Circuits & Signals
EECE2150 — Fall 2018
Final Exam

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File:12303/exam/exam

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Name: ______________________________

General Rules:

- You must do Problem 1. You may do any 5 of Problems 2 through 7. Each problem is graded on a scale of zero to 16. Writing your name is worth 4 points.
- You may make use of three sheets of notes, 8.5–by–11 inches, using both sides of the page.
- You may use a calculator.
- Present your work as clearly as possible. I give partial credit if I can figure out that you know what you are doing. I do not give credit for putting down everything you know and hoping I will find something correct in it.
- Each question has a vertical black bar providing space for your work and a line for numerical answers or box for plots or drawings. Please write your answer to each question clearly. If it happens to be correct and the work leading to it looks right, I give you points quickly and move on to the next problem. Please show your work in the space provided, or on extra pages, clearly labeled with the problem number. If the answer is wrong, this will make it easy for me to find ways to give you partial credit.
- Avoid any appearance of academic dishonesty. Do not talk to other students during the exam. Keep phones, computers, and other electronic devices other than calculators secured and out of reach.

Student: Please check “Do not grade,” for exactly one of problems 2 through 7 that you do not wish to do. That one will not be counted in your grade. The graded problems are all worth 16 points.

Problem 1 Grade: ________________ /16

Problem 2 Grade: ________________ /16 ☐ Do Not Grade

Problem 3 Grade: ________________ /16 ☐ Do Not Grade

Problem 4 Grade: ________________ /16 ☐ Do Not Grade

Problem 5 Grade: ________________ /16 ☐ Do Not Grade

Problem 6 Grade: ________________ /16 ☐ Do Not Grade

Problem 7 Grade: ________________ /16 ☐ Do Not Grade

Total Points: ________________ /96

Exam Score: ________________ /100

1 Short–Answer Questions

A sinusoidal voltage is applied to an inductor. The maximum positive current is seen at a time ...

- [ ] one quarter wave before the maximum positive voltage
- [x] one quarter wave after the maximum positive voltage
- [ ] one half wave before the maximum positive voltage
- [ ] one half wave after the maximum positive voltage
- [ ] at the same time as the maximum positive voltage

The current into the (+) input of an op–amp depends on the value of the feedback resistor ...

☐ True ☒ False

The product of a complex number and its conjugate is always a real number ...

☒ True ☐ False

The Fourier transform of a square wave has only even harmonics ...

☐ True ☒ False

You have observed three components sitting on a table for several hours. They are not currently connected to any circuit, but you don't know what might have happened before. Which one(s) is/are safe to touch by the terminals? Check all correct answers.

☒ Resistor ☒ Inductor ☐ Capacitor

To obtain the mid–band gain, ...

☐ Short the capacitors ☐ Open the capacitors ☒ Do whichever is most favorable for the gain

A low–pass filter and high–pass filter are both linear devices and so it doesn't matter whether the signals goes through the high–pass or low pass first.

☐ True ☐ False ☒ True in theory but not always in practice

A sampled signal can always be reproduced perfectly if it is

- ☐ Sampled at more than its highest frequency
- ☒ Sampled at more than twice its highest frequency
- ☐ Sampled for an infinite time

The spectrum of a real signal ...

- ☐ is always real
- ☐ is an even function
- ☒ has an even real part and odd imaginary part
- ☐ has an odd real part and even imaginary part

A good ammeter has a ...

- ☐ high input impedance
- ☒ low input impedance

The input impedance of an oscilloscope is typically ...

☐ 72 Ω ☐ 600 Ω ☒ 1 MΩ ☐ infinite

A voltage gain of −10 is ...

