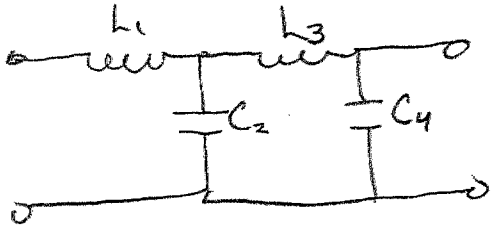
ECEU574 Assignment 7 Solutions

1.) $L(f) = 1 + \left(\frac{f}{f_c}\right)^{2n} \geq 100$ @ $f = 5\,GHz$

$n \geq 4 \Rightarrow n = 4.$



$a_1 = x_1 = \frac{X_1}{75} = \frac{\omega_c L_1}{75}$

$L_1 = \frac{(0.765)\,75}{2\pi\ 2.5\cdot 10^9}$

$L_1 = 3.65\,nH$

$a_2 = b_2 = \omega_c C_2 \cdot 75$

$C_2 = \frac{1.848}{75 \cdot 2\cdot\pi\cdot 2.5\cdot 10^9} = 1.57\,pF$

$a_3 = x_3 = \frac{\omega_c L_3}{75}$

$L_3 = \frac{(1.848)(75)}{2\pi\ 2.5\cdot 10^9} = 8.8\,nH$

$a_4 = b_4 = \omega_c C_4 \cdot 75$

$C_4 = \frac{0.765}{75(2\pi)\,10^9} = 1.6\,pF$

2. $C_0(x) = 1$

$C_1(x) = x$

$C_2(x) = 2x^2 - 1$

$C_3(x) = 4x^3 - 3x$

$L(f) = 1 + \alpha C_n^2\left(\frac{f}{f_c}\right)$

$1+\alpha = 10^{\frac{1}{20}}$

$\alpha = 0.122$

n, odd	L(5GHz)
1	1.7
3	28.3
5	

$\longrightarrow \quad n = 3$

$$\beta = \sinh\left(\frac{\tanh^{-1}\left(\frac{1}{\sqrt{1+\alpha}}\right)}{n}\right)$$

$\beta = 0.52$

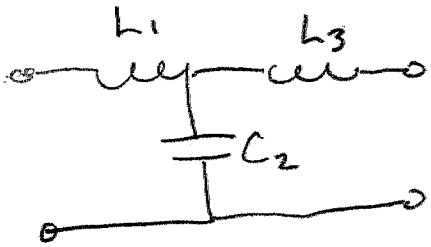


table coefficients:

$C_1 = \frac{a_1}{\beta} = \frac{1}{0.52} = 1.92$

$C_2 = \frac{a_2 a_1}{C_1\left(\beta^2 + \sin^2\left[\frac{\pi}{3}\right]\right)}$

$= \frac{2}{1.92} \cdot \frac{1}{(0.52)^2 + 0.75}$

$C_2 = 1.04 \cdot (0.98) = 1.0$

$C_3 = \frac{2}{(1.02)\left((0.52)^2 + \sin^2\left(\frac{2\pi}{5}\right)\right.}$

$= (1.96)(0.98) = 1.92$

$C_1 = \frac{\omega_c L_1}{75} \Rightarrow L_1 = \frac{(1.92)(75)}{2\pi(2.5)10^9}$

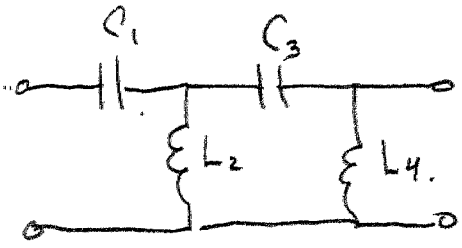
$L_1 = 9.2\,nH$

$C_2 = 75\,\omega_c C_2' \Rightarrow C_2' = \frac{1.02}{(75)(2\pi)(2.5)10^9}$

$= 0.86\,pF$

$L_3 = L_1$.

3. $L(1.25\,GHz) = 1 + (2)^{2n} \geq 100 \Rightarrow n = 4$



$a_1 = 0.765 \quad a_3 = 1.848$

$a_2 = 1.848 \quad a_4 = 0.765$

$a_1 = \frac{1}{\omega_c C_1}\frac{1}{75} \Rightarrow C_1 = \frac{1}{2\pi(2.5)10^9(0.765)(75)} = 0.9\,\mu F$

$a_2 = \frac{75}{\omega_c L_2} \Rightarrow L_2 = \frac{75}{2\pi(2.5)10^9(1.848)} = 2.6\,nH$

$a_3 = \frac{1}{\omega_c C_3} \Rightarrow C_3 = C_1 \cdot \frac{(0.765)}{1.848} = 37\,nF$

$a_4 = \frac{75}{\omega_c L_4} \Rightarrow L_4 = \frac{75}{\omega_c a_4} = L_2 \cdot \frac{a_2}{a_4} = 6.3\,nH.$

