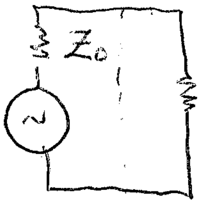


$$Z_{in} = Z_0 \frac{1+\rho e^{-j2l\frac{2\pi}{\lambda}}}{1-\rho e^{-j2l\,2\pi/\lambda}} , \quad \rho = \frac{Z_L - Z_0}{Z_L + Z_0}$$

line length l
load impedance Z_L
wavelength λ

when $Z_L = 0$, $\rho = -1$

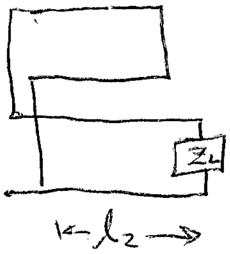
$$Z_{in} = Z_0 \cdot \frac{1 - e^{-j2l\frac{2\pi}{\lambda}}}{1 + e^{-j2l\,2\pi/\lambda}}$$

general admittance formula

$$\frac{1}{Z_{in}} = \frac{1}{Z_0} \cdot \frac{1-\rho e^{-j2l\frac{2\pi}{\lambda}}}{1+\rho e^{-j2l\frac{2\pi}{\lambda}}}$$

when $Z_L = 0$

$$\frac{1}{Z_{in}} = \frac{1}{Z_0} \frac{1+e^{-j2l\frac{2\pi}{\lambda}}}{1-e^{-j2l\,2\pi/\lambda}}$$



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; where.

$$\frac{1}{Z_{eq}} = \frac{1}{Z_0} \left\{ \frac{1-\rho e^{-j2l_2\frac{2\pi}{\lambda}}}{1+\rho e^{-j2l_2\frac{2\pi}{\lambda}}} + \frac{1+e^{-j2l_1\frac{2\pi}{\lambda}}}{1-e^{-j2l_1\frac{2\pi}{\lambda}}} \right\}$$

$$= \frac{1}{Z_0} \left\{ \frac{\left(1-\rho e^{-j2l_2\frac{2\pi}{\lambda}}\right)\left(1-e^{-j2l_1\frac{2\pi}{\lambda}}\right)}{\left(1+\rho e^{-j2l_2\frac{2\pi}{\lambda}}\right)\left(1-e^{-j2l_1\frac{2\pi}{\lambda}}\right)} + \frac{\left(1+e^{-j2l_1\frac{2\pi}{\lambda}}\right)\left(1+\rho e^{-j2l_2\frac{2\pi}{\lambda}}\right)}{\left(1+\rho e^{-j2l_2\frac{2\pi}{\lambda}}\right)\left(1-e^{-j2l_1\frac{2\pi}{\lambda}}\right)} \right\}$$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_o}\left\{ \frac{\left(1+\rho^* e^{j2l_2\frac{2\pi}{\lambda}}\right)\left(1-\rho e^{-j2l_2\frac{2\pi}{\lambda}}\right)\left(1-e^{-j2l_1\frac{2\pi}{\lambda}}\right)\left(1-e^{j2l_1\frac{2\pi}{\lambda}}\right)}{\left|1+\rho e^{-j2l_2\frac{2\pi}{\lambda}}\right|^2 \cdot \left|1-e^{-j2l_1\frac{2\pi}{\lambda}}\right|^2}\right.$$

$$\left. + \frac{\left(1-e^{j2l_1\frac{2\pi}{\lambda}}\right)\left(1+e^{-j2l_1\frac{2\pi}{\lambda}}\right)\left(1+\rho e^{-j2l_2\frac{2\pi}{\lambda}}\right)\left(1+\rho^* e^{j2l_2\frac{2\pi}{\lambda}}\right)}{\text{same denominator as above.}} \right\}$$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_o}\left\{ \frac{\left[1-\rho e^{-j2l_2\frac{2\pi}{\lambda}} + \rho^* e^{j2l_2\frac{2\pi}{\lambda}} - |\rho|^2\right]\left[1-e^{j2l_1\frac{2\pi}{\lambda}} - e^{-j2l_1\frac{2\pi}{\lambda}} + 1\right]}{\left|1+\rho e^{-j2l_2\frac{2\pi}{\lambda}}\right|^2 \cdot \left|1-e^{-j2l_1 2\pi/\lambda}\right|^2}\right.$$

$$\left. + \frac{\left[1+e^{-j2l_1\frac{2\pi}{\lambda}} - e^{j2l_1\frac{2\pi}{\lambda}} - 1\right]\left[1+\rho e^{-j2l_2\frac{2\pi}{\lambda}} + \rho^* e^{j2l_2\frac{2\pi}{\lambda}} + |\rho|^2\right]}{\left|1+\rho e^{-j2l_2\frac{2\pi}{\lambda}}\right|^2 \left|1-e^{-j2l_1 2\pi/\lambda}\right|^2} \right\}$$

for C_1, C_2 complex,

$$\mathrm{imag}(C_1 C_2) = \mathrm{Re}(C_1)\,\mathrm{Imag}(C_2) + \mathrm{Imag}(C_1)\,\mathrm{Re}(C_2)$$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_o}\,\frac{1}{\left|1+\rho e^{-j2l_2 2\pi/\lambda}\right|^2}\,\frac{1}{\left|1-e^{-j2l_1 2\pi/\lambda}\right|^2} \times$$