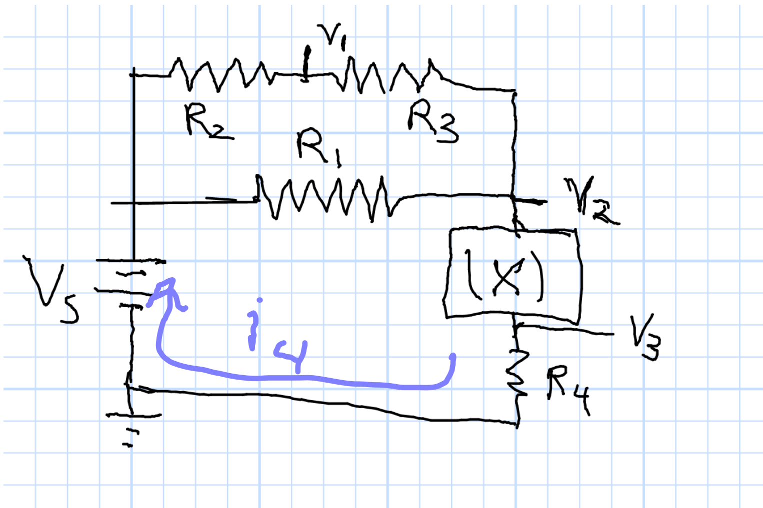
☐ 10 dB ☐ −10 dB ☒ 20 dB ☐ −20 dB

2 Kirchoff's Laws

Consider the circuit in the figure. The rectangular box marked (x) represents an unknown circuit component.

The supply is 12 Volts. $R_1 = 1$ kΩ, $R_2 = R_3 = 500$ Ω, $R_4 = 200$ Ω.

We measure $V_3 = 2$ V.



What is the current through the unknown device?

$i = i_4 = \frac{V_3}{R_4} = \frac{2V}{200\Omega} = 0.01A$

10 mA

What is the current produced by the source?

the same

10 mA

What is the voltage, V_2?

$V_2 = 12V - i_4 R_1 \| (R_2 + R_3)$
$= 12V - 0.01A \times 500\Omega$

$V_2 =$ 7 V

What is the voltage, V_1?

$V_1 = 12V - \frac{i_4}{2} R_2 = 12V - \frac{0.01A}{2} 500\Omega$

$V_1 =$ 9,5 V

How much power is produced by the source?

$iV = 0.01A \times 12V$

$P_{source} =$ 120 mW

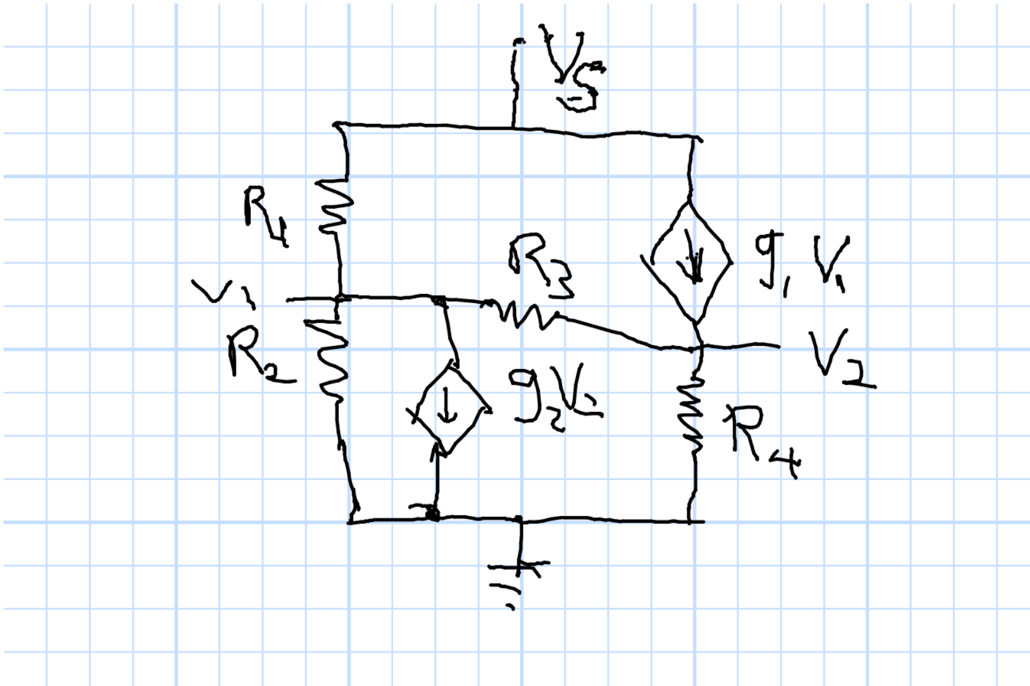
How much power is consumed by the unknown device?

