ECEG105/ECEU646 Optics for Engineers Course Notes
Part 4: Apertures, Aberrations

Prof. Charles A. DiMarzio

Northeastern University

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Advanced Geometric Optics

- Introduction
- Stops, Pupils, and Windows
- f-Number
- Examples
  - Magnifier
  - Microscope
- Aberrations
- Design Process
  - From Concept through Ray Tracing
  - Finalizing the Design
  - Fabrication and Alignment

Some Assumptions We Made

- All lenses are infinite in diameter
  - Every ray from every part of the object reaches the image
- Angles are Small:
  - $sin(\theta)=tan(\theta)=\theta$
  - $cos(\theta)=1$

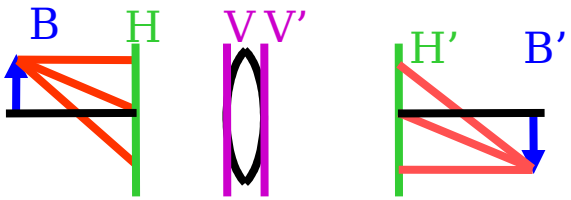
What We Have Developed

- Description of an Optical System in terms of Principal Planes, Focal Length, and Indices of Refraction

$$\frac{n'}{s'} + \frac{n}{s} = P \qquad \frac{n}{f} + \frac{n'}{f'} = P \qquad \frac{x'}{x} = \frac{s'}{s}$$

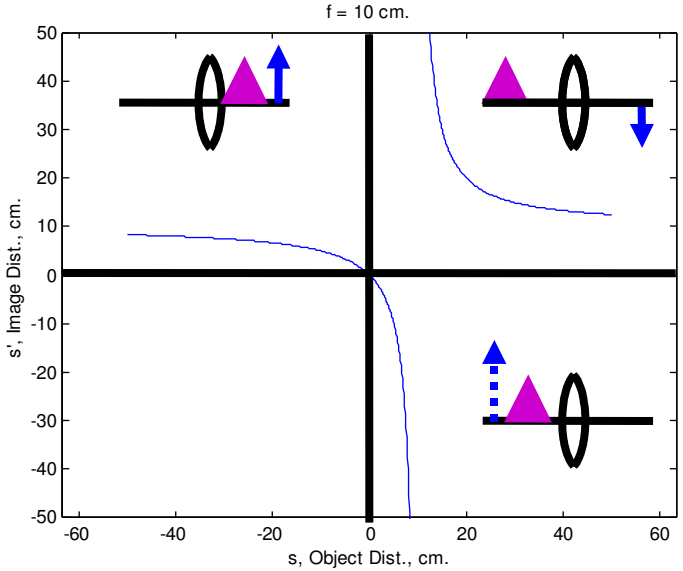
- These equations describe a mapping
  - from object space *(x,y,z)*
  - to image space *(x',y',z')*

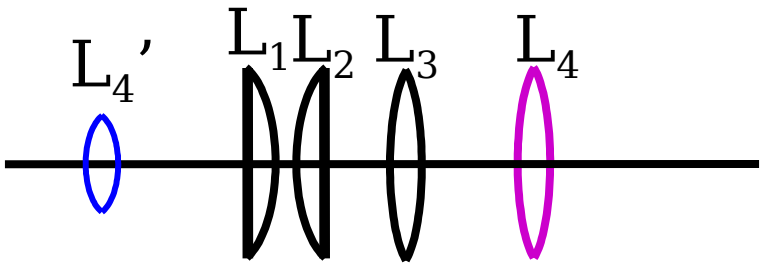
  s, s' are *z* coordinates



Lens Equation as Mapping

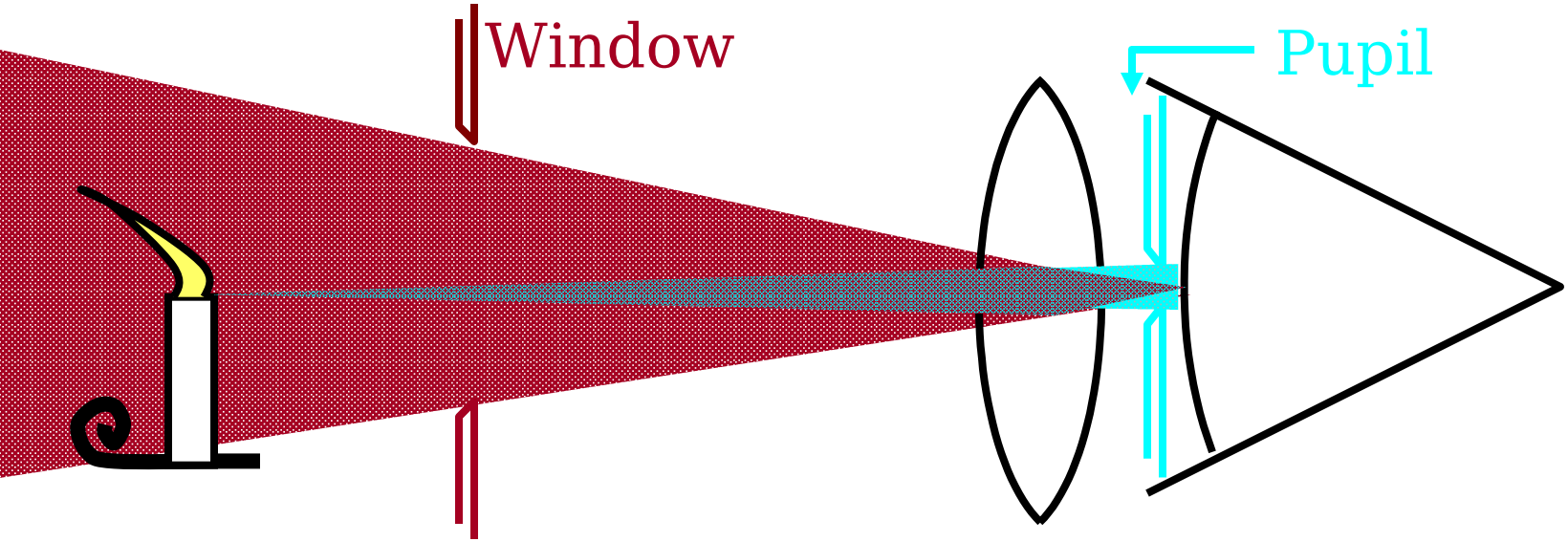
- The mapping can be applied to all ranges of z. (not just on the appropriate side of the lens)
- We can consider the whole system or any part.
- The object can be another lens





Stops, Pupils, and Windows (1)

- Intuitive Description
  - Pupil Limits Amount of Light Collected
  - Window Limits What Can Be Seen



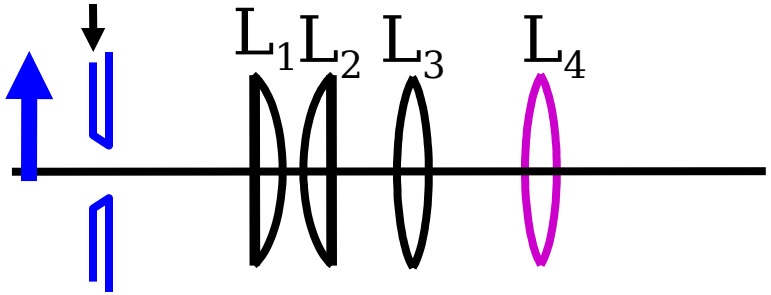
Stops, Pupils and Windows (2)

Images in Object Space	Physical Components	Images in Image Space
Entrance Pupil Limits Cone of Rays from Object	*Aperture Stop* Limits Cone of Rays from Object which Can Pass Through the System	*Exit Pupil* Limits Cone of Rays from Image
Entrance Window Limits Cone of Rays From Entrance Pupil	*Field Stop* Limits Locations of Points in Object which Can Pass Through System	*Exit Window* Limits Cone of Rays From Exit Pupil

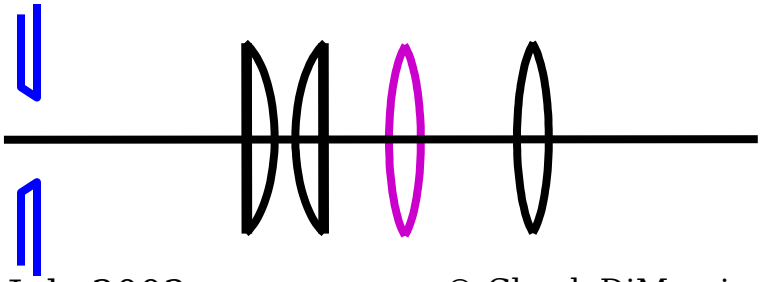
Finding the Entrance Pupil

- Find all apertures in object space

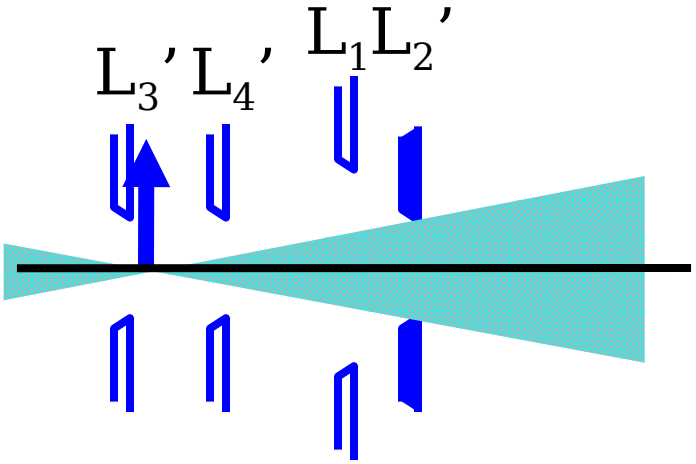
L_4' is L_4 seen through L_1-L_3



L_3' is L_3 seen through L_1-L_2

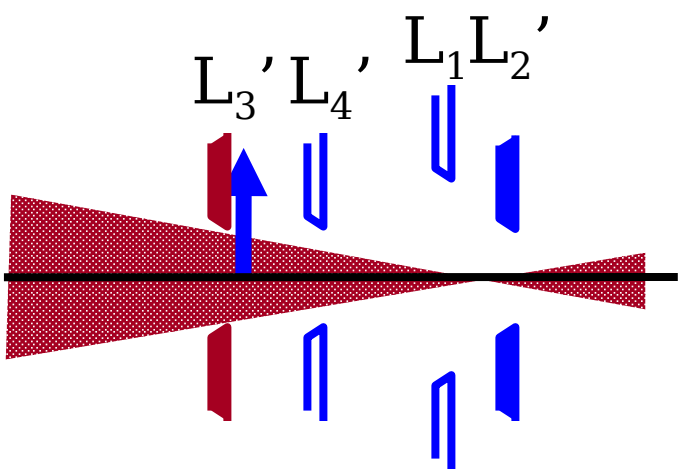


- Entrance Pupil Subtends Smallest Angle **from Object**



Finding the Entrance Window

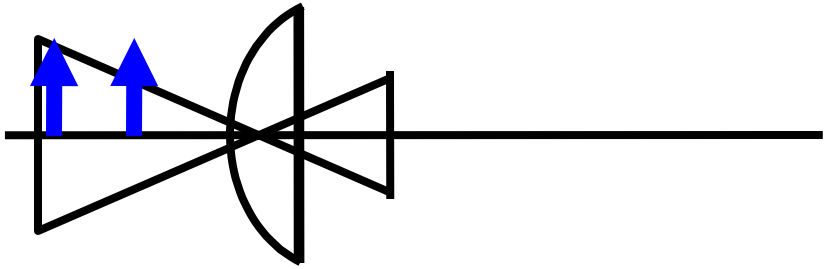
- Entrance Window Subtends Smallest Angle from Entrance Pupil



