# EECE 2150 - Circuits and Signals Final Exam - Fall 2016 - Dec 16 

## Name:

## Section:

## Instructions:

- Write your name and section number on all pages
- Closed book, closed notes; Computers and cell phones are not allowed
- You can use a single, double-sided, equation sheet
- Scientific calculators are allowed
- Complete 5 problems, if you start to work on more than 5 problems be sure to make clear which 5 problems you want graded. Otherwise the first 5 problems which you started answering will be graded.
- All problems will have an equal value of $20 \%$.
- Show all work and place a box around all your final answers
- Show your work clearly and in detail for partial credit
- You may write on both sides of the pages

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## Problem 1)

a. Find the Thevenin's equivalent of the circuit between terminals "a" and "b".
b. A $120 \Omega$ load resistor is connected between terminals "a" and "b". Using the Thevenin's equivalent circuit found in part a, calculate the power absorbed by the load.


## Problem 2)

In the circuit shown, the switch has been open for a long time before closing at $t=0$
a. Find the initial voltage across the capacitor at $t=0$
b. Find the time constant of the circuit after the switch is closed
c. Write an expression for the voltage across the capacitor as a function of time for $t>0$
d. Write an expression for the output voltage, $v_{o}(t)$ as a function of time for $t>0$


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## Problem 3)

For the circuit shown,
a. Find the current going through $R_{1}$ for $V_{s}=10 \mathrm{~V}$ and $I_{s}=0 \mathrm{~A}$. Explain your answer using the behavior of the capacitor and inductor for constant (DC) sources.
b. Find the current going through $\mathrm{R}_{2}$ for $V_{s}=0 \mathrm{~V}$ and $I_{s}=2 \cos \left(10^{10} t+\frac{\pi}{3}\right) \mathrm{A}$. (You can assume that this frequency is high enough that you can let $\omega \rightarrow \infty$ ). Explain your answer using the behavior of the capacitor and inductor at very high frequencies.
c. Using sinusoidal steady state analysis, find the current going through $\mathrm{R}_{2}$ for $V_{s}=10 \cos \left(10^{3} t+\frac{\pi}{2}\right)$ volts and $I_{s}=\cos \left(10^{3} t\right) A$.

$$
\begin{aligned}
& R_{1}=10 \Omega, R_{2}=10 \Omega \\
& L=10 \mathrm{mH} \quad C=100 \mu F
\end{aligned}
$$



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## Problem 4)

The parts of this problem are independent. You can use the tables of Fourier Transform pairs and properties anywhere they are useful as long as you state which property or pair you are using.
a. Find the Fourier Transform of a signal $x(t)$ if

$$
x(t)=\left\{\begin{array}{cc}
-1, & -1<\mathrm{t}<3 \\
0 & \text { otherwise }
\end{array}\right.
$$

b. Find the Fourier Transform of a signal $y(t)$ if

$$
y(t)= \begin{cases}1.5 e^{-0.5(t+2)} & t>-2 \\ 0 & t \leq-2\end{cases}
$$

Note that if the notation for the step function is familiar to you, you may find it helpful that this is the same as saying that $y(t)=1.5 e^{-0.5(t+2)} u(t+2)$
c. If a system with input $x(t)$ and output $y(t)$ is described by the differential equation

$$
y^{\prime \prime \prime}(t)+2 y^{\prime \prime}(t)+3 y^{\prime}(t)-4 y(t)=3 x^{\prime \prime}(t)-x^{\prime}(t)+2 x(t)
$$

give an expression for the frequency response of the system $Y(\omega) / X(\omega)$.
d. A linear time invariant system has input $x(t)$ and output $y(t)$. If the Fourier Transform of the input is $X(\omega)=\frac{2-j \omega}{1+j \omega}$ and the frequency response of the system is $H(\omega)=\frac{-4}{3+j 2 \omega}$, give an expression for $\mathrm{Y}(\omega)$, the Fourier Transform of $y(t)$. Note that you do not need to evaluate this Fourier Transform in any way, just tell us what it is.