$i_4 (V_2 - V_3) = 0.01A \times 5V$

$P_{device} =$ 50 mW

3 Node Analysis: Dependent Sources

Consider the circuit shown in the figure.



$$R_1 = 10 \text{ kOhms} \qquad R_2 = R_1$$

$$R_3 = 2 \text{ kOhms} \qquad R_4 = 5 \text{ kOhms}$$

$$g_1 = 40\ \mu\text{A/V} \qquad g_2 = 30\ \mu\text{A/V} \qquad V_s = 12 \text{ V}$$

Write two node equations for the unknown voltages V_1 and V_2.

$$\frac{V_s - V_1}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_3} + V_2 g_2 = 0$$

$$-g_1 V_1 + \frac{V_2}{R_4} + \frac{V_2 - V_1}{R_3} = 0$$

Simplify the equations. That is, group unknowns. Keep in symbolic form.

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V_1 + \left(\frac{-1}{R_3} + g_2\right) V_2 = \frac{V_s}{R_1}$$

$$\left(-g_1 - \frac{1}{R_3}\right) V_1 + \left(\frac{1}{R_4} + \frac{1}{R_3}\right) V_2 = 0$$

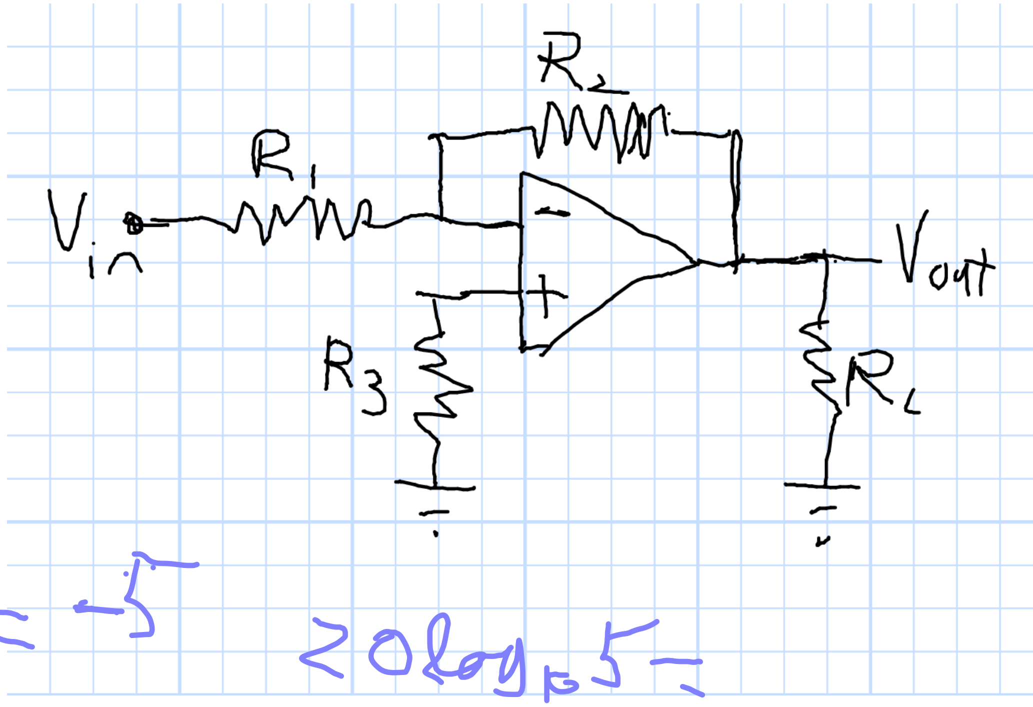
3. Convert the equations to matrix form, $\mathcal{M}\mathbf{x} = \mathbf{y}$

$$\begin{pmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_3} + g_2 \\ -g_1 - \frac{1}{R_3} & \frac{1}{R_4} + \frac{1}{R_3} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} V_s/R_1 \\ 0 \end{pmatrix}$$

You do not need to solve the matrix equation for $\mathbf{x}$. You may leave the matrix in symbolic form.

4 Op–Amps

The circuit shown is an inverting amplifier, with $R_1 = 3$ kΩ, $R_2 = 15$ kΩ, $R_3 = 1$ kΩ, $R_L = 5$ kΩ



What is the voltage gain of the amplifier? Answer as a ratio and in decibels.