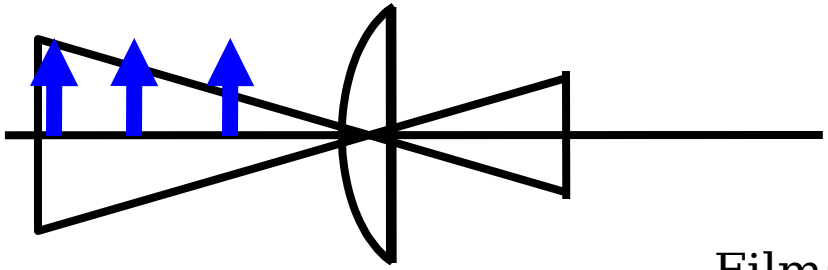
- Aperture Stop is the physical object conjugate to the entrance pupil
- Field Stop is the physical object conjugate to the entrance window
- All other apertures are irrelevant

Field of View

f=28 mm

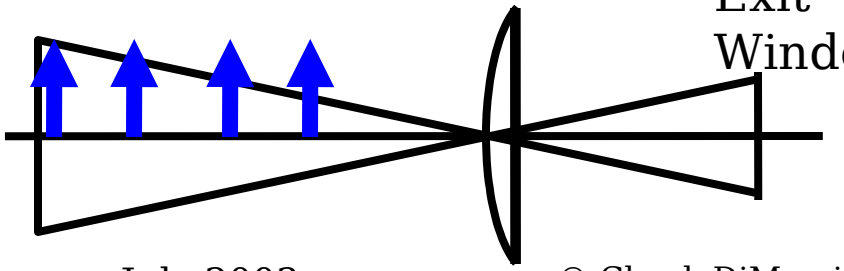


f=55 mm

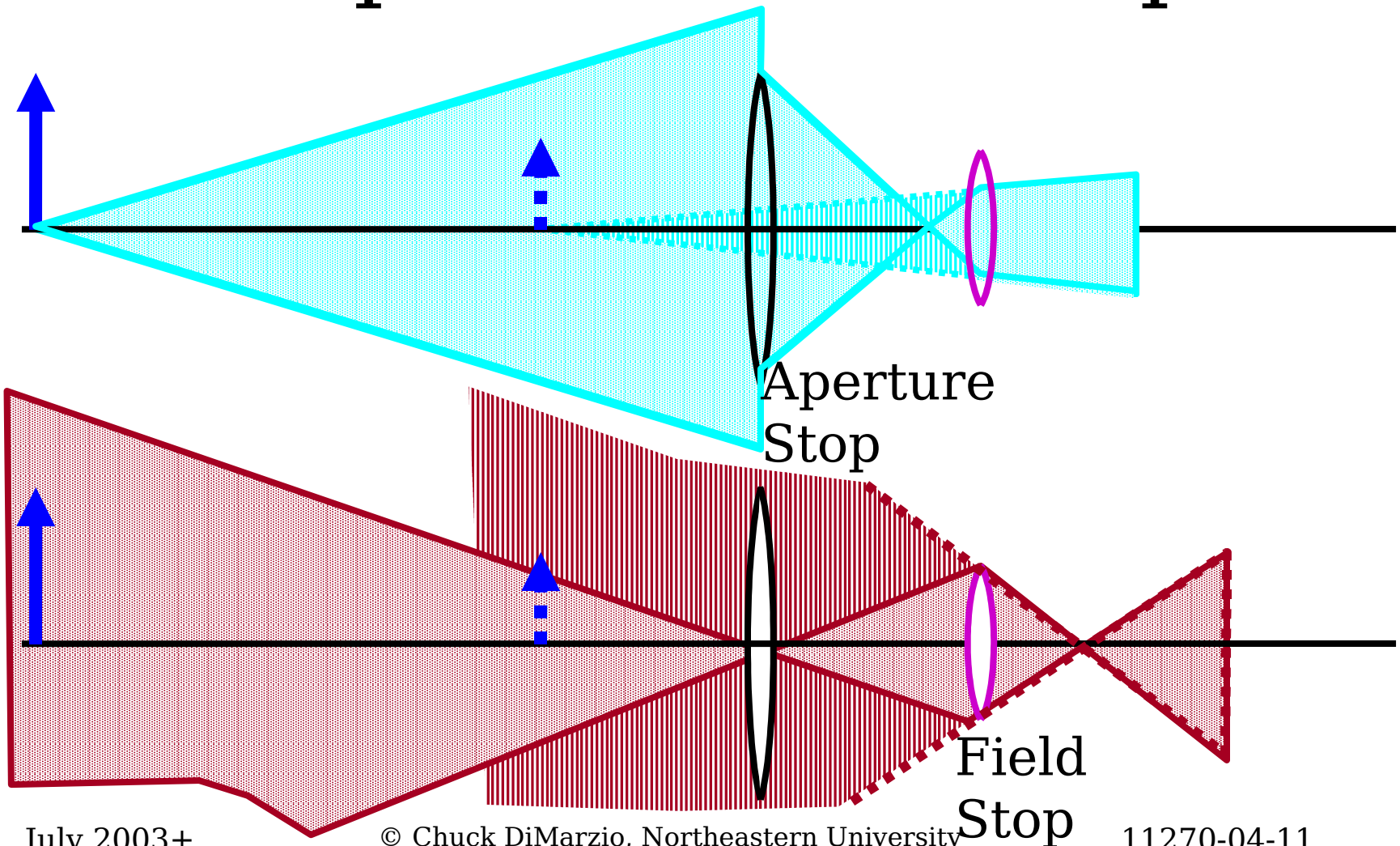


f=200 mm

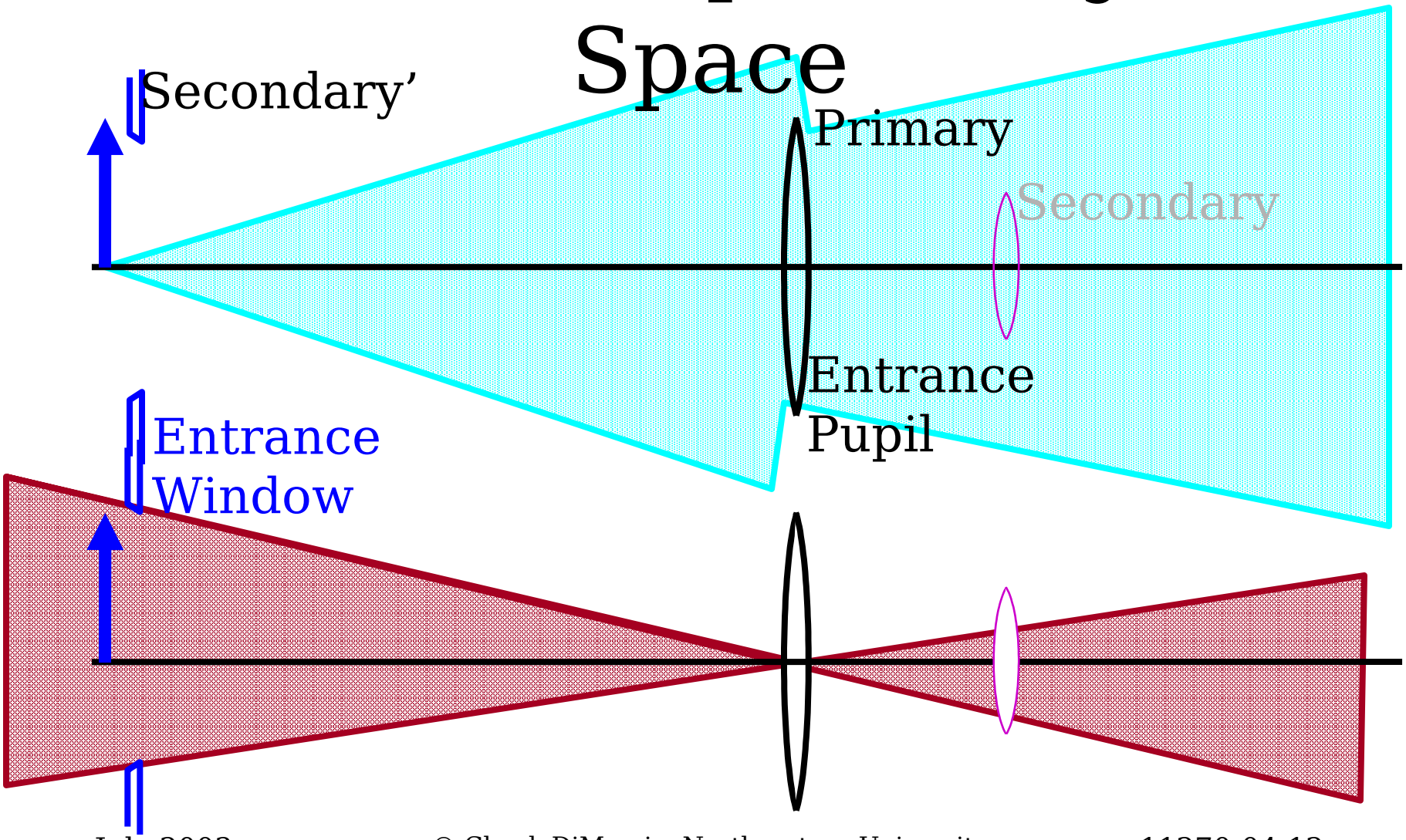
Film=
Exit
Window



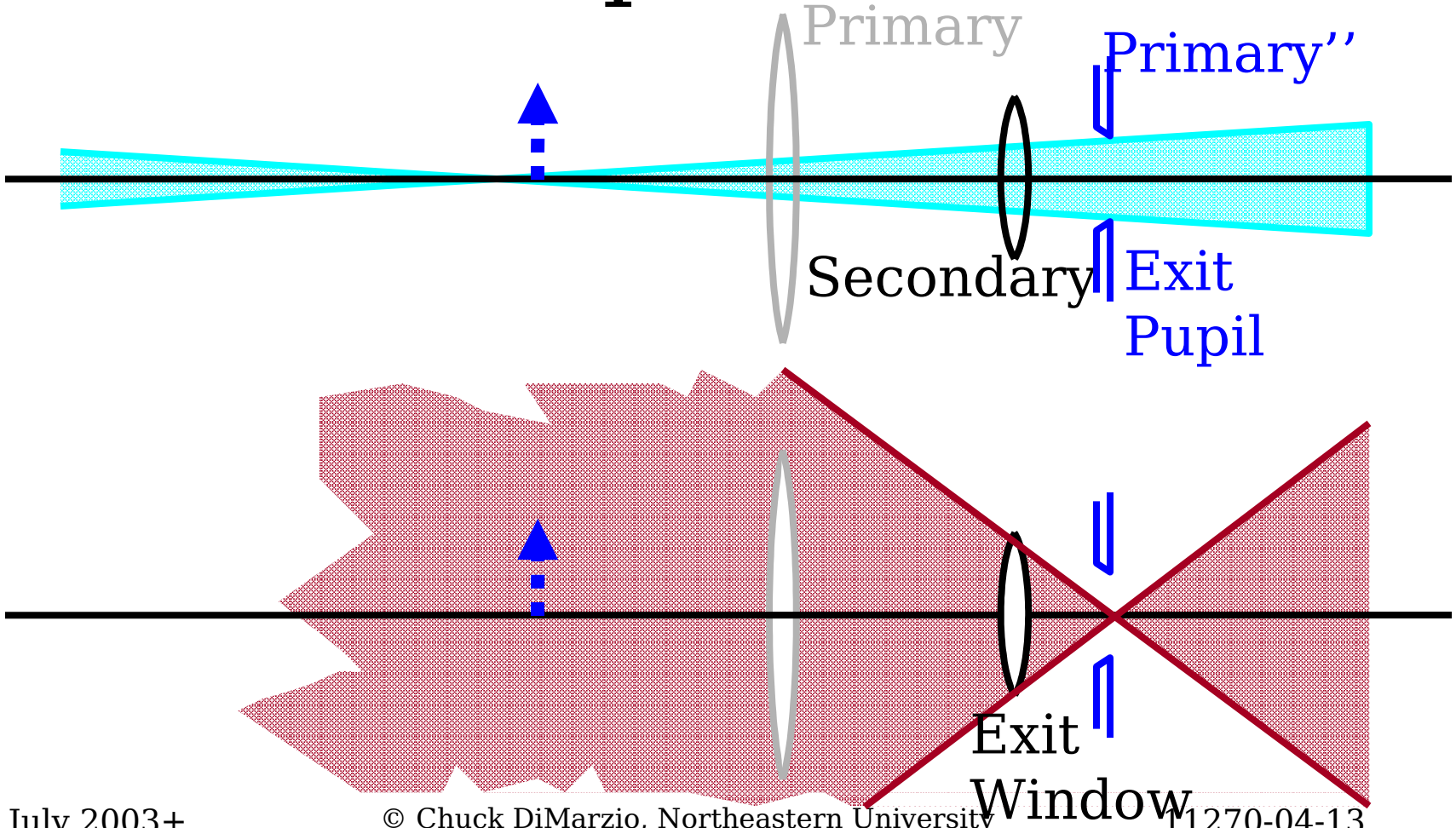
Example: The Telescope



The Telescope in Object Space



The Telescope in Image Space



Stopped 26 Sep 03

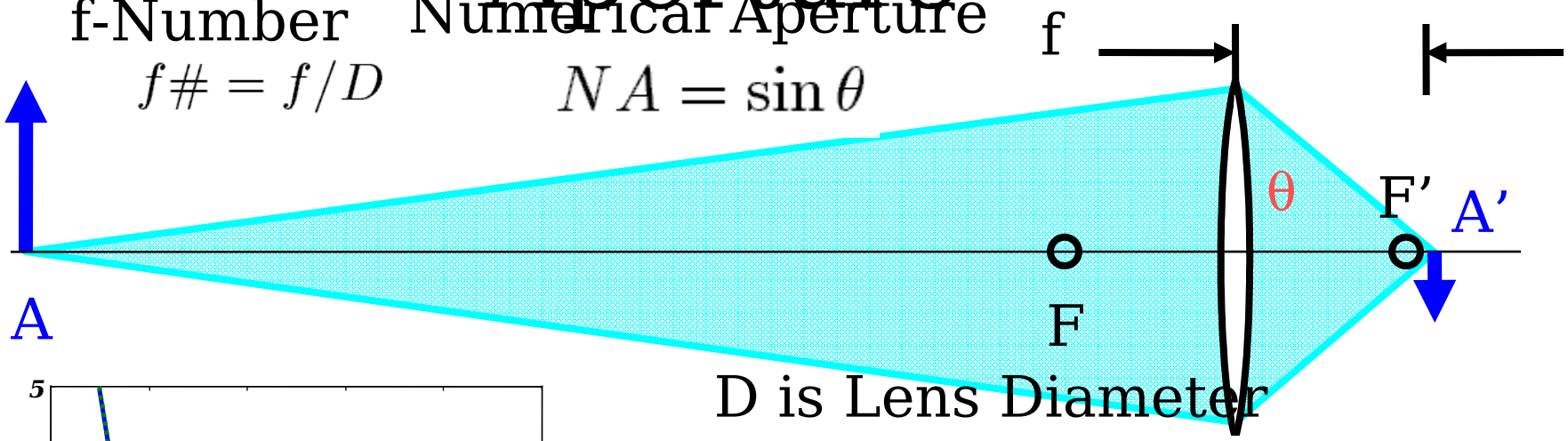
f-Number & Numerical Aperture

f-Number

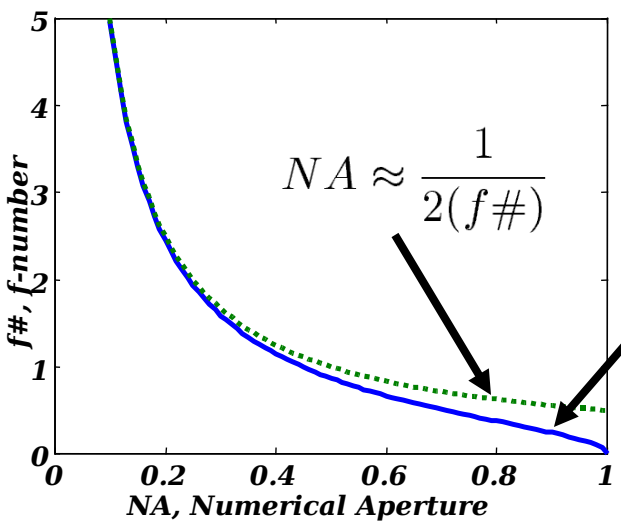
$$f\# = f/D$$

Numerical Aperture

$$NA = \sin\theta$$



D is Lens Diameter



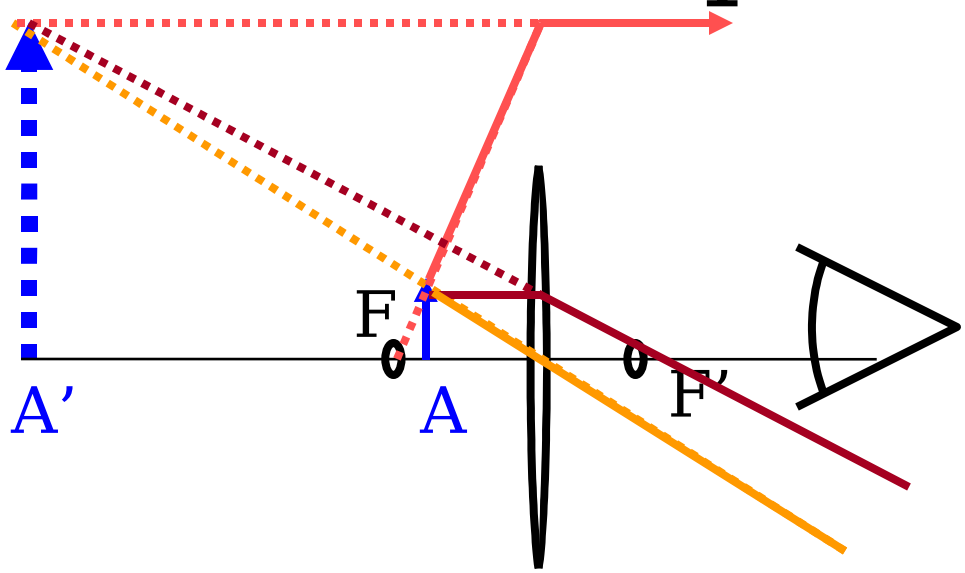
$$\sin\theta = \frac{D}{2\sqrt{f^2 + \left(\frac{D}{2}\right)^2}} = \frac{1}{2(f\#)\sqrt{(f\#)^2 + \left(\frac{1}{2(f\#)}\right)^2}}$$

$$f\# = \frac{\sqrt{1-(NA)^2}}{2(NA)}$$

