# EECE 2150 - Circuits and Signals Final Exam - Fall 2016 - Dec 12 

## Name:

## Section:

## Instructions:

- Write your name and section number on all pages
- Closed book, closed notes; Computers and cell phones are not allowed
- You can use a single, double-sided, equation sheet
- Scientific calculators are allowed
- Complete 5 problems, if you start to work on more than 5 problems be sure to make clear which 5 problems you want graded. Otherwise the first 5 problems which you started answering will be graded.
- All problems will have an equal value of $20 \%$.
- Show all work and place a box around all your final answers
- Show your work clearly and in detail for partial credit
- You may write on both sides of the pages


## Problem 1)

Use Node-Voltage or Mesh-Current method and calculate current $i_{o}$ in the circuit.


Name:
Section:

## Problem 2)

For the circuit shown, the capacitor stays in position (a) for a long time before it is switched to position (b) at $t=0$
a. Find the initial voltage across the capacitor at $t=0$
b. Find the time constant of the circuit after switching to position (b)
c. Write a complete expression for the voltage across the capacitor as a function of time, $t \geq 0$
d. Write a complete expression for the output voltage, $v_{o}(t)$ as a function of time, $t \geq 0$


Name:
Section:

## Problem 3)

For the circuit of the figure below,
a. Find the current going through $\mathrm{R}_{2}$ for $V_{S}=10$ volts. Explain your answer using the behavior of the capacitor and inductor for constant (DC) sources.
b. Find the current going through $\mathrm{R}_{2}$ for $V_{s}=15 \cos \left(10^{10} t+\frac{\pi}{3}\right)$ volts. (You can assume that this frequency is high enough that you can let $\omega \rightarrow \infty$ ). Explain your answer using the behavior of the capacitor and inductor at very high frequencies.
c. Using sinusoidal steady state analysis, find the current going through $\mathrm{R}_{1}$ for $V_{s}=$ $3 \cos \left(10^{3} t+\frac{\pi}{4}\right)$ volts.

$$
R_{1}=10 \Omega, \quad R_{2}=20 \Omega
$$

$R_{2}=10 \Omega$
$L=10 \mathrm{mH}$
$C=100 \mu \mathrm{~F}$


Name:
Section:

## Problem 4)

The parts of this problem are independent. You can use the tables of Fourier Transform pairs and properties anywhere they are useful as long as you state which property or pair you are using.
a. Find the Fourier Transform of a signal $x(t)$ if

$$
x(t)= \begin{cases}1.5 & 2<t<4 \\ 0 & \text { otherwise }\end{cases}
$$

b. Find the Fourier Transform of a signal $y(t)$ if

$$
y(t)= \begin{cases}-3 e^{-4 t} & t>1 \\ 0 & t \leq 1\end{cases}
$$

Note that if the notation for the step function is familiar to you, you may find it helpful that this is the same as saying that $y(t)=-3 e^{-4 t} u(t-1)$
c. If a system with input $x(t)$ and output $y(t)$ is described by the differential equation

$$
y^{\prime \prime}(t)+5 y^{\prime}(t)-y(t)=-x^{\prime}(t)+2 x(t),
$$

give an expression for the frequency response of the system $\mathrm{Y}(\omega)$ / $\mathrm{X}(\omega)$.
d. A linear time invariant system has input $x(t)$ and output $y(t)$. If the Fourier Transform of the input is $X(\omega)=\frac{2}{2-j 3 \omega}$ and the frequency response of the system is $H(\omega)=\frac{j 2 \omega}{3+j 4 \omega}$, give an expression for $\mathrm{Y}(\omega)$, the Fourier Transform of $\mathrm{y}(\mathrm{t})$. Note that you do not need to evaluate this Fourier Transform in any way, just tell us what it is.

## Problem 5)

For a particular active (Op-Amp) filter, the frequency response is:

$$
H(\omega)=\frac{V_{o}(\omega)}{V_{s}(\omega)}=\frac{\frac{j \omega}{R_{1} C}}{(j \omega)^{2}+j \omega\left(\frac{2}{R_{3} C}\right)+\left(\frac{1}{R_{2} R_{3} C^{2}}\right)}
$$

a. Based on this expression, what is the value of this frequency response when the frequency $\omega=0$ ? Show your work to receive full credit!
b. Based on this expression, what is the value of this frequency response when the frequency $\omega \rightarrow \infty$ ? Show your work to receive full credit.
c. Based on your answers to the questions above, what kind of filter is this? Lowpass, highpass, bandpass, band reject or none of those? Explain your reasoning to receive full credit.
d. If the element values for the circuit are chosen such that $\mathrm{R}_{1} \mathrm{C}=1, \mathrm{R}_{2} \mathrm{C}=0.5$ and $\mathrm{R}_{3} \mathrm{C}=0.5$. what is the output of this filter, $\mathrm{v}_{\mathrm{o}}(\mathrm{t})$, if $\mathrm{v}_{\mathrm{s}}(\mathrm{t})=4+2 \cos (2 \mathrm{t}+\pi / 3)+$ $4 \cos (1000 t+\pi / 4)$ ?
Note that if, in carrying out this computation, you come across a complex number where the real and imaginary parts differ by more than 2 orders of magnitude (in other words by more than a factor of 100 ) you can approximate by neglecting the smaller term.

## Problem 6)

The trigonometric Fourier series for a square wave at $\omega 0=100 \pi \mathrm{rad} / \mathrm{s}$ oscillating from 0 to 1 is given by

$$
f(t)=a_{0}+\sum_{n=1}^{\infty} a_{n} \cos n \omega_{0} t+b_{n} \sin n \omega_{0} t
$$

where $a 0=0.5$, and otherwise:

$$
a_{n}=(-1)^{\frac{n+3}{2}} \frac{2}{n \pi}, \mathrm{n} \text { odd; } 0, \mathrm{n} \text { even }
$$

and $b n=0$.
a. Plot the sum of a0 and a1.
b. Plot the sum of a0, a1, and a2.

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c. Plot the output if you put the signal (the original square wave) into an ideal high-pass filter with a cutoff (or cut-on) frequency $\omega \mathrm{c}=50 \pi \mathrm{rad} / \mathrm{sec}$, and an in-band gain of 1 . Explain your reasoning.

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## Problem 7)

You have an audio signal from a microphone that contains frequency components from 20 Hz to 60 kHz . The voltage of the signal ranges from 0 to $\pm 100 \mathrm{mV}$ as a function of time. You plan to sample the signal at 50 kHz using a $10-\mathrm{bit} \mathrm{A} / \mathrm{D}$ (analog to digital) converter with an input range of -10 to 10 V .
a. To preserve all frequencies over the human hearing range ( 20 Hz to 20 kHz ) while minimizing aliasing (distortion) of your sampled signal, you need to filter the signal before the A/D conversion. Approximately what should the cutoff frequency of the filter be?
b. How many voltage levels are output by a 10 -bit $\mathrm{A} / \mathrm{D}$ ? How large is each voltage level for this $\mathrm{A} / \mathrm{D}$ ?
c. Now, instead of the audio signal, assume that the input to your $\mathrm{A} / \mathrm{D}$ is $0.05 \cos (10000 \pi \mathrm{t})$ volts. Sketch the digital output below for 1 cycle, including quantization effects (quantization is the mapping of the analog signal onto a discrete digital value at each sampling time).
d. To minimize quantization effects, your signal should be amplified before A/D conversion. What should the gain of your amplifier be to minimize quantization effects?

