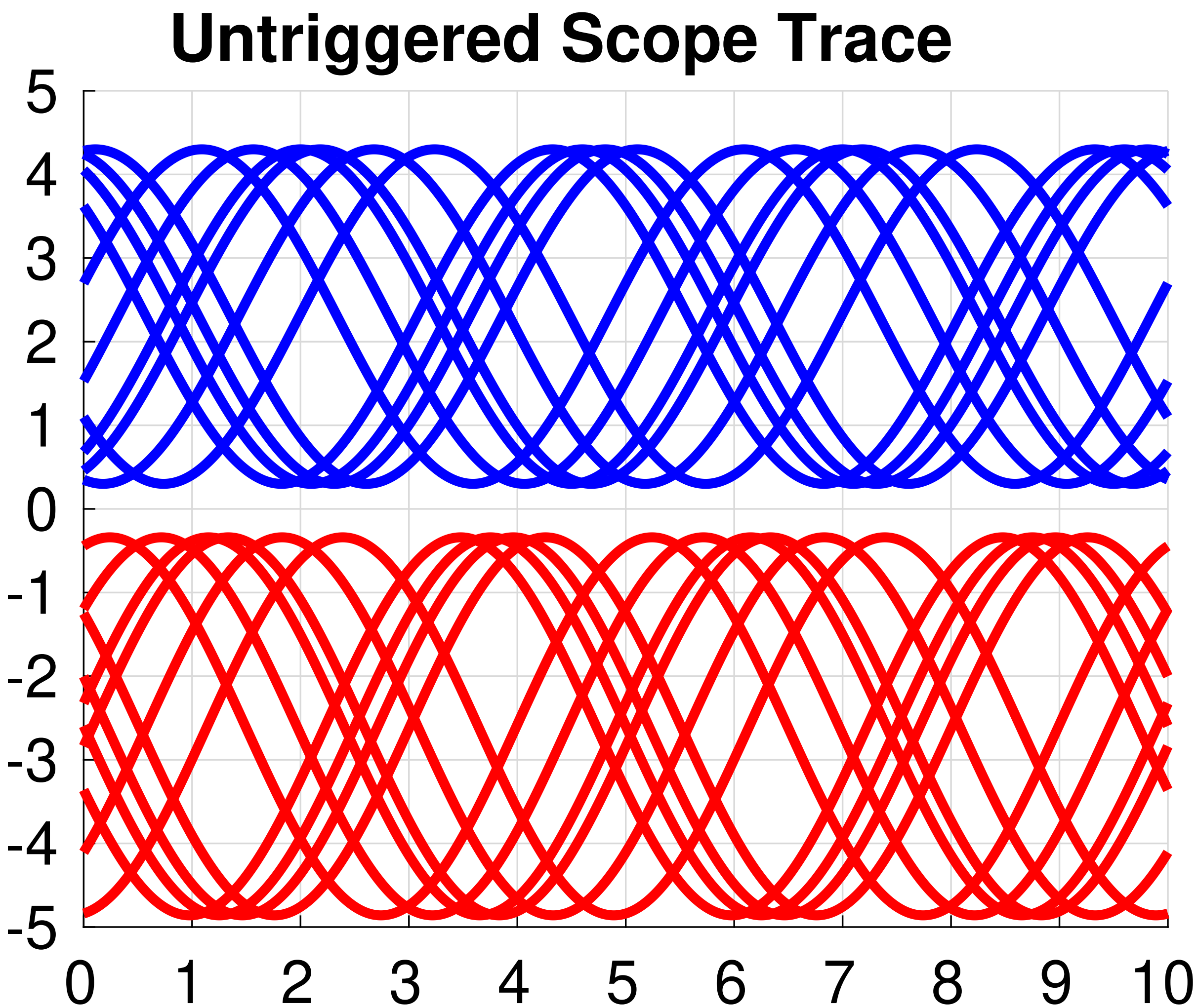
```
>> vAcheck=vBcheck+x(2)*R2-x(3)*R2
vAcheck =    10
```