Importance of Aperture

- ``Fast'' System
  - Low f-number, High NA *(NA→1, f# →1)*
  - Good Light Collection (can use short exposure)
  - Small Diffraction Limit *(λ/D)*
  - Propensity for Aberrations *(*sin $\theta \neq \theta$*)*
    - Corrections may require multiple elements
  - Big Diameter →
    - Big Thickness → Weight, Cost
    - Tight Tolerance over Large Area

The Simple Magnifier



$$s < f \qquad \text{but} \qquad s \approx f$$

$$m = \frac{x'}{x} = -\frac{s'}{s} \approx \frac{-s'}{f}$$

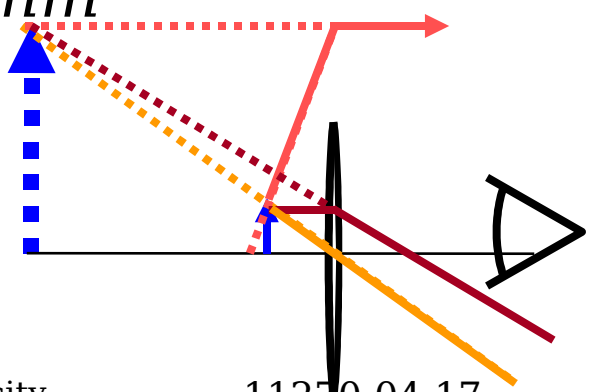
$$s < f \quad \text{means} \quad s' < 0$$

$$s \approx f \quad \text{means} \quad s' \to -\infty$$

$$M_m = \frac{250mm}{f}$$

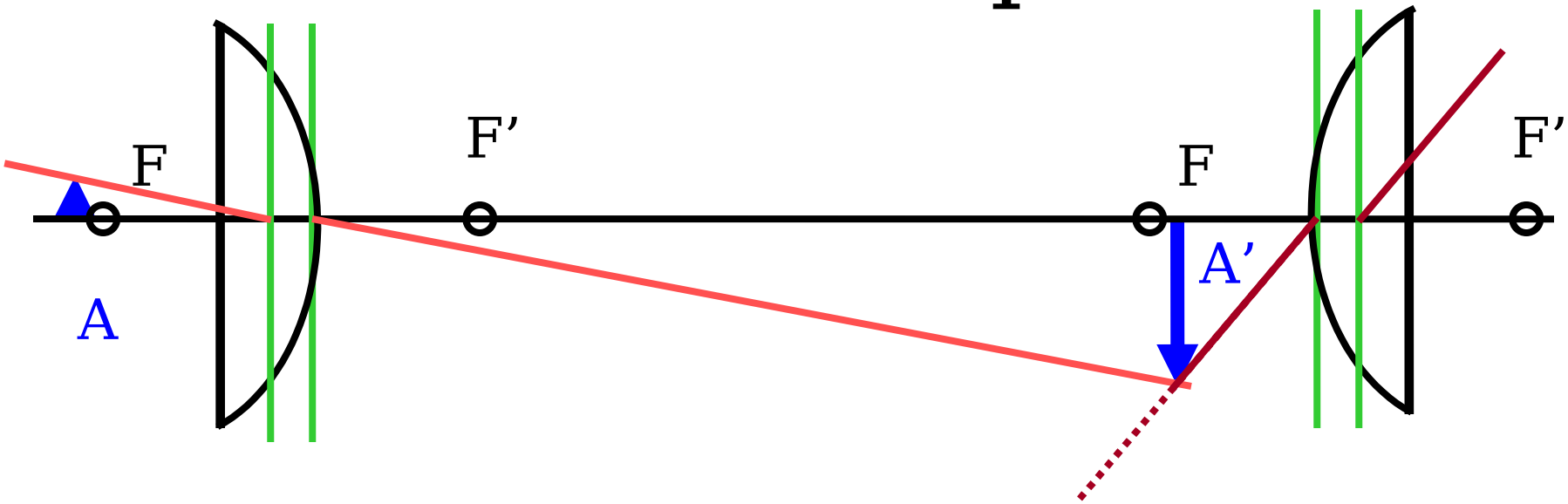
The Simple Magnifier (2)

- Image Size on Retina Determined by x'/s'
- No Reason to go beyond $s' = 250\ mm$
- Magnification Defined as $$M_m = \frac{250mm}{f}$$
- No Reason to go beyond $D=10\ mm$
- $f\# \rightarrow 1$ Means $f=10\ mm$
- *Maximum* $M_m=25$



