1.a. Take 3.8 (a) and find equivalent impedance, using 3.8 (b). That is,

$$R_s + jX_s = \frac{jR_pX_p}{R_p + jX_p}\,\frac{R_p - jX_p}{R_p - jX_p} = \frac{R_pX_p^2 - jR_p^2X_p}{R_p^2 + X_p^2}$$

so

$$R_s = \frac{R_pX_p^2}{R_p^2 + X_p^2} = \frac{R_p}{Q_p^2 + 1}, \text{ since } Q_p = R_p/X_p$$

and

$$X_s = \frac{R_p^2X_p}{R_p^2 + X_p^2} = \frac{X_p}{1 + Q_p^{-2}}$$

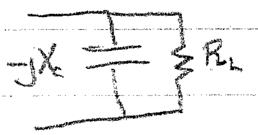
$$Q_s = \frac{X_s}{R_s} = \frac{X_p}{1 + Q_p^{-2}}\,\frac{Q_p^2 + 1}{R_p} = \frac{Q_p^2 + 1}{1 + Q_p^{-2}}\,\frac{1}{Q_p}$$

$$= \frac{(Q_p^2 + 1)\cdot Q_p}{(Q_p^2 + 1)}$$

$$= Q_p.$$

b. $R_s \sim R_p$ when $Q = Q_p$ is small

$X_s \sim X_p$ when $Q = Q_p$ is large.



want $\mathrm{Re}\left[\frac{-jX_cR_L}{R_L - jX_c}\cdot\frac{R_L + jX_c}{R_L + jX_c}\right] = 5\Omega.$

$$\frac{X_c^2 R_L}{R_L^2 + X_c^2} = 5\Omega.$$

$$\frac{R_L}{5} = \frac{R_L^2}{X_c^2} + 1$$

$$\frac{1}{X_c^2} = \frac{1}{R_L^2}\left[\frac{R_L}{5} - 1\right]$$

for $R_L = 50\Omega$, $X_c = \frac{50}{3}\Omega$

imaginary part: $\frac{-X_c R_L^2}{R_L^2 + X_c^2}$

so

$$X_L = \frac{X_c R_L^2}{R_L^2 + X_c^2} = \frac{X_c}{1 + \frac{X_c^2}{R_L^2}} = \frac{X_c}{1 + (\frac{1}{3})^2} = \frac{50}{3 + \frac{1}{3}}$$

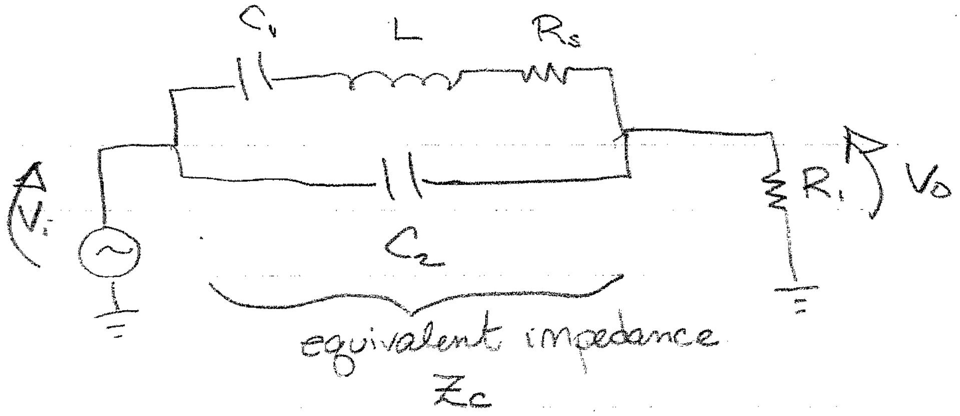
example $f = 7MHz$

$X_c = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_c} = \frac{3}{2\pi(7 \cdot 10^6)\, 50} = 1.36\ nF$

$X_L = \omega L$

$\Rightarrow L = \frac{X_L}{2\pi(7 \cdot 10^6)} = 341\ nH$

2)