Reading an Oscilloscope



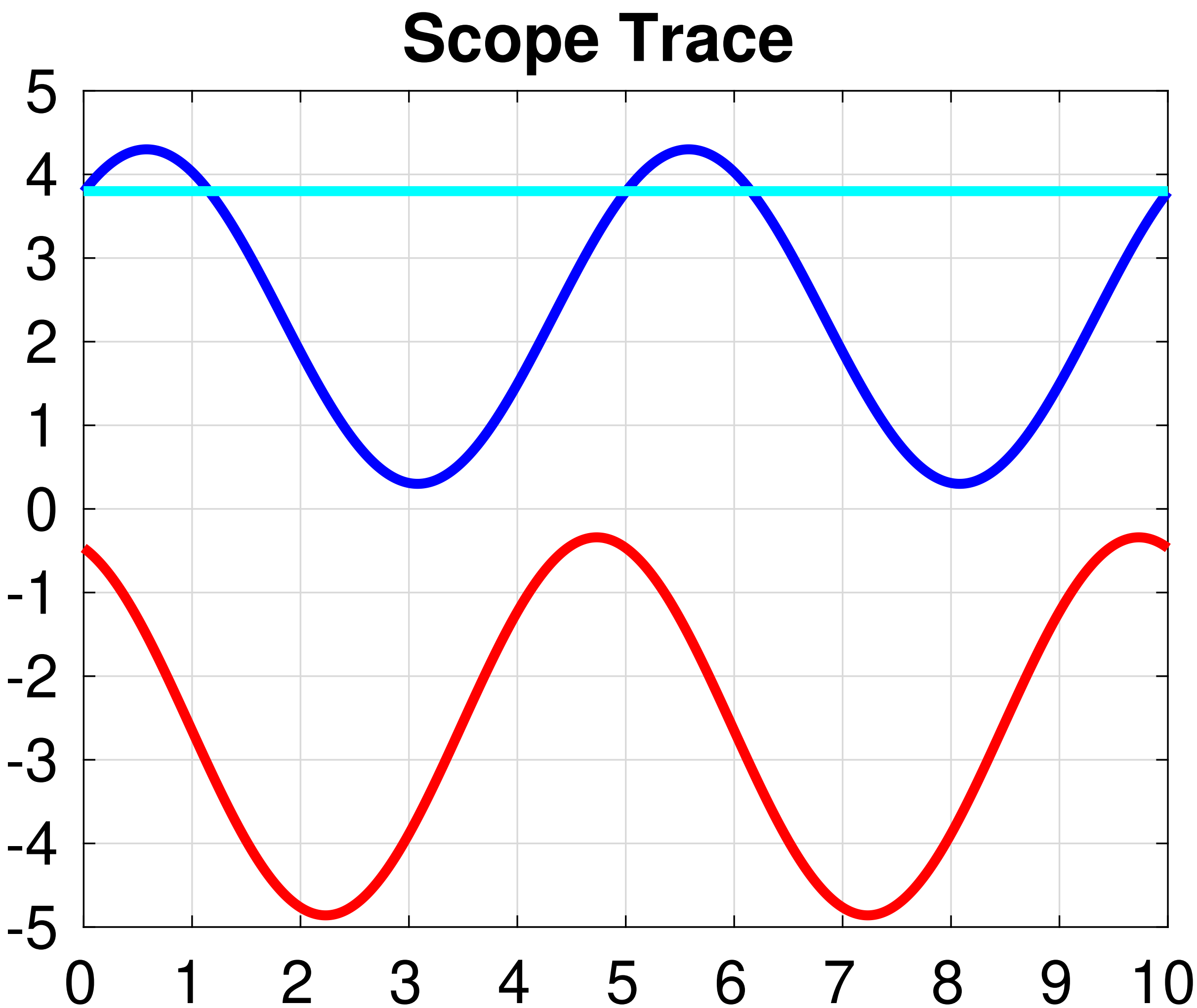
Ch1: V 10V/cm Ch2: V_B, 5V/cm Time base 20μs