## Problem 5)

For a particular active (Op-Amp) filter, the frequency response is:

$$
H(\omega)=\frac{V_{o}(\omega)}{V_{s}(\omega)}=\frac{-j 2 \omega R_{2} R_{3} C}{R_{1}+R_{2}+j 2 \omega R_{1} R_{2} C}
$$

a. Based on this expression, what is the value of this frequency response when the frequency $\omega=0$ ? Show your work to receive full credit!
b. Based on this expression, what is the value of this frequency response when the frequency $\omega \rightarrow \infty$ ? Show your work to receive full credit.
c. Based on your answers to the questions above, what kind of filter is this? Lowpass, highpass, bandpass, band reject or none of those? Explain your reasoning to receive full credit.
d. If the element values for the circuit are chosen such that $R_{1}+R_{2}=10^{6}$, $R_{1} R_{2} C=2.5 \times 10^{5}$ and $R_{2} R_{3} C=10^{6}$, what is the output of this filter, $v_{o}(t)$, if $v_{s}(t)=3+2 \cos \left(2 t+\frac{\pi}{4}\right)-3 \cos (1000 t+\pi / 6)$ ?
Note that if, in carrying out this computation, you come across a complex number where the real and imaginary parts differ by more than 2 orders of magnitude (in other words by more than a factor of 100) you can approximate by neglecting the smaller term.

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## Problem 6)

The trigonometric Fourier series for a square wave at $\omega_{0}=200 \pi \mathrm{rad} / \mathrm{s}$ oscillating from 0 to 1 is given by

$$
f(t)=a_{0}+\sum_{n=1}^{\infty} a_{n} \cos n \omega_{0} t+b_{n} \sin n \omega_{0} t
$$

where $a_{0}=0.5$, and otherwise:

$$
a_{n}=\left\{\begin{array}{cc}
(-1)^{\frac{n+3}{2}} \frac{2}{n \pi} & n \text { odd } \\
0 & n \text { even }
\end{array}\right\}
$$

and $b_{n}=0$.
a. Plot the sum of terms with $\mathrm{n}=0$ and 1 (the first two terms) as a function of time.

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b. Plot the output as a function of time if you put the signal (the original square wave) into an ideal band-pass filter with cutoff frequencies of $\omega_{c_{l}}=300 \pi \mathrm{rad} / \mathrm{sec}$, and $\omega_{c_{h}}=800 \pi \mathrm{rad} / \mathrm{sec}$ and an in-band gain of 1 . Explain your reasoning.

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## Problem 7)

You have an ECG signal that, including noise and interference, contains frequency components from 0.1 Hz to 15 kHz . You know that the pure ECG signal only has frequency components up to 250 Hz . The voltage of the signal ranges from -50 to +50 mV as a function of time. You plan to sample the signal at 40 kHz using a 4-bit $\mathrm{A} / \mathrm{D}$ (analog to digital) converter with an input range of -1 to $1 V$.
a. Should you sample the signal first, and then use digital filtering to remove the unwanted parts or should you use an analog filter before sampling the signal? Explain why the method you chose is better. Note that there may not be a single correct answer!!
b. How many voltage levels are output by a 4-bit $\mathrm{A} / \mathrm{D}$ ? How large is the difference between voltage levels for this A/D?
c. To minimize quantization effects (quantization is the mapping of the analog signal onto a discrete digital value at each sampling time), your signal should be amplified before A/D conversion. What should the gain of your amplifier be to minimize quantization effects?
d. Now, instead of the ECG signal, assume that the input signal to your $\mathrm{A} / \mathrm{D}$ is $0.25 \cos (1000 \pi t) V$. Sketch the digital output below for 1 cycle, including quantization effects (quantization is the mapping of the analog signal onto a discrete digital value at each sampling time).

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