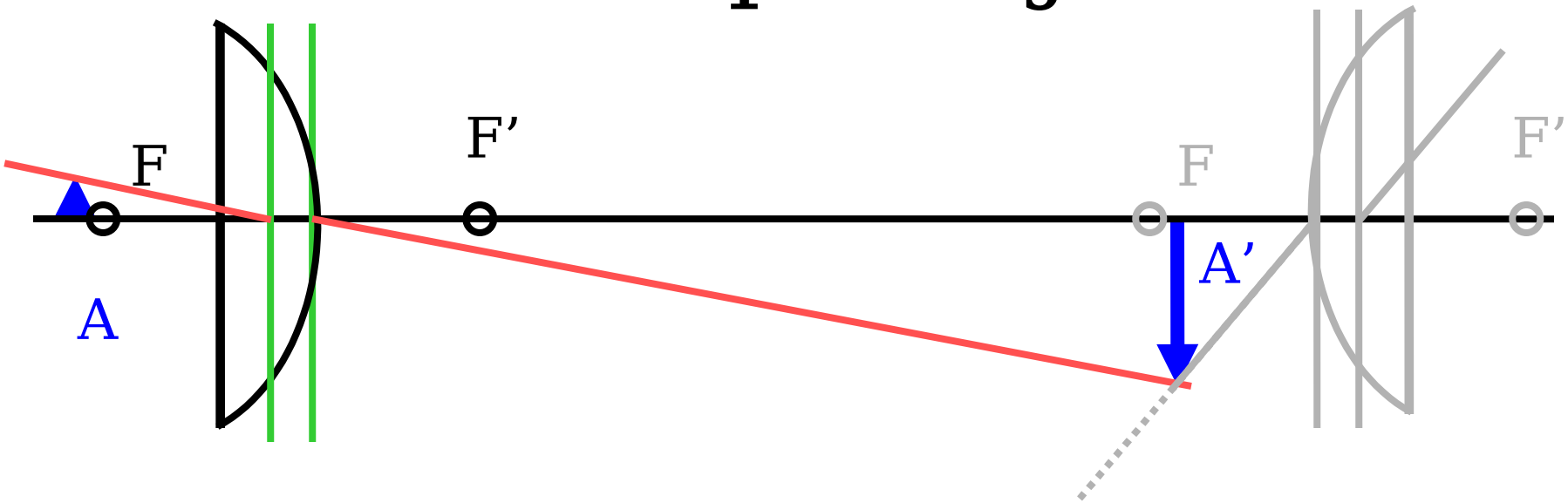
For the Interested Student: What if $s>f$?

Microscope

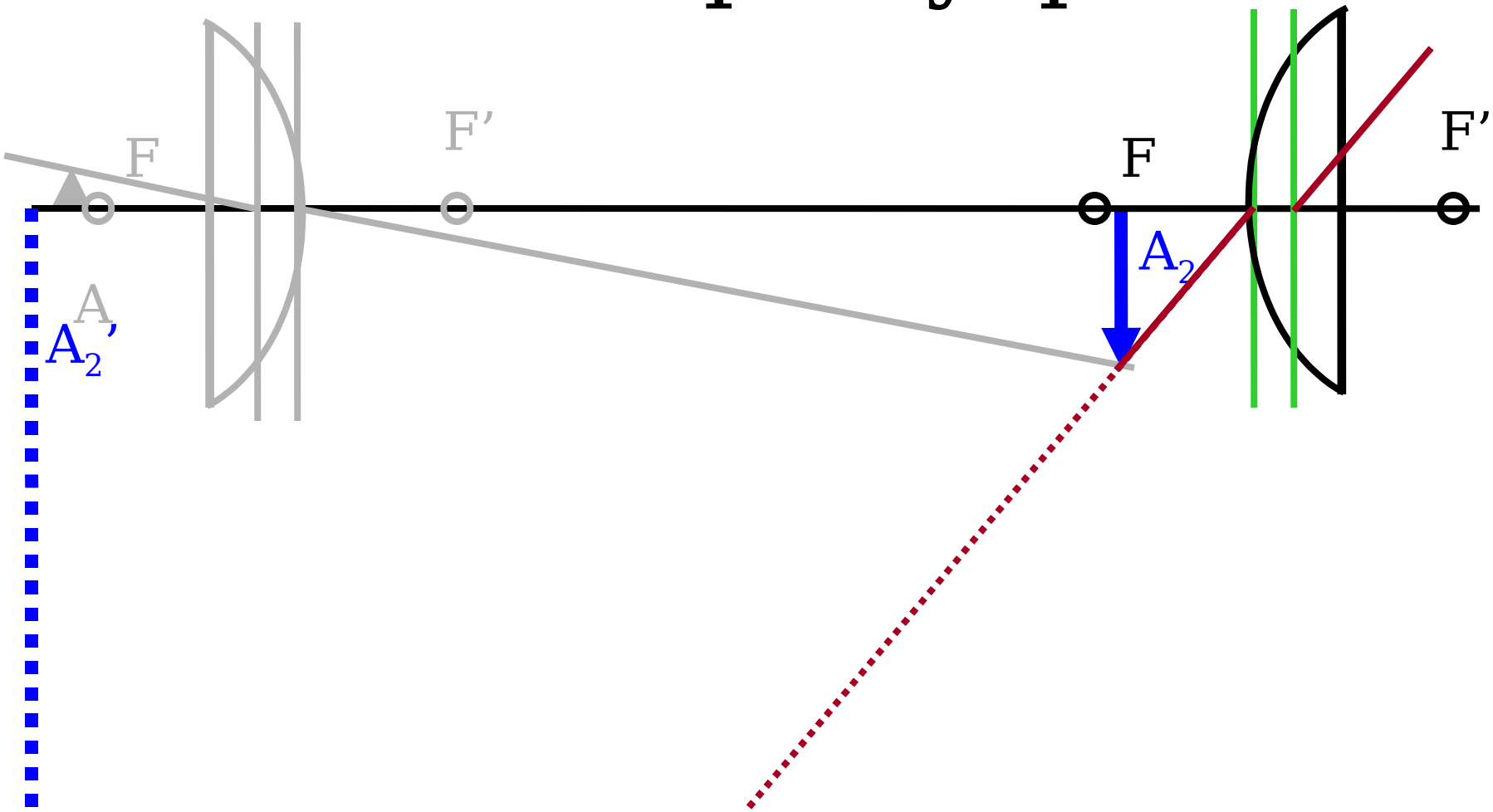


- Two-Step Magnification
  - Objective Makes a Real Image
  - Eyepiece Used as a Simple Magnifier