$A_V =$ −5 ; 14 dB

i = Vin/R1

If we connect an ideal voltage source, $V_{in} = 100$ mV $\cos 2\pi f t$, to the input, what is the current from that source. Assume positive current is from left to right in R_1.

$i_1 =$ i = 33 μA cos 2πft

How much RMS power is produced by the source?

1.64 μW $\frac{334\,\mu A \times 10\,mV}{2}$

For the same input, what is the output voltage?

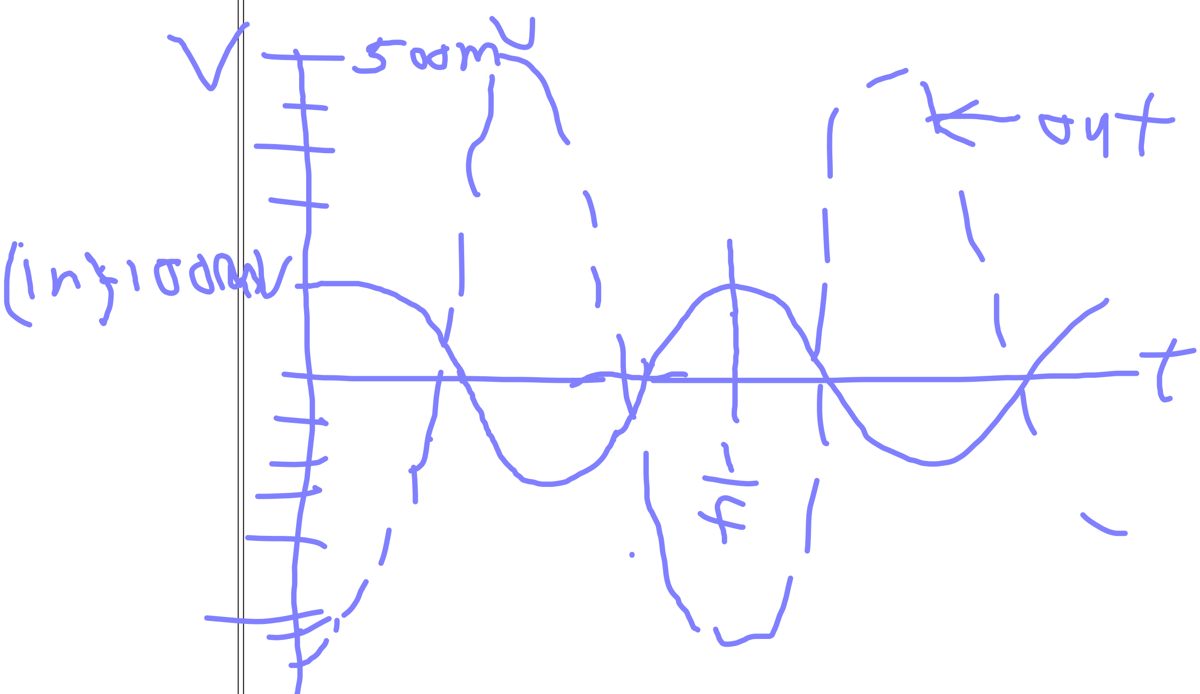
500 mV cos $2\pi f T$

How much RMS power is absorbed by the load?

25 μW $\frac{1}{2}\frac{(500\,mV)^2}{5K}$

Sketch oscilloscope traces of the input and output voltages. Label them and show time and voltage axes.

(Input and Output:)



5 Digitization

We wish to convert an analog signal to a digital one. The signal looks like

$$v(t) = V_0 + v_1(t),$$

$V = 4 + V_1$

where $V_0 = 4$ Volts and v_1 represents a varying signal that contains the important information. We will use a 10–bit digitizer that has an input range of -10 to 10 Volts.

$10 = 4 + V_{1max}$

First, assume we do no processing of the signal, but deliver it directly to the digitizer.

$-10 = 4 + V_{min}$

What are the maximum and minimum values of v_1 that will allow us to digitize the signal correctly?

$v_{1min} =$ -14 Volts

$v_{1max} =$ 6 Volts

What will happen if the voltage exceeds these limits?

It will be limited to -10 to 10.

What is the smallest change in v_1 that can be observed?

