ECEG105/ECEU646
Optics for Engineers
Course Notes
Part 4: Apertures, Aberrations
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Advanced Geometric Optics

• Introduction
• Stops, Pupils, and Windows
• f-Number
• Examples
  – Magnifier
  – Microscope
• Aberrations

• Design Process
  – From Concept through Ray Tracing
  – Finalizing the Design
  – Fabrication and Alignment
Some Assumptions We Made

• All lenses are infinite in diameter
  – Every ray from every part of the object reaches the image

• Angles are Small:
  – $\sin(\theta) = \tan(\theta) = \theta$
  – $\cos(\theta) = 1$
What We Have Developed

• Description of an Optical System in terms of Principal Planes, Focal Length, and Indices of Refraction

\[
\frac{n'}{s'} + \frac{n}{s} = P \quad \frac{n}{f} + \frac{n'}{f'} = P \quad \frac{x'}{x} = \frac{s'}{s}
\]

• These equations describe a mapping
  – from object space \((x,y,z)\)
  – to image space \((x',y',z')\)

\(s, s'\) are \(z\) coordinates
Lens Equation as Mapping

• The mapping can be applied to all ranges of z. (not just on the appropriate side of the lens)
• We can consider the whole system or any part.
• The object can be another lens
Stops, Pupils, and Windows (1)

- Intuitive Description
  - Pupil Limits Amount of Light Collected
  - Window Limits What Can Be Seen
Stops, Pupils and Windows (2)

Images in Object Space

- Entrance Pupil
  Limits Cone of Rays from Object

- Entrance Window
  Limits Cone of Rays from Entrance Pupil

Physical Components

- Aperture Stop
  Limits Cone of Rays from Object which Can Pass Through the System

- Field Stop
  Limits Locations of Points in Object which Can Pass Through System

Images in Image Space

- Exit Pupil
  Limits Cone of Rays from Image

- Exit Window
  Limits Cone of Rays From Exit Pupil
Finding the Entrance Pupil

- Find all apertures in object space
  \( L_4' \) is \( L_4 \) seen through \( L_1-L_3 \)

- Entrance Pupil Subtends Smallest Angle from Object

\( L_3' \) is \( L_3 \) seen through \( L_1-L_2 \)
Finding the Entrance Window

- Entrance Window Subtends Smallest Angle from Entrance Pupil
- Aperture Stop is the physical object conjugate to the entrance pupil
- Field Stop is the physical object conjugate to the entrance window
- All other apertures are irrelevant
Field of View

f = 28 mm

f = 55 mm

f = 200 mm

Film = Exit Window
Example: The Telescope

Aperture Stop

Field Stop
The Telescope in Object Space

Secondary

Secondary'

Entrance Pupil

Entrance Window

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The Telescope in Image Space

Primary

Secondary

Primary''

Exit Pupil

Exit Window

Stopped 26 Sep 03
**f-Number & Numerical Aperture**

- **f-Number:** $f\# = \frac{f}{D}$
- **Numerical Aperture:** $NA = \sin \theta$

**Graph**

- $f\# \approx \frac{1}{2f\#}$
- $D$ is Lens Diameter

**Equations**

\[
\sin \theta = \frac{D}{\sqrt{f^2 + \left(\frac{D}{2}\right)^2}} = \frac{1}{2(f\#)\sqrt{(f\#)^2 + \left(\frac{1}{2(f\#)}\right)^2}}
\]

\[
f\# = \frac{\sqrt{1 - (NA)^2}}{2(NA)}
\]
Importance of Aperture

• "Fast" System
  – Low f-number, High NA \((NA \rightarrow 1, f\# \rightarrow 1)\)
  – Good Light Collection (can use short exposure)
  – Small Diffraction Limit \((\lambda/D)\)
  – Propensity for Aberrations \((\sin \theta \neq \theta)\)
    • Corrections may require multiple elements
  – Big Diameter \(\rightarrow\)
    • Big Thickness \(\rightarrow\) Weight, Cost
    • Tight Tolerance over Large Area
The Simple Magnifier

$s < f \quad \text{but} \quad s \approx f$

\[
m = \frac{x'}{x} = -\frac{s'}{s} \approx \frac{-s'}{f}
\]

$s < f$ means $s' < 0$

$s \approx f$ means $s' \rightarrow -\infty$

\[
M_m = \frac{250\text{mm}}{f}
\]
The Simple Magnifier (2)