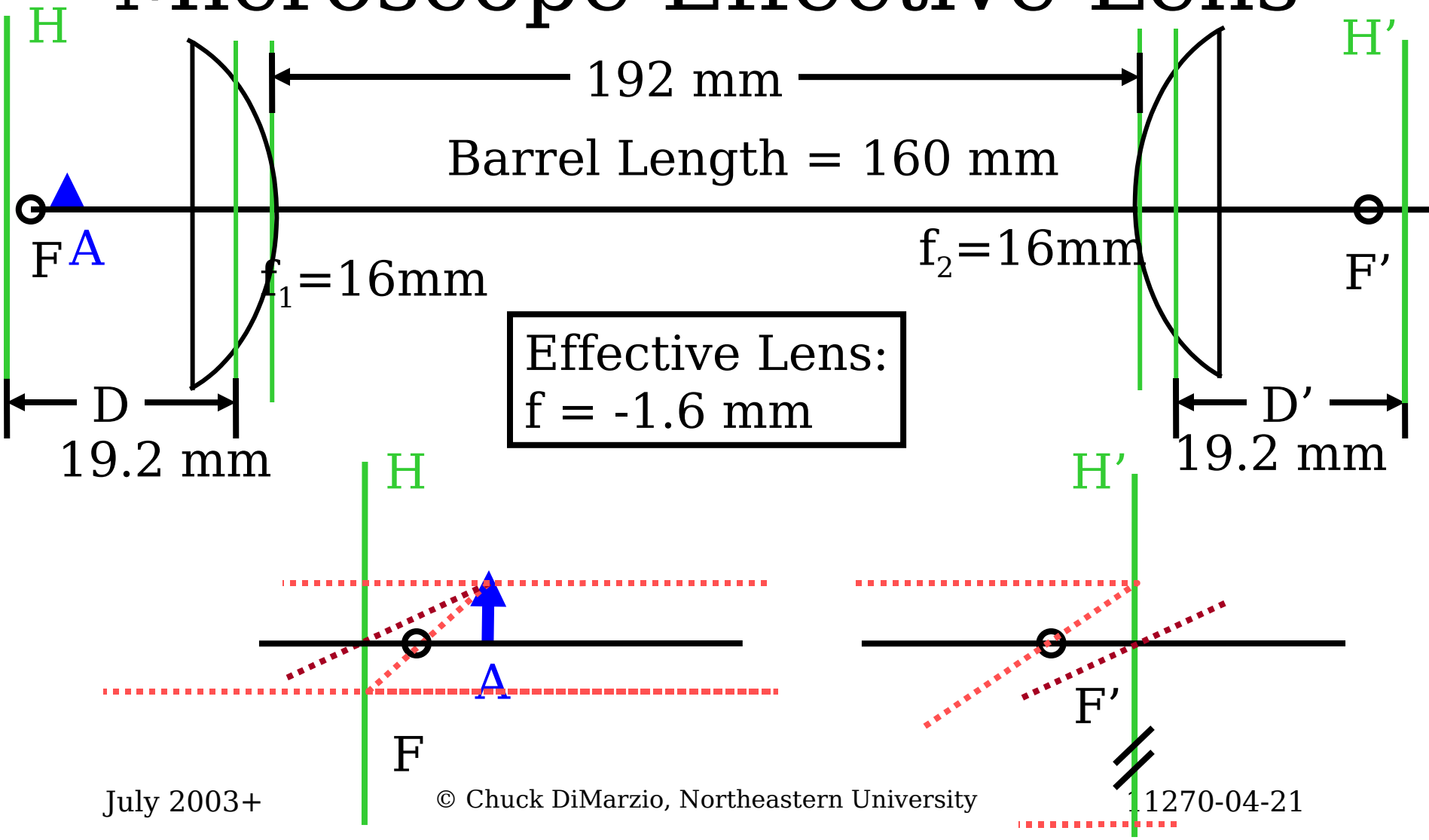
Microscope Objective



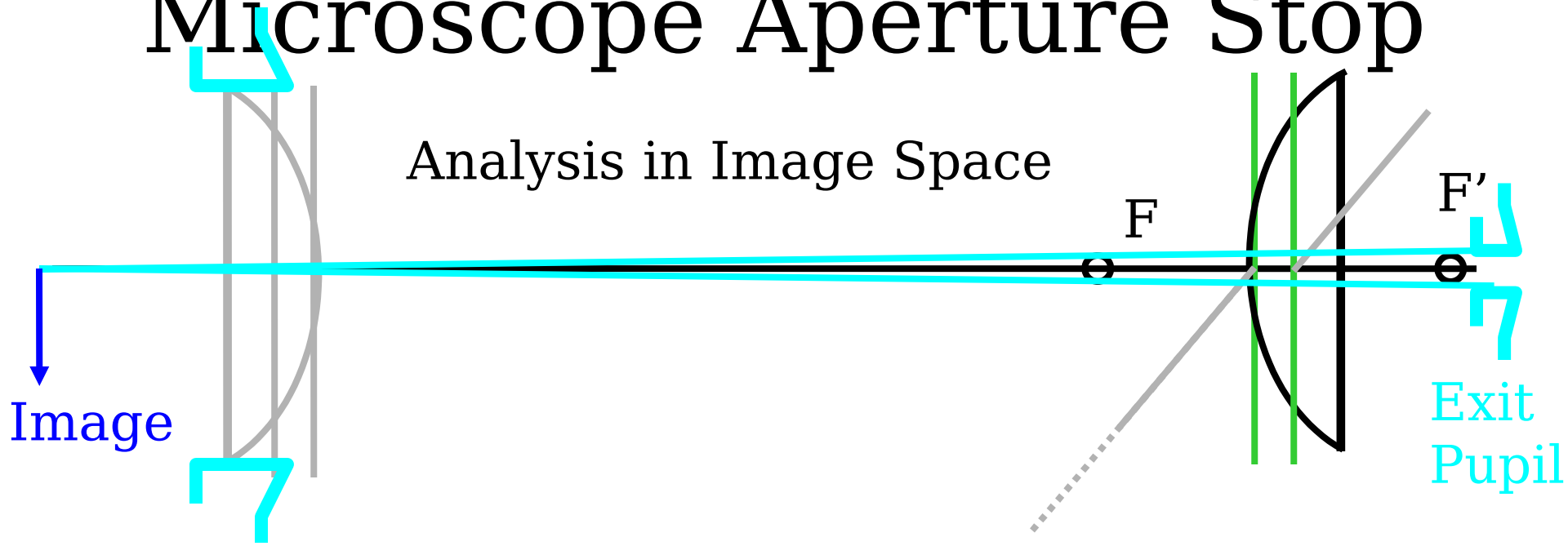
Microscope Eyepiece



Microscope Effective Lens



Microscope Aperture Stop



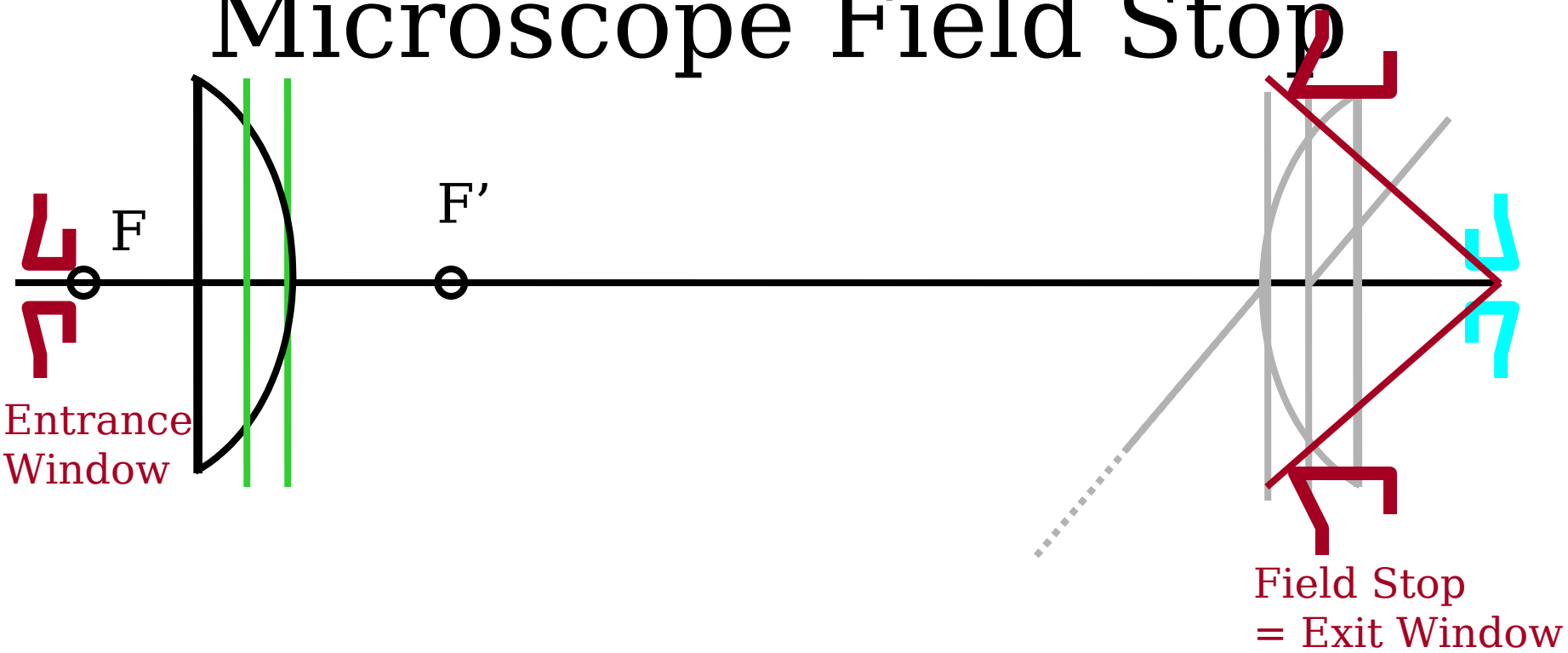
Aperture
Stop
=Entrance
Pupil

Put the Entrance Pupil of your eye at the Exit Pupil of the System, Not at the Eyepiece, because

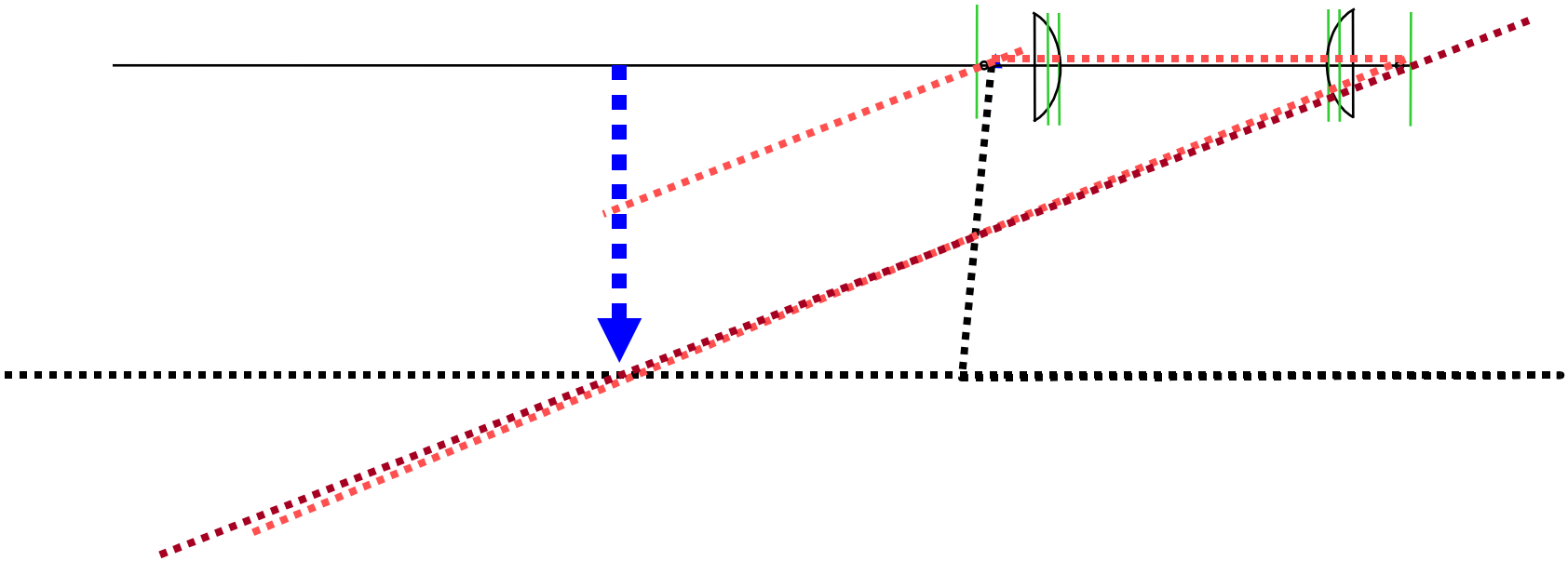
1) It tickles (and more if it's a rifle scope)

2) The Pupil begins to act like a window

Microscope Field Stop



Microscope Effective Lens



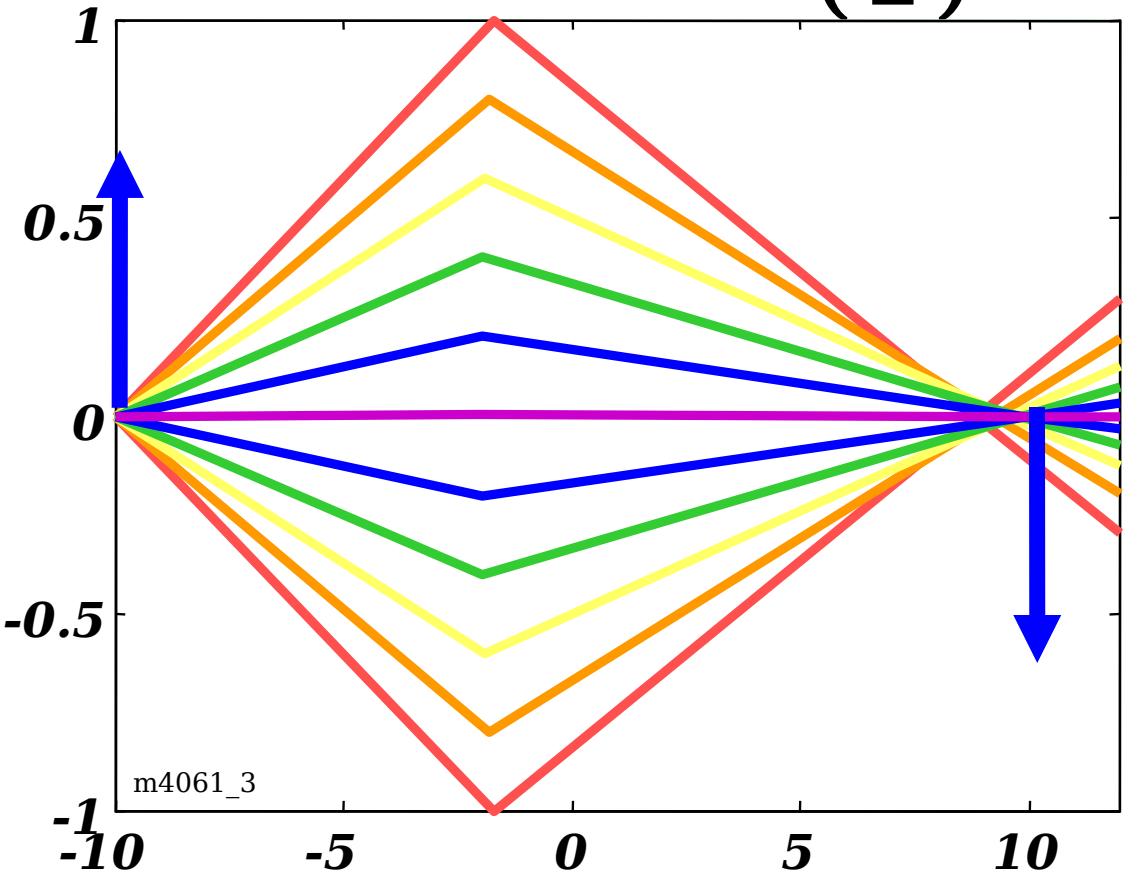
Apertures Summary

- Object and Image Space
- Locating All the Elements
- Finding the Pupil
  - Computing the Pupil Size and NA or f#
- Finding the Window
  - Computing the Field of View

Aberrations

- Failure of Paraxial Optics Assumptions
  - Ray Optics Based On $sin(\theta)=tan(\theta)=\theta$
  - Spherical Waves $\phi=\phi_0+2\pi x^2/\rho\lambda$
- Next Level of Complexity
  - Ray Approach: $sin(\theta)=\theta+\theta^3/3!$
  - Wave Approach: $\phi=\phi_0+2\pi x^2/\rho\lambda+c\rho^4+...$
- A Further Level of Complexity
  - Ray Tracing

Examples of Aberrations (1)