$$\left\{ \left[1-|\rho|^2 - j\cdot 2\,\mathrm{Im}\left(\rho e^{-j2l_2\frac{2\pi}{\lambda}}\right)\right]\overbrace{\left[2 - 2\,\mathrm{Re}\left[e^{j2l_1\frac{2\pi}{\lambda}}\right]\right]}^{\text{real}} + \underbrace{\left[j\cdot 2\cdot \mathrm{Im}\left(e^{-j2l_1\frac{2\pi}{\lambda}}\right)\right]}_{\text{imaginary.}}\underbrace{\left[1+|\rho|^2 + 2\,\mathrm{Re}\left[\rho e^{-j2l_2\frac{2\pi}{\lambda}}\right]\right]}_{\text{real}} \right\}$$

so the imaginary part of $\frac{1}{Z_{eq}}$ is

$$\mathrm{Im}\left(\frac{1}{Z_{eq}}\right) = \frac{1}{Z_0} \frac{1}{|1+\rho e^{-j2l_2\frac{2\pi}{\lambda}}|^2} \frac{1}{|1-e^{-j2l_1\frac{2\pi}{\lambda}}|^2} \times$$

$$\Big[2\,\mathrm{Im}\left(e^{-j2l_1\frac{2\pi}{\lambda}}\right)\left[1+|\rho|^2+2\mathrm{Re}\left[\rho\, e^{-j2l_2\frac{2\pi}{\lambda}}\right]\right]$$

$$- 2\,\mathrm{Im}\left(\rho\, e^{-j2l_2\frac{2\pi}{\lambda}}\right)\left[2-2\mathrm{Re}\left[e^{j2l_1\frac{2\pi}{\lambda}}\right]\right]\Big]$$

The imaginary part vanishes when

$$\mathrm{Im}\left(e^{-j2l_1\frac{2\pi}{\lambda}}\right)\left[1+|\rho|^2+2\mathrm{Re}\left(\rho\, e^{-j2l_2\frac{2\pi}{\lambda}}\right)\right] = \mathrm{Im}\left(\rho\, e^{-j2l_2\frac{2\pi}{\lambda}}\right) \times$$

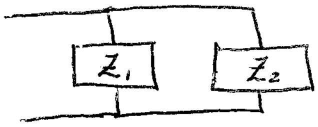
$$2\left[1-\mathrm{Re}\left(e^{j2l_1\frac{2\pi}{\lambda}}\right)\right]$$

and in this case, the admittance is

$$\mathrm{Re}\left(\frac{1}{Z_{eq}}\right) = \frac{1}{Z_0} \frac{1}{|1+\rho e^{-j2l_2\frac{2\pi}{\lambda}}|^2} \frac{1}{|1-e^{-j2l_1 2\pi/\lambda}|^2} \times$$

$$\left\{ \left[1-|\rho|^2\right]\cdot 2\left[1-\mathrm{Re}\left[e^{j2l_1\frac{2\pi}{\lambda}}\right]\right] \right\}$$

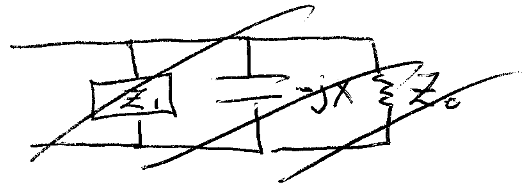
when $l_2 = 0$, the case is simpler:

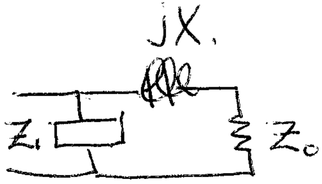


where $Z_2 = Z_L = Z_0 + jX$

and Z_1 is due to the shorted line of length l_1.

The equivalent circuit is





since Z_1 is ~~always reactive, we must~~ find

~~$Z_1 = jX$ for~~

$Z_1 \| (jX_1 + Z_0)$ is real.

$$\frac{Z_1 (jX + Z_0)}{Z_1 + jX + Z_0} = \frac{-|Z_1|X + jZ_0|Z_1|}{j(|Z_1| + X) + Z_0}, \quad Z_1 \text{ imaginary}$$

$$= \frac{\left(-|Z_1|X + jZ_0|Z_1|\right)\left(Z_0 - j\left(|Z_1| + X\right)\right)}{\left[(|Z_1| + X)^2 + Z_0^2\right]}$$

$$Z_0^2 \cancel{|Z_1|} + (|Z_1| + X)\cancel{|Z_1|}X = 0$$

$$\mathrm{Im}(Z_1) = \frac{-Z_0^2}{X} - X$$

$$\mathrm{Re}\left(Z_1 \,\|\, (jX + Z_0)\right) = \frac{-\mathrm{Im}(Z_1)\, X Z_0 + Z_0\, \mathrm{Im}(Z_1)\left[\mathrm{Im}\, Z_1 + X\right]}{\left(\mathrm{Im}(Z_1) + X\right)^2 + Z_0^2}$$

plug in for $\mathrm{Im}(Z_1)$:

$$\mathrm{Re}(\;''\;) = \frac{\left(\frac{Z_0^2}{X} + X\right) X Z_0 - Z_0 \left(\frac{Z_0^2}{X} + X\right)\left(-\frac{Z_0^2}{X}\right)}{\frac{Z_0^4}{X^2} + Z_0^2}$$

$$= \frac{Z_0^3 + X^2 Z_0 + Z_0^3\left(\frac{Z_0^2}{X^2} + 1\right)}{\frac{Z_0^4}{X^2} + Z_0^2}$$

$$=$$