- Image Size on Retina Determined by $x'/s'$
- No Reason to go beyond $s' = 250 \text{ mm}$

- Magnification Defined as $M_m = \frac{250\text{mm}}{f}$
- No Reason to go beyond $D=10 \text{ mm}$
- $f\# \rightarrow 1$ Means $f=10 \text{ mm}$
- Maximum $M_m = 25$

For the Interested Student: What if $s>f$?
Microscope

- Two-Step Magnification
  - Objective Makes a Real Image
  - Eyepiece Used as a Simple Magnifier
Microscope Objective
Microscope Eyepiece
Microscope Effective Lens

Barrel Length = 160 mm

Effective Lens: $f = -1.6 \text{ mm}$
Microscope Aperture Stop

Analysis in Image Space

Put the Entrance Pupil of your eye at the Exit Pupil of the System, Not at the Eyepiece, because
1) It tickles (and more if it’s a rifle scope)
2) The Pupil begins to act like a window
Microscope Field Stop

Entrance Window

F

F'

Field Stop = Exit Window
Microscope Effective Lens
Apertures Summary

- Object and Image Space
- Locating All the Elements
- Finding the Pupil
  - Computing the Pupil Size and NA or f#
- Finding the Window
  - Computing the Field of View
Aberrations

• Failure of Paraxial Optics Assumptions
  – Ray Optics Based On $\sin(\theta)=\tan(\theta)=\theta$
  – Spherical Waves $\phi=\phi_0+2\pi x^2/\rho \lambda$

• Next Level of Complexity
  – Ray Approach: $\sin(\theta)=\theta+\theta^3/3!$
  – Wave Approach: $\phi=\phi_0+2\pi x^2/\rho \lambda+c\rho^4+...$

• A Further Level of Complexity
  – Ray Tracing
Examples of Aberrations

Paraxial Imaging

\[ \frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{R} \]

\( R = 2, \quad n=1.00, \quad n'=1.50 \)
\( s=10, \quad s'=10 \)

In this example for a ray having height \( h \) at the surface, \( s'(h) < s'(0) \).
Example of Aberrations

Longitudinal Aberration = $\Delta z$

Transverse Aberration = $\Delta x$

Where Exactly is the image?

What is its diameter?
Spherical Aberration
Thin Lens in Air

Definition

\[ L_s = \frac{1}{s'(h)} - \frac{1}{s'(0)} \]

Equation

\[ L_s = \frac{h^2}{8f^3 n(n-1)} \times \left( \frac{n+2}{n-1} q^2 + 4(n+1)pq \right) \]
\[ + (3n+2)(n-1)p^2 + \frac{n^3}{n-1} \]

Coddington Position Factor

\[ p = \frac{s' - s}{s' + s} \]

Coddington Shape Factor

\[ q = \frac{R_2 + R_1}{R_2 - R_1} \]

Minimization

\[ \frac{dL_s}{dq} = 0 \quad q = -\frac{2 \left( n^2 - 1 \right) p}{n + 2} \]
Transverse Spherical Aberration

\[ \Delta x \approx \frac{h \Delta s}{s'(0)} \]

\[ L_s = \frac{1}{s'(h)} - \frac{1}{s'(0)} \approx \frac{\Delta s}{[s'(0)]^2} \]

\[ \Delta x = \frac{h^3 s'(0)}{8f^3} \frac{1}{n(n-1)} \times \left[ \frac{n+2}{n-1} q^2 + 4(n+1)pq \right. \]

\[ + (3n+2)(n-1)p^2 + \frac{n^3}{n-1} \]

Depends on \((1/f\#)^3\)
Evaluating Transverse SA

**Optimization**

\[ \Delta x = \frac{h^3 s'(0)}{8 f^3} \frac{1}{n (n-1)} \times \]

\[
\frac{n+2}{n-1} q^2 + 4(n+1)pq \\
+ (3n+2)(n-1)p^2 + \frac{n^3}{n-1}
\]

\[ q = -\frac{2(n^2-1)p}{n+2} \]

\[ \Delta x_{min} = \frac{h^3 z'_0}{8 f^3} \left[ -\frac{np^2}{n+2} + \frac{n^2}{(n-1)^2} \right] \]