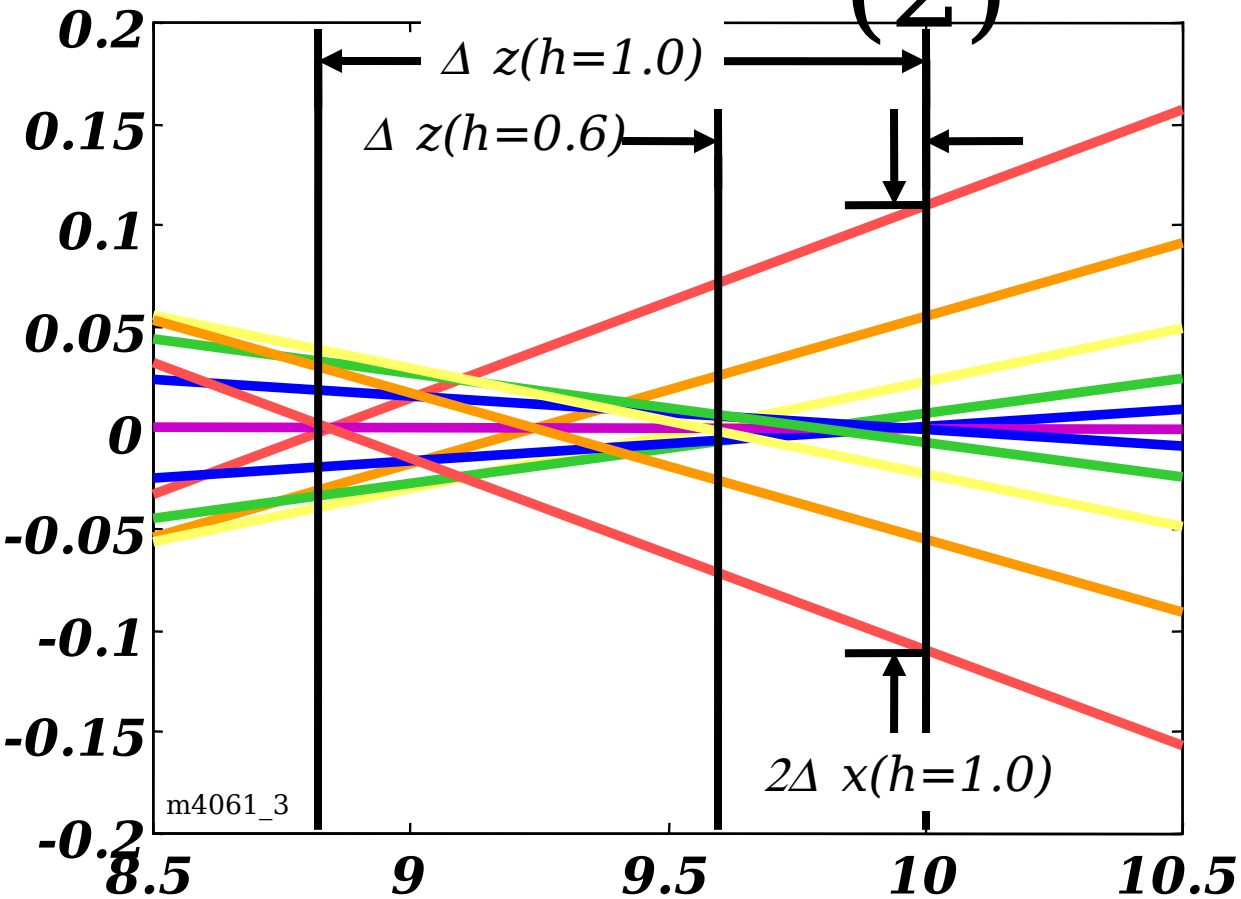
Paraxial Imaging

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{R}$$

R = 2,
n=1.00, n'=1.50
s=10, s'=10

In this example for a ray having height *h* at the surface, *s'(h)<s'(0).*

Example of Aberrations (2)



Longitudinal Aberration = Δz

Transverse Aberration = Δx

Where Exactly is the image?

What is its diameter?

Spherical Aberration

Thin Lens in Air

Definition

$$L_s = \frac{1}{s'(h)} - \frac{1}{s'(0)}$$

Equation

$$L_s = \frac{h^2}{8f^3}\frac{1}{n\,(n-1)}\times$$

$$\left(\frac{n+2}{n-1}q^2 + 4\,(n+1)\,pq\right.$$

$$\left. + (3n+2)\,(n-1)\,p^2 + \frac{n^3}{n-1}\right)$$

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Coddington Position Factor

$$p = \frac{s'-s}{s'+s}$$

Coddington Shape Factor

$$q = \frac{R_2+R_1}{R_2-R_1}$$

Minimization

$$\frac{dL_s}{dq} = 0 \qquad q = -\frac{2\left(n^2-1\right)p}{n+2}$$

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Transverse Spherical Aberration

$$\Delta x \approx \frac{h\Delta s}{s'(0)}$$

$$L_s = \frac{1}{s'(h)} - \frac{1}{s'(0)} \approx \frac{\Delta s}{[s'(0)]^2}$$

$$\Delta x = \frac{h^3 s'(0)}{8f^3}\frac{1}{n(n-1)}\times$$

$$\left[\frac{n+2}{n-1}q^2 + 4(n+1)pq\right.$$

$$\left. + (3n+2)(n-1)p^2 + \frac{n^3}{n-1}\right]$$

Depends on $(1/f\#)^3$

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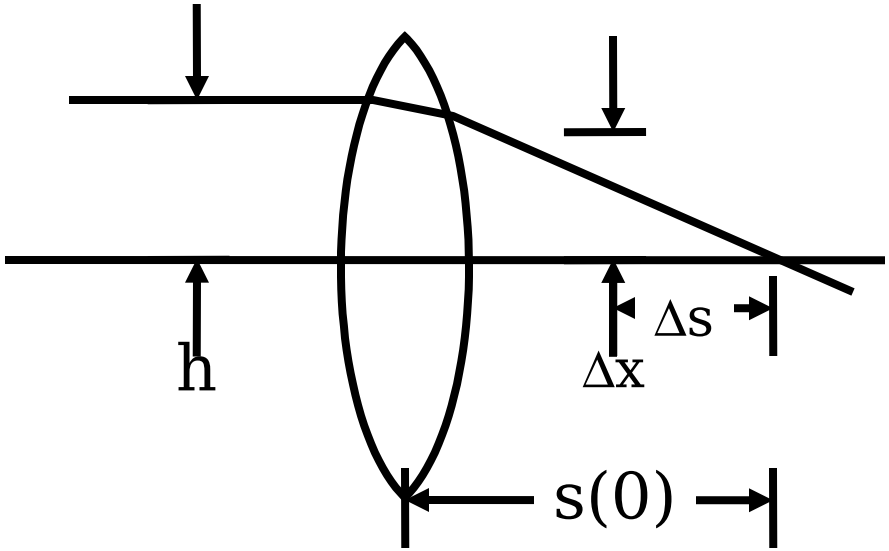

Evaluating Transverse SA

Optimization

$$\Delta x = \frac{h^3 s'(0)}{8f^3}\frac{1}{n(n-1)}\times$$

$$\left[\frac{n+2}{n-1}q^2 + 4(n+1)pq\right.$$

$$\left. + (3n+2)(n-1)p^2 + \frac{n^3}{n-1}\right]$$

$$q = -\frac{2(n^2-1)p}{n+2}$$

$$\Delta x_{min} = \frac{h^3 z_0'}{8f^3}\left[\frac{-np^2}{n+2} + \frac{n^2}{(n-1)^2}\right]$$

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Diffraction Limit

Smallest Possible Spot

Proof Later

$$\text{D. L.} = \frac{\lambda}{2h}z_0' \qquad \text{where} \qquad z_0' = f\left(\frac{2p}{p-1}\right)$$

Ratio ("XDL")

$$\frac{\Delta x_{min}}{\text{D. L.}} = \frac{h^4}{4\lambda f^3}\left[\frac{-np^2}{n+2} + \frac{n^2}{(n-1)^2}\right]$$