Untriggered Oscilloscope



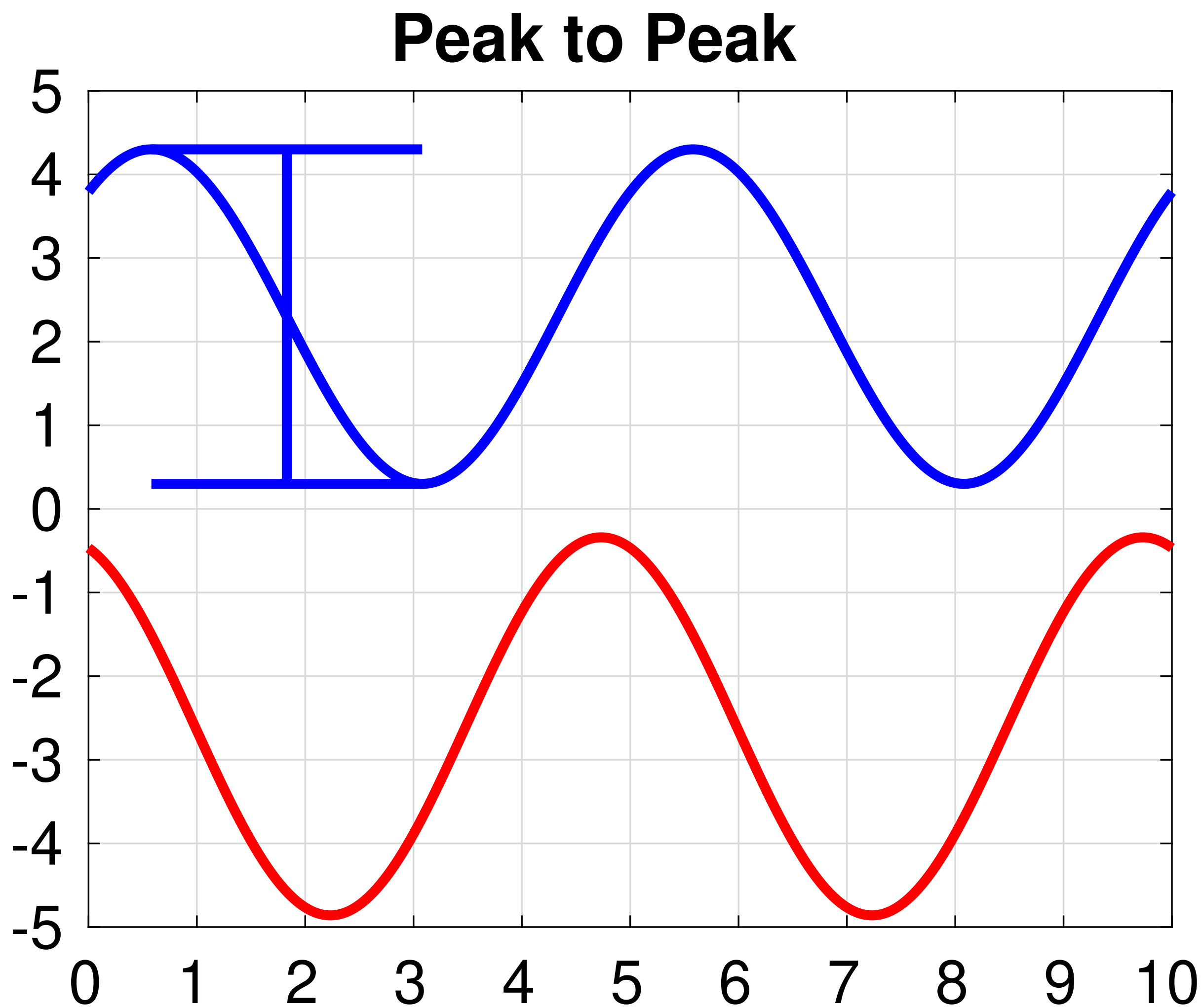
Ch1: V 10V/cm   Ch2: V_B, 5V/cm   Time base 20μs
Trigger Mode: Auto

