Homework Set 4

Problem discussion: In class Wed. Mar. 10
Due: By the start of class on Mon. Mar. 22

This homework has only one problem. Please turn in a complete answer. In particular you should include any narrative required to understand your method and comment on your results where appropriate, label and place captions on any figures you include, and in general think of your homework as a short report. The intent is not to have you do a lot of writing for the sake of writing, and you can certainly assume that we know what the problem statement was, but your homework should be professionally done and we should not have to guess at how to interpret what you hand in.

Note that this homework again requires you to work in Prof. DiMarzio’s lab, as discussed in class. You will need to coordinate with Tianchen Shi (tshi@ece.neu.edu) to find a mutually agreeable time to take your data in the lab. As before, the hyperspectral camera you need to use is housed in 464 Egan building, and the phone number there there, if you need to call, is 617 373-2117. Please keep in mind that this is a research lab, and that the graduate students and undergraduates working with Prof. DiMarzio also have busy schedules and thus will expect you to take seriously your appointments with them to take data.

Problem 1: DOT Lab Problem

This assignment will allow you to simulate a diffuse optical tomography (DOT) scenario, take data, and try to reconstruct the optical absorption in a volume from your data. This is the first time we have tried this experiment, and we are really quite excited—we believe it is really a rather unusual, if not unique, opportunity for students to be able to perform an experiment at this level of sophistication. Since this is the first time we are running the experiment, unexpected problems may occur, and we ask you in advance to be willing to work with us to make this problem work well for you.

The big picture for this problem is that you will need to:

1. Go into the lab and take data required for DOT imaging.
2. Work with Tianchen Shi to get the background parameters you will need to solve the forward problem.
3. Run a Born forward model routine (we will provide the software) to predict the measurements you have acquired and compare.

4. Use your Born model in a couple different inverse solution methods and discuss your results.

Lab Experiment:

A drawing of the experimental setup is shown in Figure 1. The goal is to image two small clay balls (one green, the other brown) immersed in a milky-looking suspension of Titanium Oxide (TiO2) in water in a 15x30x20 cm fish tank. TiO2 is a powdery material used to provide a scattering medium with minimal added absorption—for instance, it’s what is used to make white paint white. You will use an optical fiber to deliver light from a Tungsten bulb into the tank, and the hyperspectral imaging system used in the last lab session to take the images of light intensity distribution at the air-medium interface through the lens. You will place the fiber in six different locations to obtain images of the volume from different source positions, change the filter setting to obtain images at different wavelengths, and use averages of pixels on the CCD array to simulate an array of discrete detectors.
In order to use the Born model, you need the background scattering and absorption coefficients, the index of refraction of light in the medium, and an image of the diffuse light scattered by the balls without the “incident” light coming directly from the source. We will give you the background absorption coefficient (which depends on wavelength) and the index of refraction (which is assumed independent of wavelength). You will obtain the background scattering coefficient and the appropriate image from your experimental measurements. In particular, you will make two sets of measurements for each condition of interest: one without the balls in place, and one with them in place. Mr. Shi will help you get the background scattering coefficient from the first of these measurements. The measured image you will use with the Born model for your reconstructions will be a “difference image” that you will calculate by subtracting the “without-target” image from the “with-target” image, where here the target is the two clay balls. Thus this difference image will be an experimentally-determined image of just the diffuse light scattered by the clay balls. Thus, the steps you need to take in the lab are:

1. To get the background-only data, you will sink the balls deeply enough into the tank that we can safely assume their presence is not observed at the surface. Then you can place the light source at one of corners of the camera’s field of view (FOV), and carefully align the fiber into position with HSI in real-time mode. Take background-only images at each wavelength, with the HSI GUI toolbox. Mr. Shi will tell you which wavelengths to use. Our current plans are to use 550, 600, and 650 nm.

2. Now raise the balls back up into position. Remember to mark the location and depth where you put your targets, as you will need to put them back into that position again. Again take images at each wavelength. The difference between these images and the corresponding background-only images is the data you will use in your reconstructions.

3. Move the fiber to five other positions and repeat the whole process. The six positions to use are the four corners and the mid-points of the two short sides of the FOV.

Forward Model:

You will use software provided by us (details still to be announced) to calculate a Born forward model for each wavelength, using the six fibers and 117 detectors—a $9 \times 13$ array formed by averaging $50 \times 50$ blocks of CCD pixels for each detector. To use the software to calculate the Born model you will need the following information:

1. The value of the background absorption at each wavelength. These values come directly from the absorption spectrum for water (since we assume the TiO2 is non-absorbing) and you can get them from Mr. Shi.