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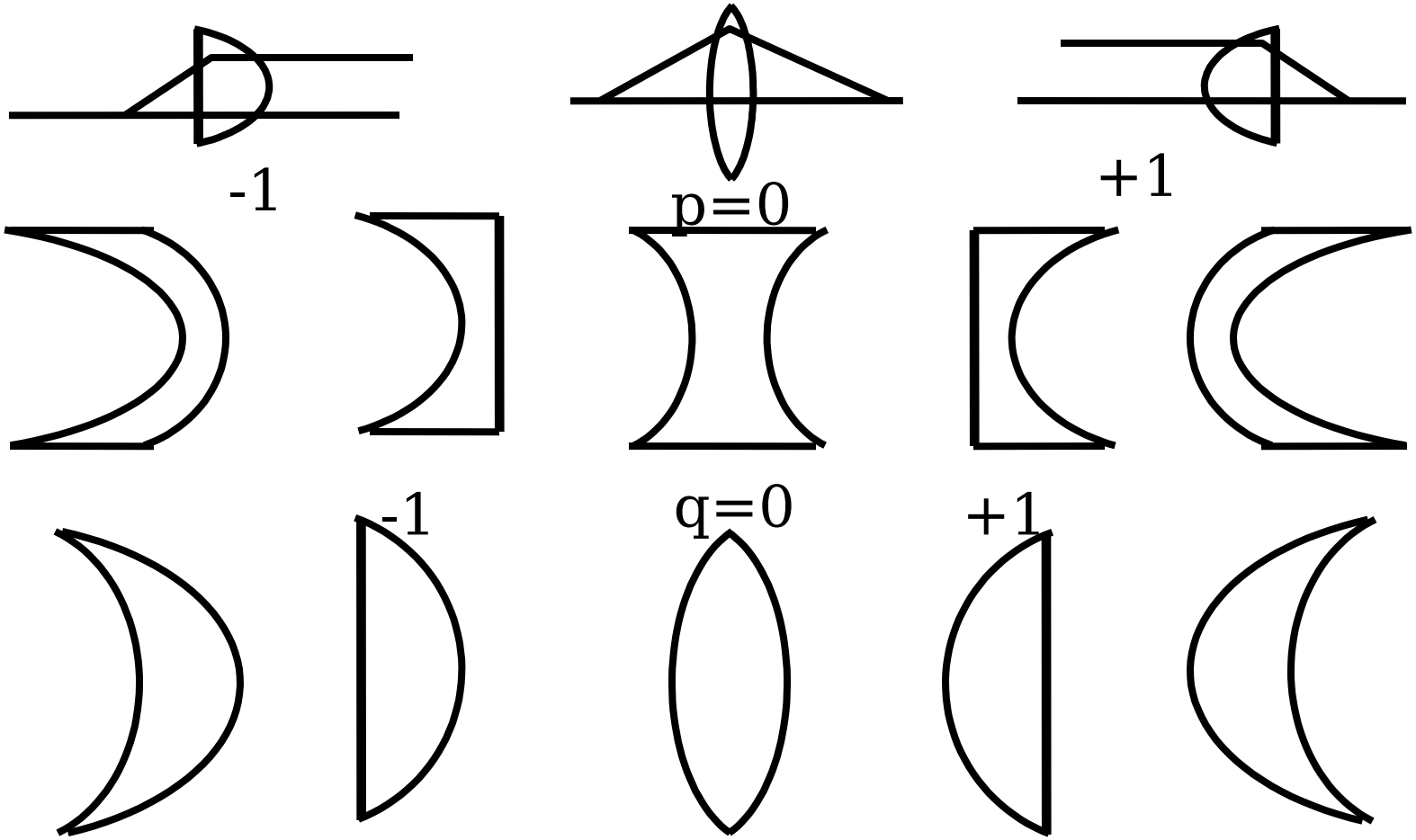
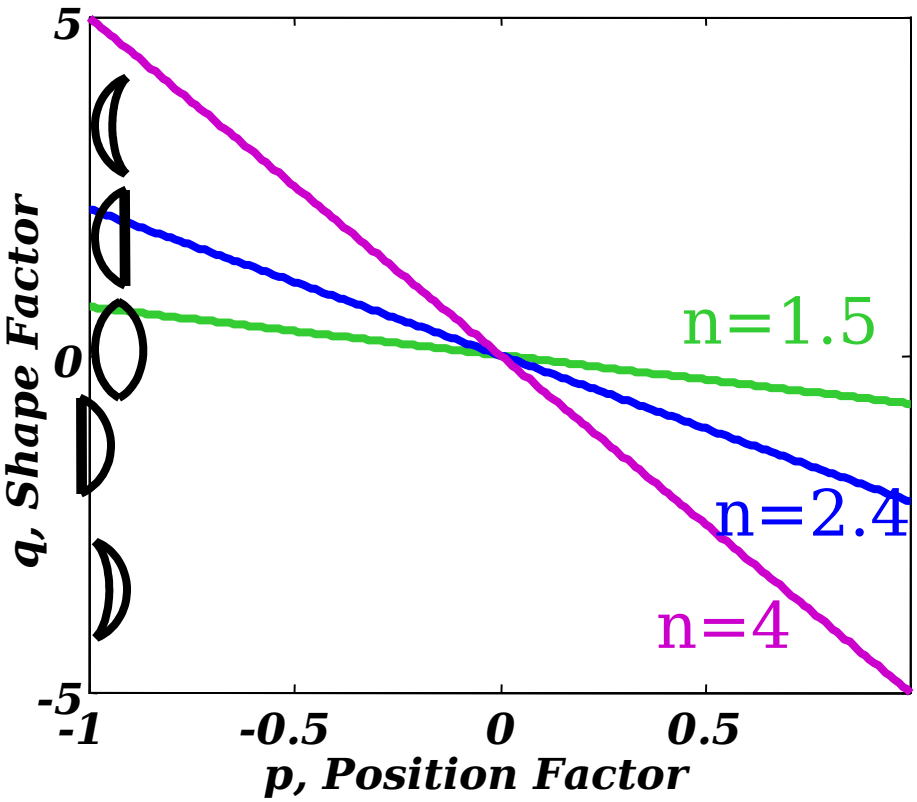
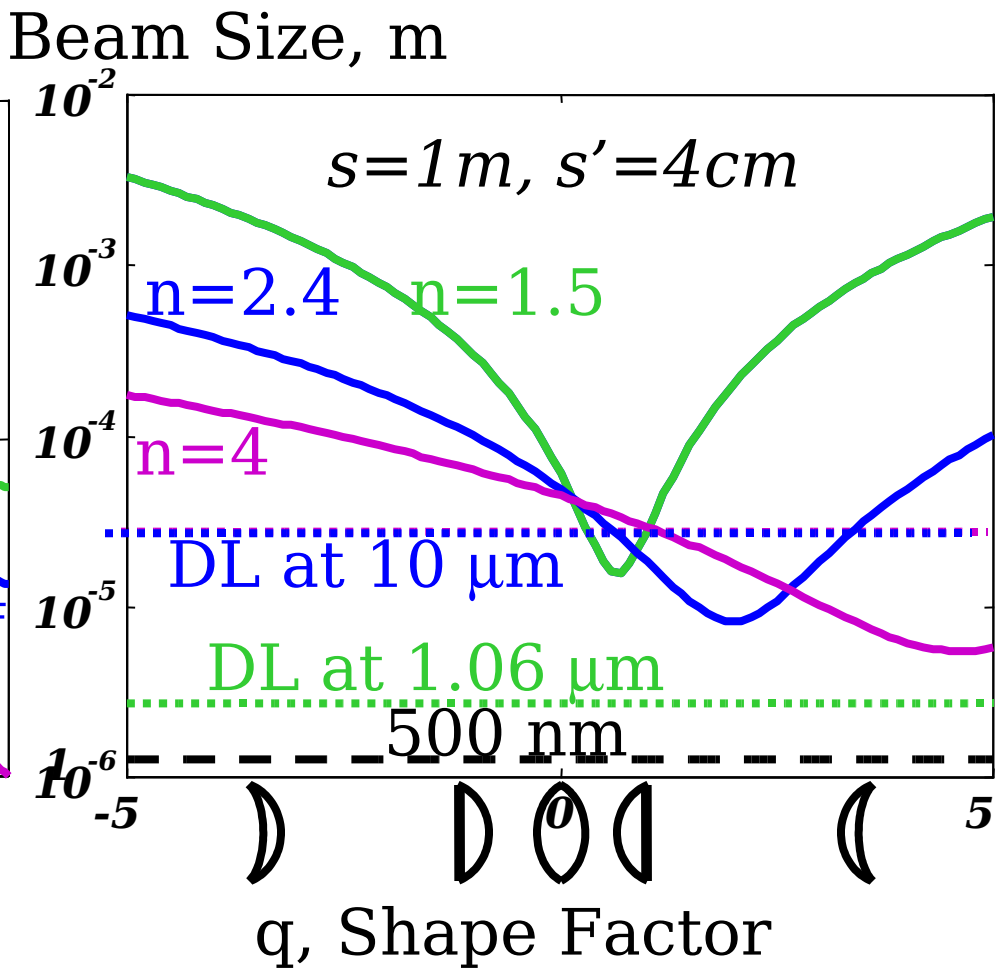
Coddington Shape Factors

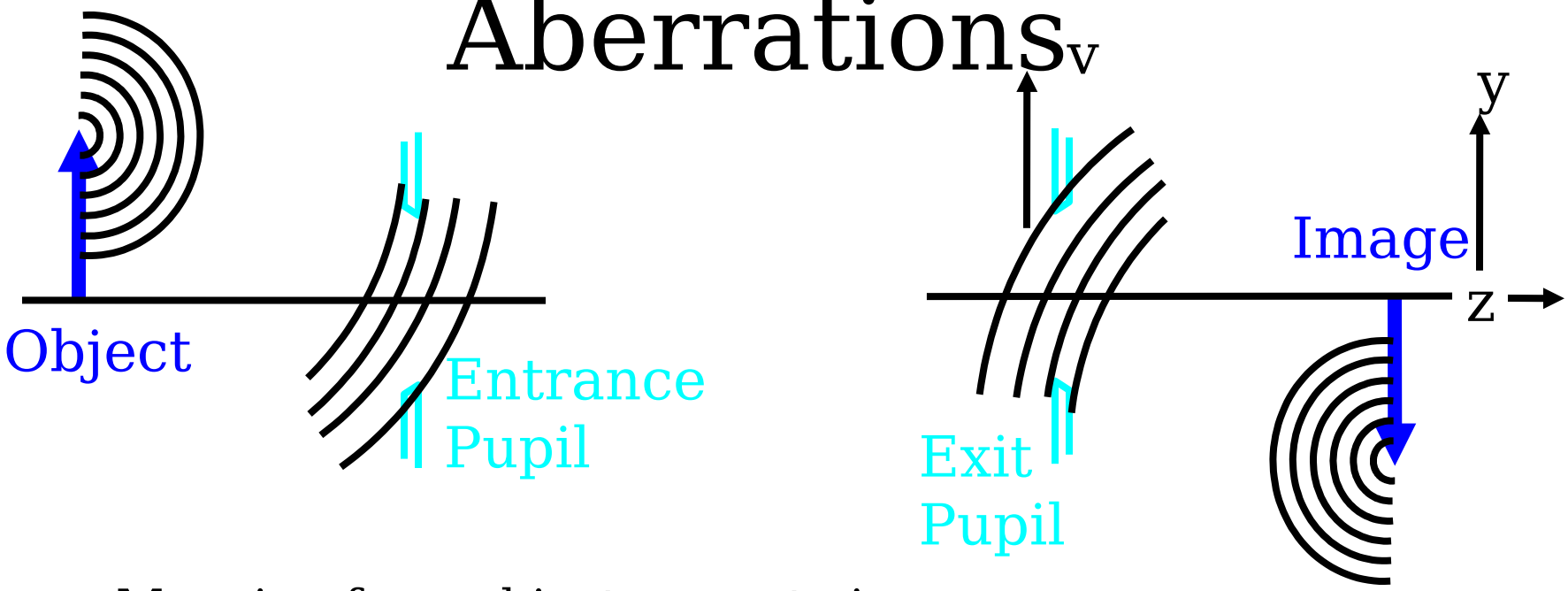


Numerical Examples



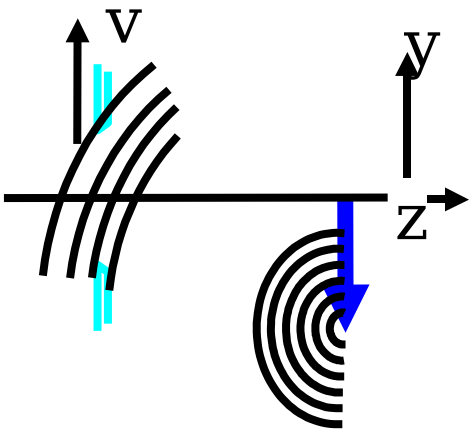


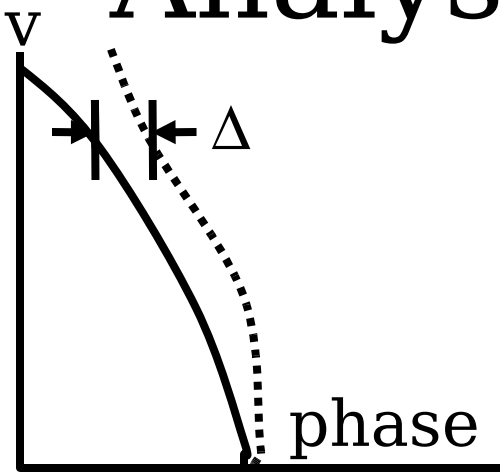
Phase Description of Aberrations



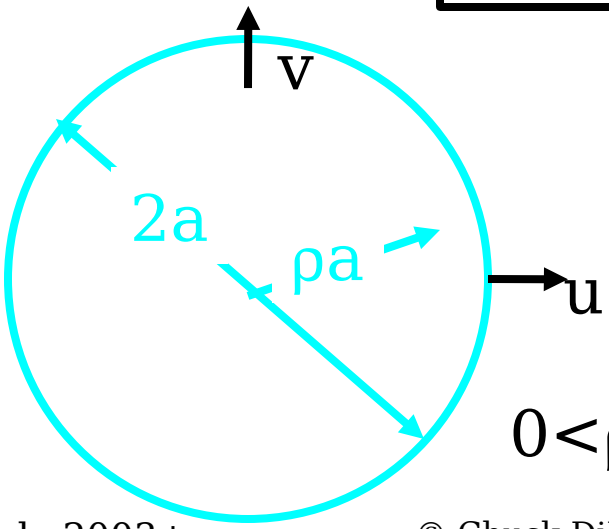
- Mapping from object space to image space
- Phase changes introduced in pupil plane
  - Different in different parts of plane
  - Can change mapping or blur images

Coordinates for Phase Analysis





Solid Line is phase of a spherical wave toward the image point.
Dotted line is actual phase.
Our goal is to find $\Delta(\rho,\phi,h)$.