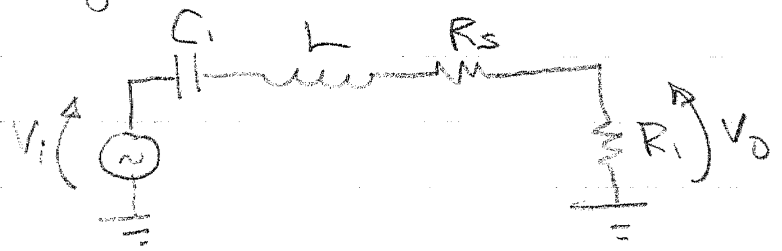
$$Z_c = \frac{\left(\frac{1}{j\omega C_1} + j\omega L + R_s\right)\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1} + j\omega L + R_s + \frac{1}{j\omega C_2}}$$

$$\mathrm{Im}(Z_c) = \frac{-R_s^2}{\omega C_2} + \frac{1}{\omega C_2}\left(\omega L - \frac{1}{\omega C_1}\right)\left(\frac{1}{\omega C_1} + \frac{1}{\omega C_2} - \omega L\right)$$

a.) condition for $\mathrm{Im}(Z_c) = 0$

$$\boxed{R_s^2 = \left(\omega L - \frac{1}{\omega C_1}\right)\left(\frac{1}{\omega C_1} + \frac{1}{\omega C_2} - \omega L\right)}$$

2b equivalent circuit if C_2 can be neglected.



series resonance

$$\frac{V_o}{V_i} = \frac{R_1}{R_1 + R_s + j\omega L + \frac{1}{j\omega C_1}} = \frac{R_1\left[(R_1+R_s) - j\left(\omega L - \frac{1}{\omega C_1}\right)\right]}{(R_1+R_s)^2 + \left(\omega L - \frac{1}{\omega C_1}\right)^2}$$

at resonance,

$$\omega L = \frac{1}{\omega C_1}, \quad \text{so}$$

$$\frac{V_o}{V_i} = \frac{R_1}{(R_1+R_s)} \quad \text{@ resonance.}$$

|reactance| = resistance condition:

$$R_1 + R_s = \left|\omega L - \frac{1}{\omega C_1}\right|$$

$$(R_1+R_s)^2 = \left(\omega L - \frac{1}{\omega C_1}\right)^2$$

$$\omega^2 (R_1+R_s)^2 = \left(\omega^2 L - \frac{1}{C_1}\right)^2$$

2 solutions for $x = \omega^2$:

$$0 = x^2 L^2 - \frac{2}{C_1} L \cdot x - (R_1+R_s)^2 x + \frac{1}{C_1^2}$$

$$x = \frac{\left(\frac{2L}{C_1} + (R_1+R_s)^2\right)}{2L^2} \pm \sqrt{\frac{\left(\frac{2L}{C_1} + (R_1+R_s)^2\right)^2}{4L^4} - \frac{1}{C_1^2 L^2}}$$

or

$$\underbrace{\omega^2}_{\text{|reactance| = resistance condition}} = \frac{1}{LC_1} + \frac{(R_1+R_s)^2}{2L^2} \pm \sqrt{\left(\frac{1}{LC_1} + \frac{(R_1+R_s)^2}{2L^2}\right)^2 - \frac{1}{L^2C_1^2}}$$

3dB condition:

find ω

$$\left|\frac{V_o}{V_i}\right|^2 = \frac{1}{2}\,\frac{R_1^2}{(R_1+R_s)^2} \stackrel{?}{=} \frac{R_1^2\left((R_1+R_s)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)}{\left[(R_1+R_s)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^2}$$

or

$$(R_1+R_s)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2\,(R_1+R_s)^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = (R_1+R_s)^2$$

so 3dB condition is identical to the |reactance| = resistance condition

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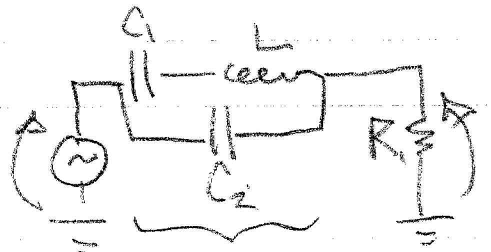
2b, cont'd) equivalent circuit if R_s can be neglected