2. The index of refraction of the water with the TiO2 in it. Use $n = 1.33$. 

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3. The value of the background scattering, which we assume is wavelength-independent. Mr. Shi will help you calculate this value directly from your background images.

4. The locations of the six sources and the 117 detectors. Our suggestion is that you “create” the detector measurements from the CCD values by the following approach:

(a) The CCD array is 3 cm by 4 cm, with a pitch of 160 pixels/cm.
(b) Avoid the edges to make sure you don’t get the image of the light source on the detectors for any light source position.
(c) Use 50x50 blocks of pixels.
(d) Use the pixel centers as the detector locations in the Born model.

Thus we suggest that you set the locations of the pixel centers every 0.25 cm in both directions, starting at 0.5 cm from the edge. If we assume that (0, 0) is in the upper left corner of the CCD array, the following Matlab code will calculate the $x$ and $y$ locations of all 117 detectors:

```matlab
x_centers = [0.5: 0.25: 3.5];
y_centers = [0.5: 0.25: 2.5];
det_x, det_y = meshgrid(x_centers, y_centers);
```

Using the corresponding locations in terms of pixel number you can figure out which pixels to average to create each “detector” measurement. The 6 source locations you can obtain from observation of your CCD images.

5. The grid to use to discretize the volume. We recommend a grid of 8 x 11 per $z$ value and 6 $z$ levels.

The Born program will return a matrix containing the Born forward model and a vector of the “background image”, or incident field, fluence values predicted by the Born model for each source-detector pair. Note that you should create three Born models; one for each wavelength. Each matrix should have a number of rows equal the number of source positions times the number of synthesized detectors, and a number of columns equal to the total number of voxels in your discretized geometry.

When we post information on the software you will use, we will describe how to arrange your measured data values (that is, the difference image values) for each wavelength so that the order matches the order of the source-detector pairs in the Born matrix. You should do the following comparisons for one wavelength and a couple source positions (pick source positions at extreme positions to maximize the difference in the images):
1. Compare your background measurements (display it as an image on the detector plane to visualize it) to the “incident field” fluence predicted by the Born model and comment.

2. Create a non-quantitative vector of the absorption perturbations that the balls represent by creating a vector of absorption perturbations using any positive constant you like for the absorption of the balls. The key item here is to make a vector of all zeros whose length is the number of voxels, and then change the zero values to your chosen constant value in the voxels which are located in the true location of the two balls.

3. Using this non-quantitative vector of absorption perturbations and the matrix returned by the Born model, calculate a non-quantitative Born prediction of the scattered field, or difference, image. Since it is non-quantitative, you can only compare it qualitatively to your measured difference image. Do so and comment on the result.

**Finding the Objects via Inverse Reconstructions**

Now use your Born forward matrix and your data to try to reconstruct the location of the objects. Be sure to use the difference images as your input data. As with the forward modeling, we will supply you with some of the software you need to do this, and you will have to do some coding yourself to finish the job. We will announce the details soon.

1. Begin by computing the SVD of one of your Born matrices, and looking at a graph of the singular values (note you can extract the diagonal of a matrix using the `diag` command, use `help` to see how to use it). Plot the singular values on a log scale. What is the approximate condition number? (You can approximate this to within one order of magnitude directly off the graph.)

2. Looking at the graph of the singular values, decide on what you think is a reasonable number of singular values to use. The goal is to keep as many singular values as you can without using too many very small ones. Use the truncated Singular Value Decomposition (TSVD) method at one wavelength to do a reconstruction and look at the results. Search for a good truncation level to use by trying at least 10 different truncation levels and looking at the results. Start with fairly extreme small and large truncation levels and then see if you can narrow down on a reasonable compromise. Discuss your results: what happens if you use too few singular values? if you use too many? Could you find a reasonable value, one where you could at least see the objects?

3. Using the same truncation parameter, repeat the reconstruction at the other wavelengths. Do you see any difference? Does it accurately reflect the different colors of the balls?

4. Note: we may decide as things progress to drop this last part of the assignment, but we’ll include it for now and when we see how difficult this
assignment is we’ll decide whether to drop it or not. Be sure to ask as we get nearer to the due date. Now try using a Tikhonov reconstruction method. Here, instead of choosing a good truncation level you need to choose a good value of the regularization parameter. Again, pick one wavelength and try at least 10 values, starting with very large and very small ones (you can use the filter factor analysis explained in class and your plot of the singular values to figure out what “very large” and “very small” should mean here). Once you find what seems like a reasonable compromise for this parameter, repeat the reconstruction for the other two wavelengths and again discuss all your results.

5. Evaluate your results. What approximations were made that you think may limit the quality of your reconstructions?