4.) $\alpha = 0.122$

$C_0(z) = 1$

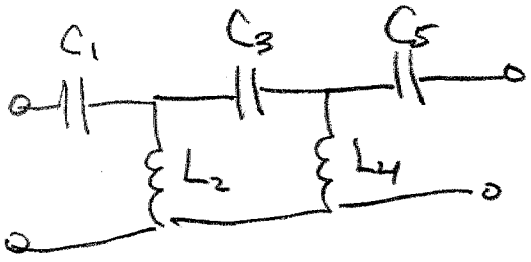
$C_1(z) = z$

$C_2(z) = 4.2 - 1 = 7$

$C_3(z) = 4(7) - 2 = 26$

$C_4(z) = 4(26) - 7 = 97$

$C_5(z) = 4(97) - 26 = 362.$



$$C_1 = \frac{1}{\omega_c C_1'} \frac{1}{75} \Rightarrow C_1' = \frac{1}{(2\pi)(2.5)10^9(1.7)(75)}$$

$$C_1' = \frac{1}{2}\,pF$$

$$C_2 = \frac{75}{\omega_c L_2} \Rightarrow L_2 = \frac{75}{2\pi(2.5)10^9(1.23)}$$

$$L_2 = 3.88\,nH$$

$$C_3 = \frac{1}{\omega_c C_3'} \frac{1}{75} \Rightarrow C_3' = C_1' \cdot \frac{C_1}{C_3}$$

$$= 0.33\,pF$$

$$C_4 = \frac{75}{\omega_c L_4} \Rightarrow L_4 = L_2 \cdot \frac{C_2}{C_4} = 3.88\,nH$$

$$C_5 = \frac{1}{\omega_c C_5' 75} \Rightarrow C_5' = C_1' \cdot \frac{C_1}{C_5}$$

$$= \frac{1}{2}\,pF$$

$$L(1.25) = 1 + \alpha C_n^2(z)$$

n	
1	1.48
3	83
5	~16000

$n = 5$

$$\beta = \sinh\left(\frac{\tanh^{-1}\left(\frac{1}{\sqrt{1+\alpha}}\right)}{5}\right)$$

$\beta = 0.362.$

$$C_1 = \frac{Q_1}{\beta} = \frac{0.618}{0.362} = 1.7$$

$$C_2 = \frac{(0.618)(1.618)}{(1.7)} \frac{1}{(0.362)^2 + \sin^2\left(\frac{\pi}{5}\right.}$$

$C_2 = 1.23$

$$C_3 = \frac{2(1.618)}{1.23} \frac{1}{\left[(0.362)^2 + \sin^2\left(\frac{2\pi}{5}\right.\right.}$$

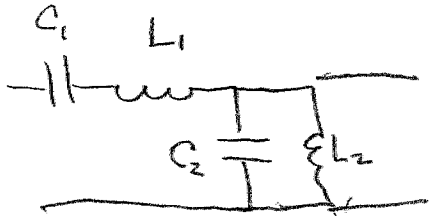
$C_3 = 2.54$

$$C_4 = \frac{(1.618)(2)}{(2.54)} \frac{1}{\left[(0.362)^2 + \sin^2\left(\frac{3\pi}{5}\right.\right.}$$

$C_4 = 1.23$

$C_5 = 1.7$

5)



$n = 2$ $\quad \Delta\omega = 2\pi\,(50\,KHz)$

$a_1 = \sqrt{2}$ $\quad \omega_0 = 2\pi\,(300KHz)$.

$a_2 = \sqrt{2}$

$$a_1 = \frac{\Delta\omega L_1}{75} \Rightarrow L_1 = \frac{a_1 \cdot 75}{\Delta\omega} = \frac{\sqrt{2}\cdot 75}{2\pi\, 50\cdot 10^3} = \frac{1}{3}\,mH$$

$$C_1 = \frac{1}{\omega_0^2 L_1} = \frac{1}{(2\pi)^2 (3)^2 (10^5)^2\, 0.33\cdot 10^{-3}}$$

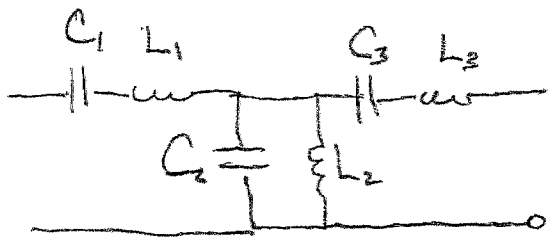
$$C_1 = 853\,\mu F$$

$$a_2 = \frac{75}{\Delta\omega C_2} \Rightarrow C_2 = \frac{75}{\Delta\omega\, a_2} = \frac{75}{2\pi 50\cdot 10^3 \cdot \sqrt{2}} = 168\,\mu F$$

$$L_2 = \frac{1}{\omega_0^2 C_2} = \frac{1}{(2\pi)^2 (3)^2 (10^5)^2\, 168\cdot 10^{-6}}$$

$$L_2 = 1.67\,nH$$

5b.)



$a_1 = 1$

$a_3 = 1$

$a_2 = 2$

$$a_1 = \frac{\Delta\omega L_1}{75} \Rightarrow L_1 = \frac{75}{2\pi\, 5\cdot 10^4} = 239\,\mu H$$

$$C_1 = \frac{1}{\omega_0^2 \cdot L_1} = \frac{1}{(355)10^{10}(239)10^{-6}} = 1.18\,nF$$

$$a_2 = \Delta\omega\, C_2 \cdot 75 \Rightarrow C_2 = \frac{2}{2\pi 5\cdot 10^4 (75)}$$

$$= 85nF$$

$$h_2 = \frac{1}{\omega_0^2 C_2} = \frac{1}{355\cdot 10^{10}(85)\cdot 10^{-9}}$$

$$h_2 = 3.3\,\mu H$$

$$h_3 = L_1$$

$$C_3 = C_1$$

3rd order filter requires smaller inductances.