Z_p

$$Z_p = \frac{\left[\frac{1}{j\omega C_1} + j\omega L\right]\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2} + j\omega L} = j\underbrace{\frac{\left[\omega L - \frac{1}{\omega C_1}\right]}{\frac{C_2}{C_1} + 1 - \omega^2 L C_2}}_{\text{purely reactive}}$$

resonance condition

$$Z_p = 0 \Rightarrow \omega L = \frac{1}{\omega C_1}$$

$$\frac{V_o}{V_i} = 1 \text{ at resonance}$$

cutoff condition

$$|Z_p| = R_1$$

get $\left[\omega L - \frac{1}{\omega C_1}\right]^2 = R_1^2\left(\frac{C_2}{C_1} + 1 - \omega^2 L C_2\right)^2$

consider ω such that $\omega^2 L C_2 << \frac{C_1+C_2}{C_1}$

(Low frequencies)

Then, the equation becomes

$$\left[\omega L - \frac{1}{\omega C_1}\right]^2 \cong R_1^2\left(\frac{C_1+C_2}{C_1}\right)^2$$

or

$$\omega^4 L^2 - 2\omega^2 \frac{L}{C_1} - \omega^2 R_1^2\left(\frac{C_1+C_2}{C_1}\right)^2 + \frac{1}{C_1^2} = 0$$

or $(x=\omega^2)$:

$$L^2 x^2 - \left(\frac{2L}{C_1} + R_1^2\left(\frac{C_1+C_2}{C_1}\right)^2\right)x + \frac{1}{C_1^2} = 0$$

2 solutions

$$x = \frac{2LC_1 + R_1^2 (C_1+C_2)^2}{2L^2C_1^2} \pm \sqrt{\left(\frac{2LC_1 + R_1^2(C_1+C_2)^2}{2L^2C_1^2}\right)^2 - \frac{1}{L^2C_1^2}}$$

or

approximate cut off frequencies

$$\omega^2 = \frac{1}{LC_1} + \frac{R_1^2}{2L^2}\left(1+\frac{C_2}{C_1}\right)^2 \pm \sqrt{\left\{\frac{1}{LC_1} + \frac{R_1^2}{2L^2}\left(1+\frac{C_2}{C_1}\right)^2\right\}^2 - \frac{1}{L^2C_1^2}}$$

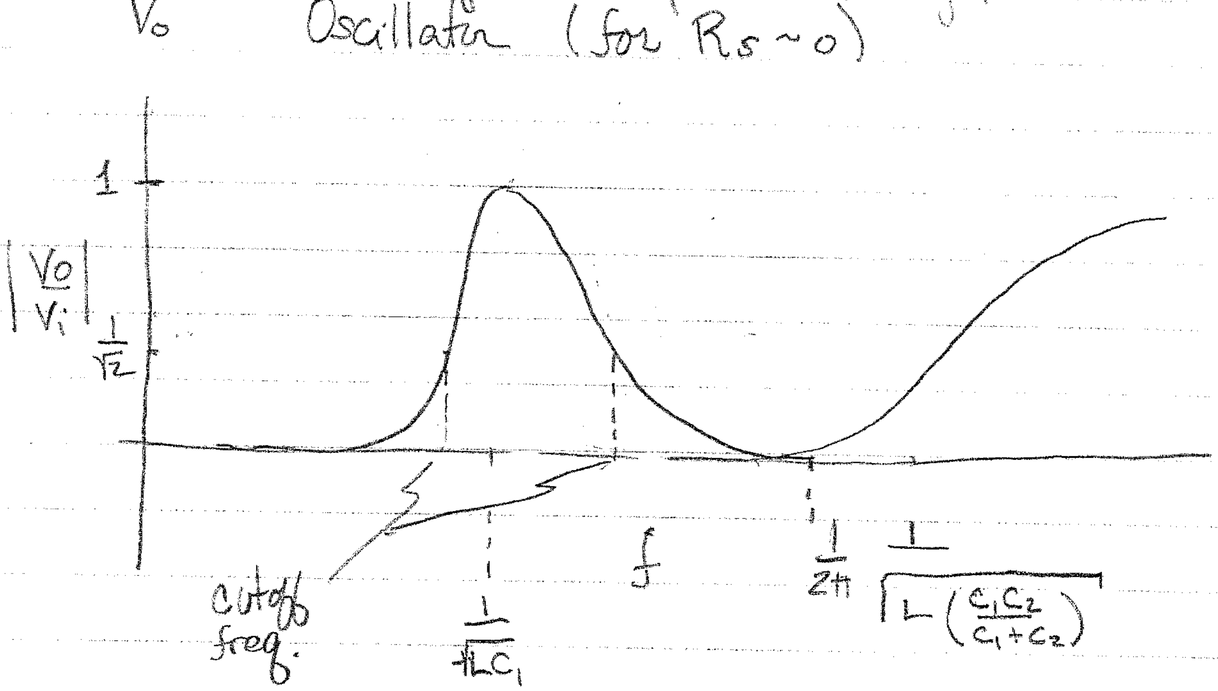
now consider frequencies such that $\omega^2 L C_2 \sim \frac{C_1+C_2}{C_1}$