$20V/2^{10}$

$\Delta v_1 =$ 0.0195 Volts

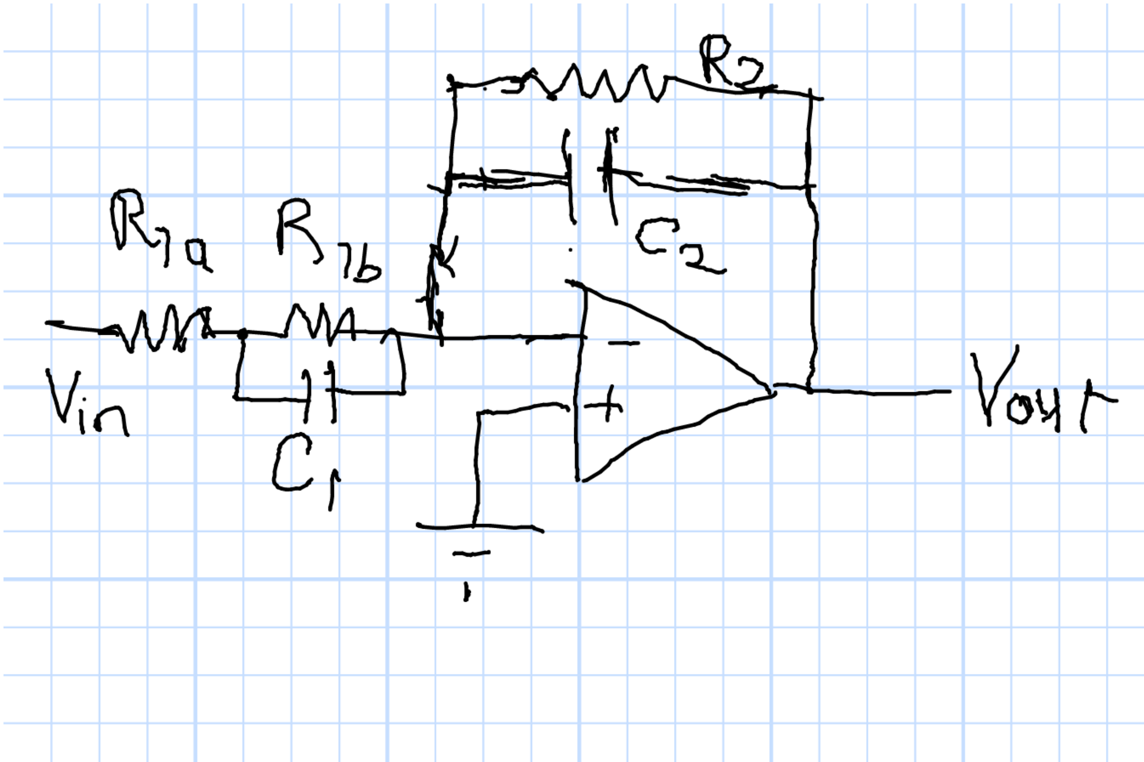
Finally, if we knew in advance that $0 \leq V_1 \leq 1$ Volt, what steps could we employ before digitization to improve the performance? Hint: I can think of one way to do it in 2 steps and another in 3.

High Pass Filter + Amplify by 40 and subtract 10 V

Or Subtract 4.5 V & Amplify by 40

6 Filter

Consider the circuit in the figure.



Write the equation for the amplifier gain. You do not need to reduce it to simplest form.

$$A_v = -\frac{R_2 \| \frac{1}{j\omega C_2}}{R_{1a} + \left(R_{1b} \| \frac{1}{j\omega C_1}\right)}$$

What is the DC gain ($f = 0$)? Open both Cs

$A_v =$ $-R_2/(R_{1a}+R_{1b})$.

What is the mid–band gain? Short C1 and open C2

$A_v =$ $-R_2/R_{1a}$.

What is the gain at $f = 1$ kHz? Express it in polar coordinates with the angle in degrees.

$A_v =$ ________ $\angle$ ________ $^\circ$.

What is the gain at $f = 1$ MHz? Express it in polar coordinates with the angle in degrees.

$A_v =$ ________ $\angle$ ________ $^\circ$.