Channel–1 Trigger



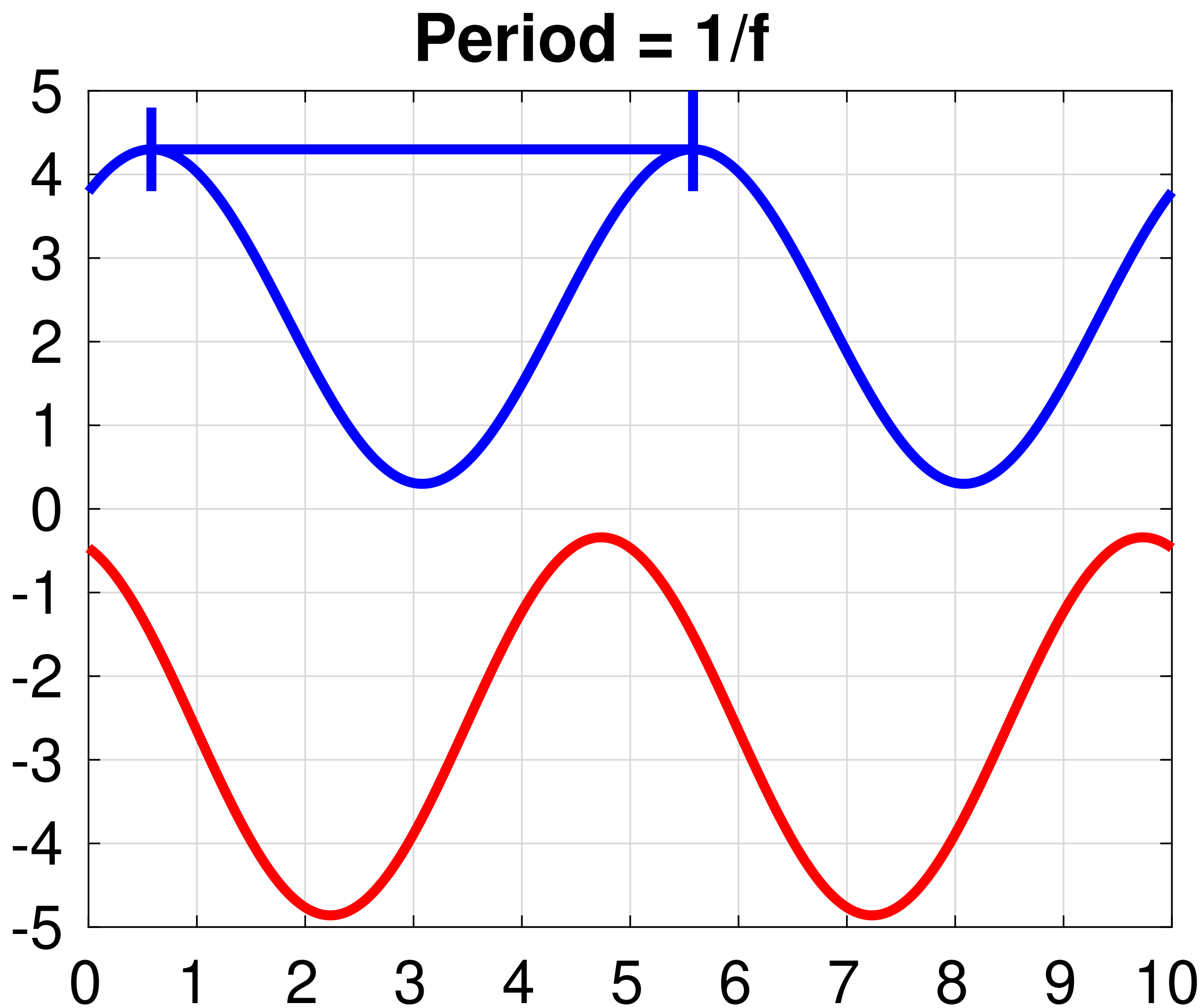
Ch1: V 10V/cm Ch2: V_B, 5V/cm Time base 20μs
Trigger Ch1, 15V, +slope