then $|Z_p| = \infty$, and $\frac{V_o}{V_i} \cong 0$ @ $\omega^2 \sim \frac{C_1+C_2}{LC_1C_2}$

and around this frequency there are 2 more cutoffs

for all crystals of interest, $\underbrace{\frac{1}{LC_1}}_{\text{resonant freq}^2} << \underbrace{\frac{C_1+C_2}{C_2}\frac{1}{LC_1}}_{\text{notch freq}^2}$

Summary of Response of Crystal Oscillator (for $R_s \sim 0$)



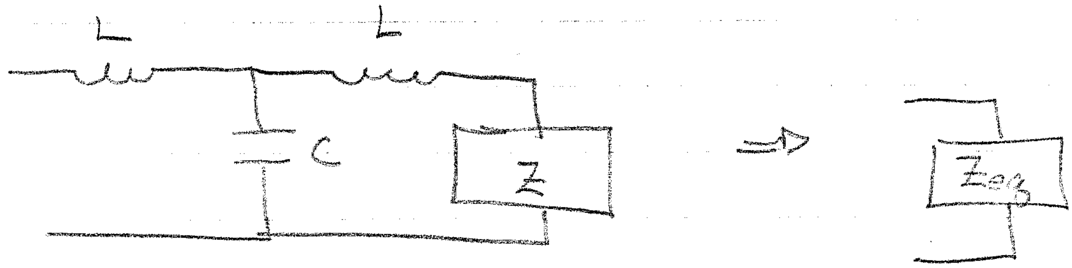
2c) if C_2 can be neglected, then this circuit acts like a series resonance device.

Use the formulas developed in 2b for $C_2=0$, setting $R_S=0$ in the formulas. So.

$$\omega^2 = \frac{1}{LC_1} \qquad \text{resonance}$$

$$\omega^2 = \frac{1}{LC_1} + \frac{R_1^2}{2L^2} \pm \sqrt{\left(\frac{1}{LC_1} + \frac{R_1^2}{2L^2}\right)^2 - \frac{1}{L^2C_1^2}} \qquad \text{cutoff}$$

3



a.) $$Z_{eq} \quad j\omega L + \frac{\frac{1}{j\omega C}\left[j\omega L + Z\right]}{Z + j\omega L + \frac{1}{j\omega C}}$$

b) at $\omega = \frac{1}{\sqrt{LC}}$,

$$Z_{eq} = j\sqrt{\frac{L}{C}} + \frac{\left[\frac{L}{C} - j\, Z\sqrt{\frac{L}{C}}\right]}{Z + 0}$$

$$= \cancel{j\sqrt{\frac{L}{C}}} + \frac{L}{C}\cdot\frac{1}{Z} - \cancel{j\sqrt{\frac{L}{C}}}$$

So, this circuit inverts the impedance Z, and is called an _impedance_ _inverter_.