7 Fourier Transforms

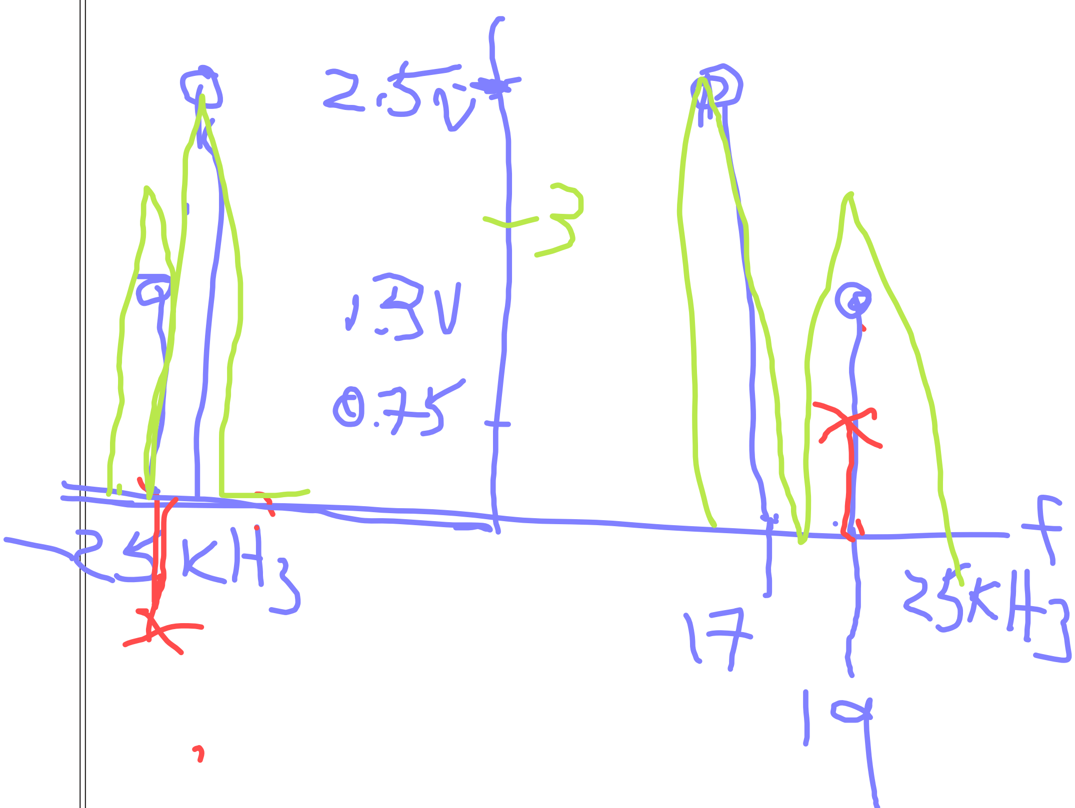
Consider a signal that has multiple frequency components that all lie between 565 and 575 Khz. This signal is digitized at a sample frequency of $f_{sample} =$ 50 kHz. There are no other inputs to the digitizer outside this passband.

After digitization, can we recover this signal perfectly, assuming that we know the limits of the passband?

Yes. All the signal will alias into the passband
Subtracting 11 X 50kHz.and we know to add 550kKz

After we collect the signal, we take the Fourier transform using the FFT algorithm. Draw the axes for the frequency–domain plot with the frequency limits we usually use (with zero frequency in the center and a width appropriate to the sampling frequency). Just draw the axes for now. The plot comes later.

(Frequency–Domain Plot:)



Assume at some particular time the signal is described by

$$v(t) = 5\ \text{Volts} \times \cos\left(2\pi \times 567\ \text{kHz} \times t\right) + 3\ \text{Volts} \times \cos\left(2\pi \times 569\ \text{kHz} \times t + \pi/3\right)$$

List all the frequencies at which we will see signals.

17, 19 and
−17, 19 kHz,

Draw these results on your plot. In keeping with our usual tradition in this section, plot an "o" for the real parts, "x" for the imaginary parts, and a line for the absolute values.

$567 - 11 \times 50 = 17$

$569 - 11 \times 50 = 19$

$$\frac{3e^{j\pi/3}}{2} = 3 \times (0.866 + j \times 0.5)/2$$

$$= \frac{2.6 + j1.5}{2}$$

$$= 1.3 + j0.75$$