# EECE 2150 – Circuits and Signals
# Final Exam – Fall 2016 – Dec 9
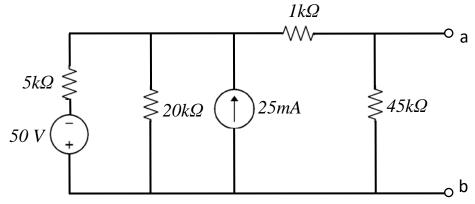
# Name:

# Section:

**Instructions:**
- Write your name and section number on all pages
- Closed book, closed notes; Computers and cell phones are not allowed
- You can use a single, double-sided, equation sheet
- Scientific calculators are allowed
- **Complete 5 problems, if you start to work on more than 5 problems be sure to make clear which 5 problems you want graded. Otherwise the first 5 problems which you started answering will be graded.**
- All problems will have an equal value of 20%.
- Show all work and **place a box around all your final answers**
- Show your work clearly and in detail for partial credit
- You may write on both sides of the pages

## Problem 1)

a. Find the Thevenin equivalent of the circuit between terminals "a" and "b".
b. A 4.5kΩ load resistor is connected to terminals "a" and "b". Using the Thevenin equivalent circuit you found in part a, calculate the power that will be absorbed by the load.
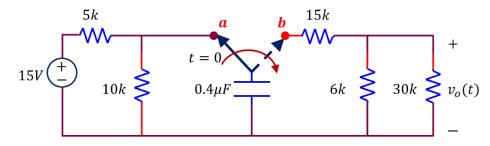
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## Problem 2)

For the circuit shown, the capacitor stays in position ($a$) for a long time before it is switched to position ($b$) at $t = 0$

   a. Find the initial voltage across the capacitor at $t = 0$.

   b. Find the initial energy stored in the capacitor at $t = 0$.

   c. Find the time constant of the circuit after switching to position ($b$).

   d. Find the voltage across the capacitor as a function of time, for $t > 0$.

   e. Find the voltage, $v_o(t)$ as a function of time, for $t > 0$.
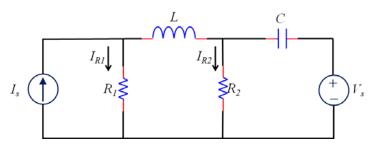
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## Problem 3)

For the circuit of the figure below,

a. Find the current going through $R_1$ for $V_s = 5$ volts and $I_s = 2A$. Explain your answer using the behavior of the capacitor and inductor for constant (DC) sources.

b. Find the current going through $R_2$ for $V_s = 10\cos(10^{10}t)$ volts, $I_s = 3\sin\left(10^{10}t + \frac{\pi}{3}\right)$A. (You can assume that this frequency is high enough that you can let $\omega\to\infty$). Explain your answer using the behavior of the capacitor and inductor at very high frequencies.

c. Using sinusoidal steady state analysis, find the current going through $R_1$ for $V_s = \cos(10^3t + \frac{\pi}{2})$ volts, $I_s = \cos(10^3t)$A.

$R_1 = 10\Omega,$ $\qquad R_2 = 10\Omega$

$L = 10mH$ $\qquad C = 100\mu F$

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## Problem 4)

The parts of this problem are independent. You can use the tables of Fourier Transform pairs and properties anywhere they are useful as long as you state which property or pair you are using.

a. Find the Fourier Transform of a signal x(t) if

$$x(t) = \begin{cases} 2 & -3 < t < -1 \\ 0 & otherwise \end{cases}$$

b. Find the Fourier Transform of a signal y(t) if:

$$y(t) = \begin{cases} 2e^{-3t} & t > 3 \\ 0 & t \leq 0 \end{cases}$$

Note that if the notation for the step function is familiar to you, you may find it helpful that this is the same as saying that $y(t) = 2e^{-3t}u(t - 3)$.

c. If a system with input x(t) and output y(t) is described by the differential equation

$$y''(t) - 2y'(t) + 3y(t) = 2x'(t) - 3x(t),$$

give an expression for the frequency response of the system Y(ω) / X(ω).

d. A linear time invariant system has input x(t) and output y(t). If the Fourier Transform of the input is $X(\omega) = \frac{1}{1+j\omega}$ and the frequency response of the system is $H(\omega) = \frac{2+j2\omega}{2-j3\omega}$, give an expression for $Y(\omega)$, the Fourier Transform of y(t). Note that you do not need to evaluate this Fourier Transform in any way, just tell us what it is.

## Problem 5)

For a particular active (Op-Amp) filter, the frequency response is:

$$H(\omega) = \frac{V_o(\omega)}{V_s(\omega)} = \frac{(j\omega)^2}{(j\omega)^2 + j\omega\left(\frac{2}{R_2 C}\right) + \left(\frac{1}{R_1 R_2 C^2}\right)},$$

a. Based on this expression, what is the value of this frequency response when the frequency $\omega = 0$? Show your work to receive full credit!

b. Based on this expression, what is the value of this frequency response when the frequency $\omega \rightarrow \infty$? Show your work to receive full credit.

c. Based on your answers to the questions above, what kind of filter is this? Lowpass, highpass, bandpass, band reject or none of those? Explain your reasoning to receive full credit.

d. If the element values for the circuit are chosen such that $R_2C = 1$ and $R_1C = 0.25$, what is the output of this filter, $v_o(t)$, if $v_s(t) = 4 + 2\cos(2t + \pi/3) + 4\cos(1000t + \pi/4)$?

Note that if, in carrying out this computation, you come across a complex number where the real and imaginary parts differ by more than 2 orders of magnitude (in other words by more than a factor of 100) you can approximate by neglecting the smaller term.

## Problem 6)

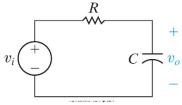The trigonometric Fourier series for a square wave oscillating from 0 to 1 is given by

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

where:

$$a_n = \begin{cases} (-1)^{\frac{n+3}{2}} \dfrac{2}{n\pi} & n \text{ is odd} \\ 0 & n \text{ is even} \end{cases}, \qquad b_n = 0$$

a.  Write the first 4 non-zero terms of the Fourier series (the constant term and 3 cosine terms) for $\omega_0 = 2\pi 100$ rad/s.

b.  If you put the signal into the following low-pass filter, will you get a square wave of a different amplitude out of the filter, or something else?  You don't need this to answer the question, but you can assume that R=1kΩ and C=1μF.

c. Support your answer to b. using a frequency-domain argument.

d. Support your answer to b. using a time-domain or circuit explanation.

## Problem 7)

You have an ECG signal that, including noise and interference, contains frequency components from <1Hz to >40kHz. The voltage of the signal ranges from 0 to +100 mV as a function of time. You plan to sample the signal at 4 kHz using a 10-bit A/D (analog to digital) converter with an input range of 0 to 1V.

a. To preserve all frequencies of interest to accurately record the ECG signal (0.5-200Hz) while minimizing aliasing (distortion) of your sampled signal, you need to filter the signal before the A/D conversion. Approximately what should the cutoff frequency of the filter be?

b. How many voltage levels are possible with a 10-bit A/D? what is the separation between adjacent voltage levels for this A/D?

c. Now, instead of the ECG signal, assume that the input voltage signal to your A/D is $0.5+0.010\cos(1000\pi t)$ volts. Sketch the digital output below for 1 cycle of this signal, including quantization effects (quantization is the mapping of the analog signal onto a discrete digital value at each sampling time).

d. To minimize quantization effects, your signal should be amplified before A/D conversion. What should the gain of your amplifier be to minimize quantization effects?