$0<\rho<1$

Aberration Terms

Taylor's Series

$$\Delta = a_0 + b_0 h^2 + b_1 \rho^2 + b_2 \rho h \cos\phi + c_0 h^4 + c_1 \rho^4 + c_2 h^2 \rho^2 \cos^2\phi + c_3 h^2 \rho^2 + c_4 h^3 \rho \cos\phi + c_5 h \rho^3 \cos^3\phi + \ldots$$

Physical Significance

Zero-Order Terms

$$\Delta = a_0 + \ldots$$

No Aberrations

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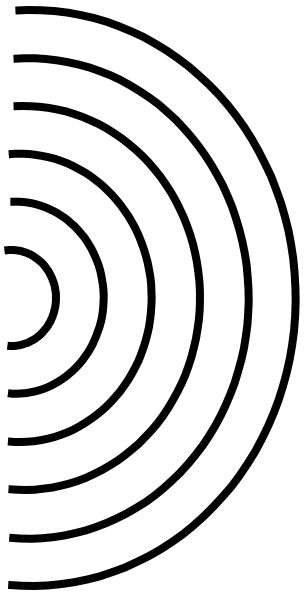
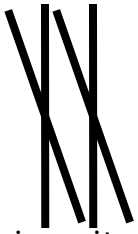


Odd Terms involve tilt, not considered here.



$$\Delta = \rho \cos\varphi$$

Second Order

$$\Delta = \ldots + b_1 \rho^2 + \ldots$$

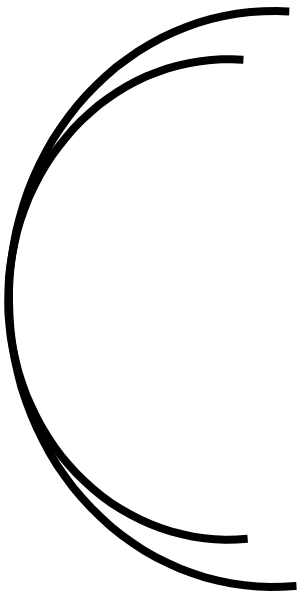


Image Position Terms:

The spherical wave is approximated by a second-order phase term, so this error is simply a change in focal length.

Fourth Order (1)

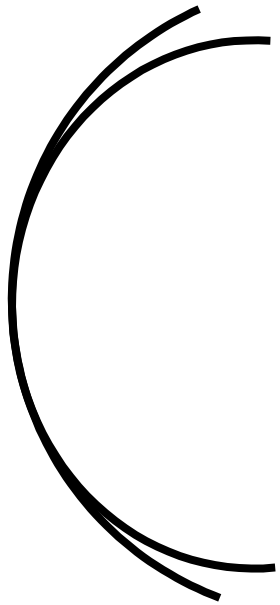
$$\Delta = \ldots + c_1 \rho^4 + \ldots$$

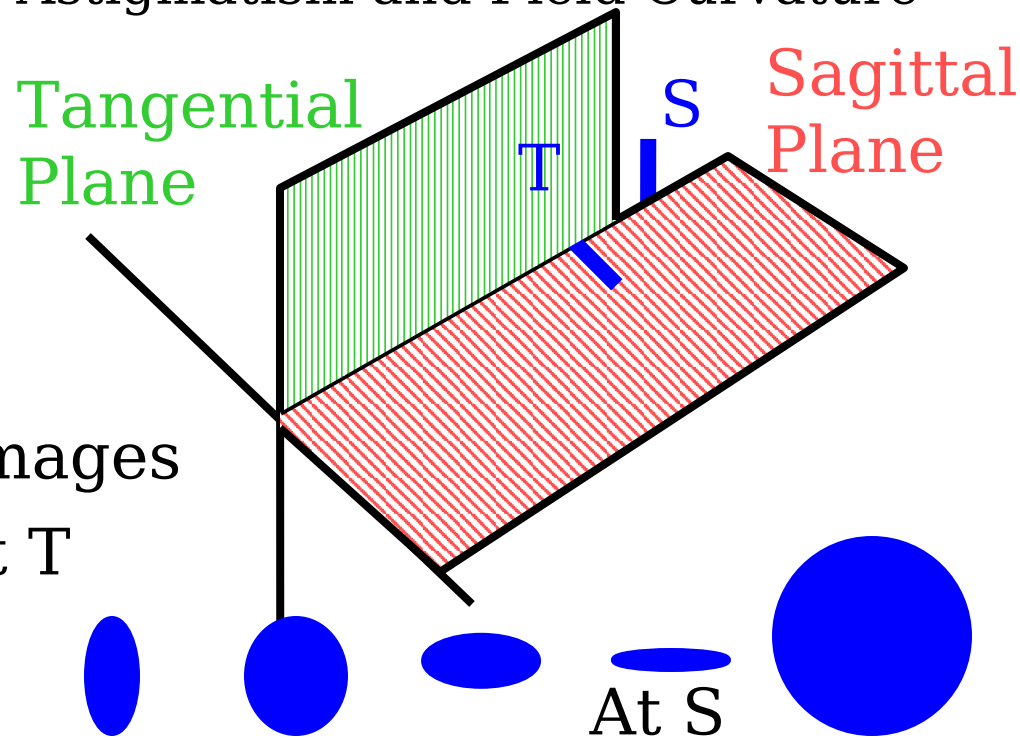
Spherical Aberration

ρ^2 is focus: depends on h^2 and $h^2 cos\phi$

$$\Delta = \ldots + c_2 h^2 \rho^2 \cos^2\phi + c_3 h^2 \rho^2 + \ldots$$

Astigmatism and Field Curvature

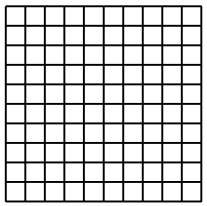




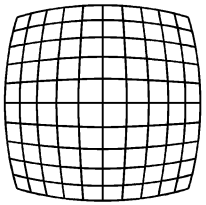
Fourth Order (2)

$$\Delta = \ldots + c_4 h^3 \rho \cos\phi + \ldots$$

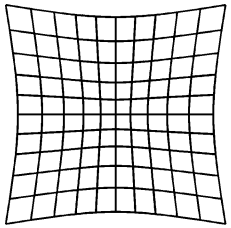
$\rho cos\phi$ is Tilt: Depends on h^3



Barrel Distortion

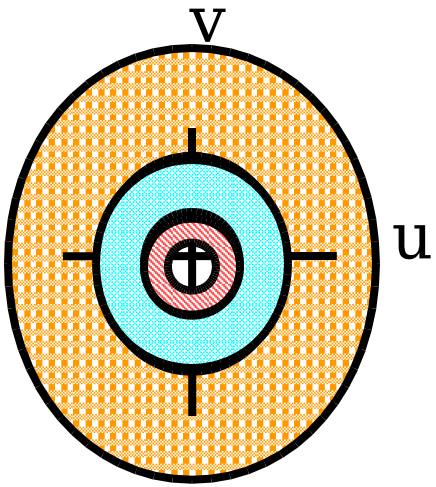


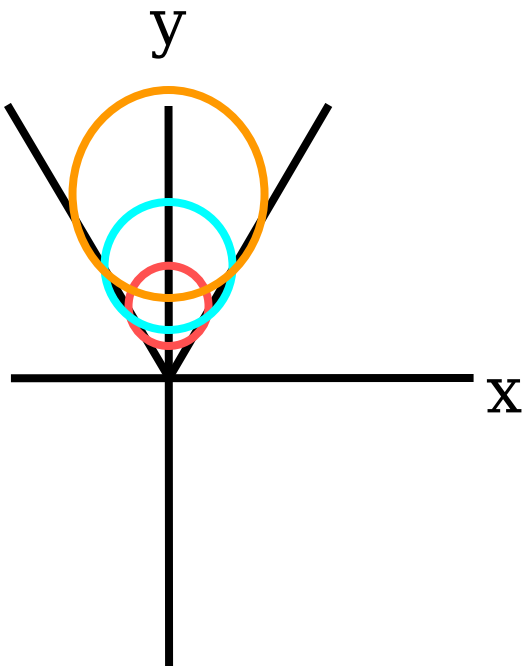
Pincushion Distortion



$$\Delta = \ldots + c_5 h \rho^3 \cos^3\phi + \ldots$$

Coma





Optical Design Process

Concept

- Thin Lens Equation

Intermediate Stage

- Thick Lenses
  - Thickness, Radii/Curvature
  - Multiple Elements
- Principal Planes
- Stops and Pupils

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Configuration

- Folding

Detailed Design

- Exact Ray Tracing

Selection of Vendors

Final Design

Fabrication and Test

- Alignment

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Ray Tracing Fundamentals

- Generate Rays
- Specify Baseline System
  - Distance, Radius,
  - Index, Diameter, etc.
- Iterate on Ray
  - Iterate on Surface
    - Find Ray–Surface Intersection
    - Find Refracted Ray
  - Close

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- Initial Layout
- Bundle of Rays
  - Chief, Marginal Rays
  - Others
- Spot Diagram
- Figure of Merit
  - Aberrations
  - Enclosed Power
  - Standard Deviation

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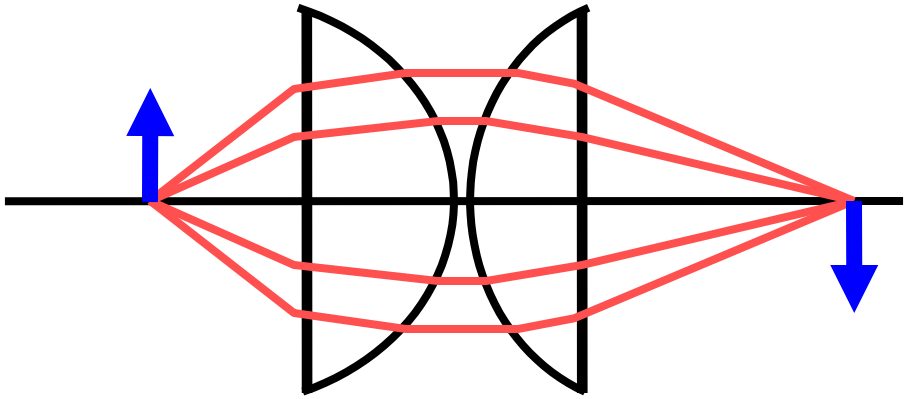
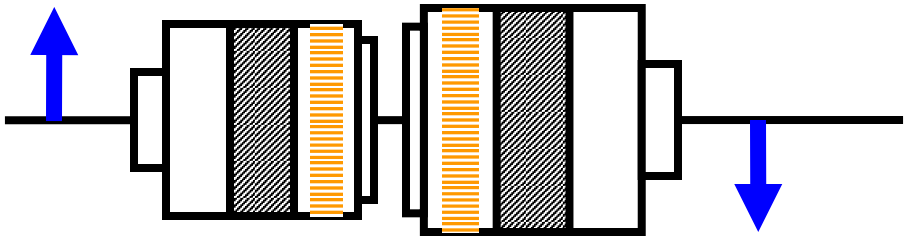
If One Element Doesn't Work...

Add Another Lens

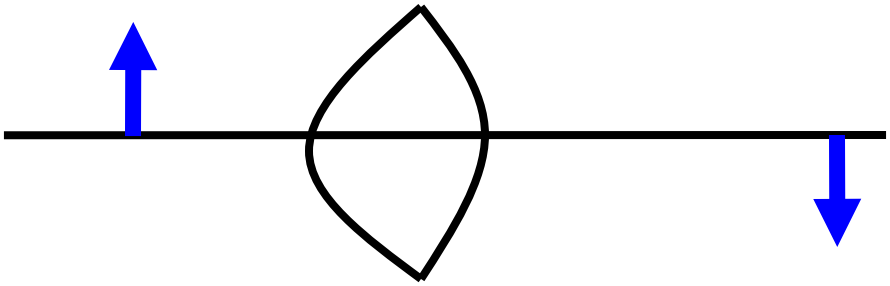


Different Index? Smaller angles with higher index. Thus germanium is better than ZnSe in IR. Not much hope in the visible.

"Let George Do It"



Aspherics



Aberrations Summary

- Origin of Aberrations
- On-Axis Aberrations
  - Change of Focus
  - Spherical Aberration
- Off-Axis Aberrations
  - Additional Blurring Effects
  - Distorting Effects

Summary of Concepts So Far

- Paraxial Optics with Thin Lenses
- Thick Lenses (Principal Planes)
- Apertures: Pupils and Windows
- Aberration Correction
  - Analytical
  - Ray Tracing
- What’s Missing? Wave Optics