**Diffraction Limit**

Smallest Possible Spot

Proof Later

\[ D. L. = \frac{\lambda}{2h} z'_0 \quad \text{where} \quad z'_0 = f \left( \frac{2p}{p-1} \right) \]

**Ratio ("XDL")**

\[ \frac{\Delta x_{min}}{D. L.} = \frac{h^4}{4\lambda f^3} \left[ -\frac{np^2}{n+2} + \frac{n^2}{(n-1)^2} \right] \]
Coddington Shape Factors

$\begin{align*}
q &= 0 \\
p &= 0 \\
q &= 0
\end{align*}$
Numerical Examples

Beam Size, m

$s=1m, s'=4cm$

$q$, Shape Factor

$p$, Position Factor

$n=2.4$, $n=1.5$

$DL$ at $10\, \mu m$

$DL$ at $1.06\, \mu m$

$500\, nm$

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Phase Description of Aberrations

- Mapping from object space to image space
- Phase changes introduced in pupil plane
  - Different in different parts of plane
  - Can change mapping or blur images
Coordinates for Phase Analysis

Solid Line is phase of a spherical wave toward the image point.
Dotted line is actual phase.
Our goal is to find $\Delta(\rho,\phi,h)$.
Aberration Terms

Taylor’s Series

$$\Delta = a_0 + b_0 h^2 + b_1 \rho^2 + b_2 \rho h \cos \phi + c_0 h^4 + c_1 \rho^4 + c_2 h^2 \rho^2 \cos^2 \phi + c_3 h^2 \rho^2 + c_4 h^3 \rho \cos \phi + c_5 h \rho^3 \cos^3 \phi + \ldots$$

Physical Significance

Zero-Order Terms

$$\Delta = a_0 + \ldots$$

No Aberrations

Odd Terms involve tilt, not considered here.
Second Order

$$\Delta = \ldots + b_1 \rho^2 + \ldots$$

Image Position Terms:

The spherical wave is approximated by a second-order phase term, so this error is simply a change in focal length.
Fourth Order (1)

\[ \Delta = \ldots + c_1 \rho^4 + \ldots \]

Spherical Aberration

\[ \rho^2 \text{ is focus: depends on } h^2 \text{ and } h^2 \cos \phi \]

\[ \Delta = \ldots + c_2 h^2 \rho^2 \cos^2 \phi + c_3 h^2 \rho^2 + \ldots \]

Astigmatism and Field Curvature

Sample Images

At T

At S

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Fourth Order (2)

\[ \Delta = \ldots + c_4 h^3 \rho \cos \phi + \ldots \]

\( \rho \cos \phi \) is Tilt: Depends on \( h^3 \)

\[ \Delta = \ldots + c_5 h \rho^3 \cos^3 \phi + \ldots \]

Coma

Barrel Distortion

Pincushion Distortion

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Optical Design Process

- Concept
  - Thin Lens Equation
- Intermediate Stage
  - Thick Lenses
    - Thickness, Radii/Curvature
  - Multiple Elements
- Principal Planes
- Stops and Pupils

- Configuration
  - Folding
- Detailed Design
  - Exact Ray Tracing
- Selection of Vendors
- Final Design
- Fabrication and Test
- Alignment
Ray Tracing Fundamentals

Generate Rays
Specify Baseline System
   Distance, Radius,
   Index, Diameter, etc.
Iterate on Ray
   Iterate on Surface
      Find Ray–Surface Intersection
      Find Refracted Ray
Close

Initial Layout
   Bundle of Rays
      Chief, Marginal Rays
      Others
Spot Diagram
   Figure of Merit
      Aberrations
      Enclosed Power
Standard Deviation
If One Element Doesn’t Work...

Add Another Lens

“Let George Do It”

Different Index?
Smaller angles with higher index. Thus germanium is better than ZnSe in IR. Not much hope in the visible.

Aspherics
Aberrations Summary

• Origin of Aberrations

• On-Axis Aberrations
  – Change of Focus
  – Spherical Aberration

• Off-Axis Aberrations
  – Additional Blurring Effects
  – Distorting Effects
Summary of Concepts So Far

• Paraxial Optics with Thin Lenses
• Thick Lenses (Principal Planes)
• Apertures: Pupils and Windows
• Aberration Correction
  – Analytical
  – Ray Tracing
• What’s Missing? Wave Optics