Reading the Voltage



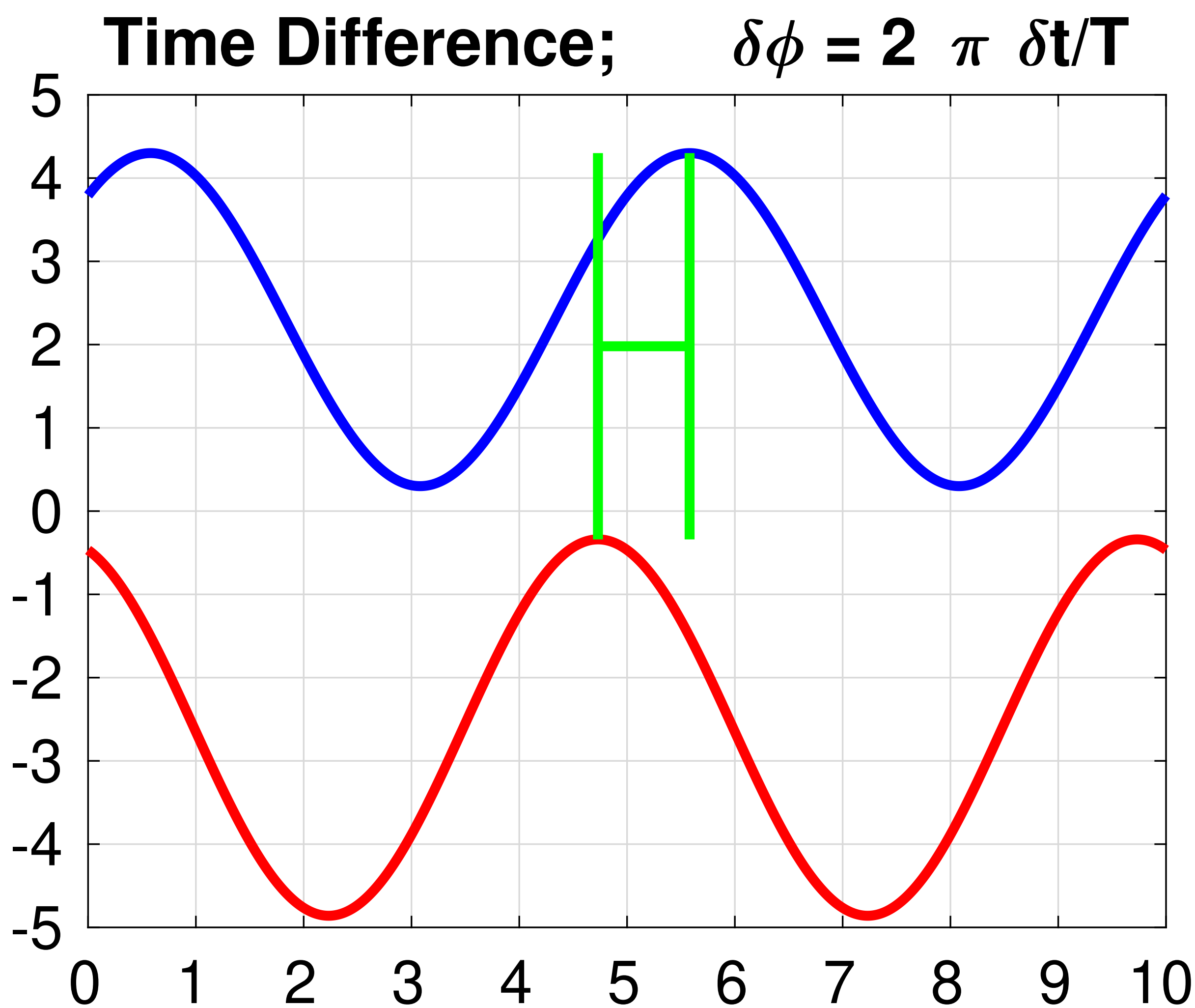
Ch1: V, 40V$_{pp}$/(10V/cm) Ch2: V_B, 22.6V$_{pp}$/(5V/cm)

Reading the Period



$T = 1/f$; 10μs/2μs/cm

Reading the Phase Difference



$\delta t = T\frac{\delta\phi}{2\pi} = T\frac{\delta\phi}{360^\circ}$; 1.7$\mu$s/2$\mu$s/cm

Trigger Ch1, 15V, +slope