ECEU574 Wireless Communication Circuits — Spring, 04
Assignment 3 Solutions — Prof. Brady

1. High frequency response of straight wire. Consider a straight segment of AWG 26 copper wire having length $2.5cm$. A sinusoidal voltage of the form $V(t) = \cos(2\pi f_{rf} t)$ is applied across the wire. An ideal current meter measures the current through the wire, and the impedance is plotted versus frequency, f_{rf}. The plots are shown below.

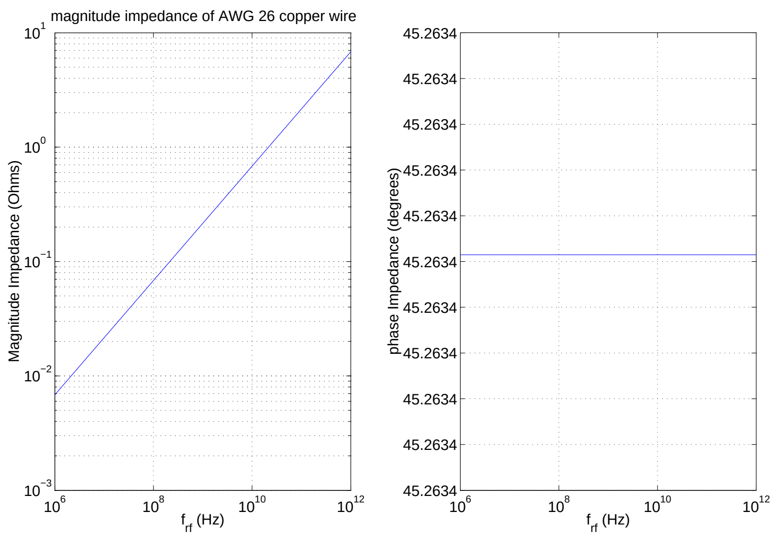

Figure 1: Impedance plots of AWG 26 copper wire.

The following schematic is proposed as a model for this wire.

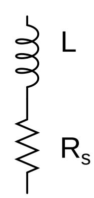

Figure 2: Proposed model for straight wire at high frequencies. All components are ideal, although the resistance $R_s = k_1\sqrt{f}$, where f is in Hz, and k_1 is a constant. Also, the inductance is frequency dependent, $L = k_2/\sqrt{f}$, where k_2 is a constant.

(a.) Using the plot, estimate the constants k_1 and k_2 for this model. Explain your method clearly.

Solution:

The impedance for this circuit is

$$Z(f) = j\omega L + R_s = \{j2\pi k_2 + k_1\}\sqrt{f} = [\sqrt{4\pi^2 k_2^2 + k_1^2}]\sqrt{f}e^{j\tan^{-1}(\frac{2\pi k_2}{k_1})},$$

which has constant phase and a magnitude which is proportional to $\sqrt{f}$. The magnitude is .07 at $\sqrt{f} = 10^4$ and .7 at $\sqrt{f} = 10^5$, and therefore we have

$$4\pi^2 k_2^2 + k_1^2 = 4.9 \times 10^{-11}.$$

The phase plot is constant at 45.26 degrees, and so the imaginary and real parts of the impedances are nearly the same. Combining these two results, we have

$$2k_1^2 = 4.9 \times 10^{-11},$$

and $k_1 = 4.9e - 6$, $k_2 = 7.87e - 7$.

2. (Metal Film Resistors). In this problem you will investigate and model the behavior of an metal film resistor at high frequencies.

A sinusoidal voltage of the form $V(t) = \cos(2\pi f_{rf} t)$ is applied to the terminals of a metal film resistor with nominal resistance of $R = 500\Omega$. An ideal current meter measures the current through the resistor, and the impedance is plotted versus frequency, f_{rf}. The plots are shown below. It is clear from these plots that while the resistor behaves in the expected manner for low frequencies, it is far from ideal at radio frequencies.

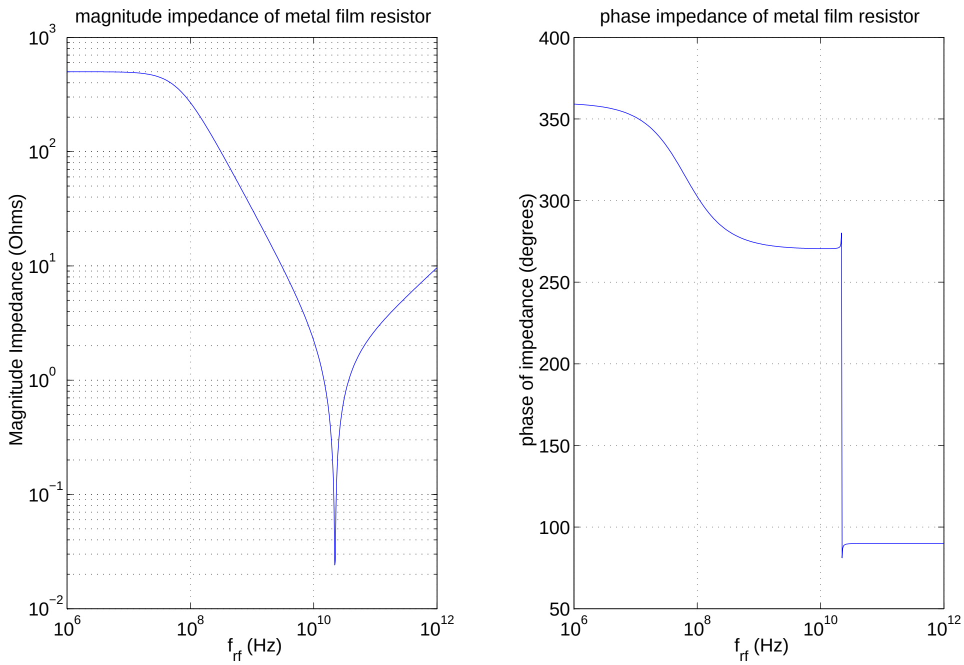

Figure 3: Impedance plots of a metal film resistor versus frequency.

Two possible circuits are proposed to model the nonideal effects of the metal film resistor, and are shown below.

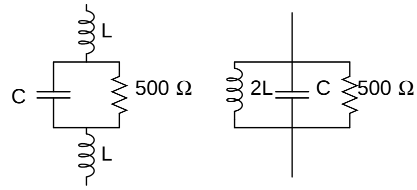

Figure 4: Proposed models for a metal film resistor at RF frequencies. All elements in these schematics are ideal, although all L's have the form $L = k_2/\sqrt{f}$, for frequency f in Hz and some unknown constant k_2.

(a.) By considering the circuit models for DC frequency, which of the two models best represents the impedance plots above, and why?

Solution:
The left circuit best represents the impedance plots, since the right plot predicts a zero impedance magnitude at DC.

(b.) For your answer in (a.), estimate all components values from the impedance plots. Be as specific as possible, and show all work clearly.

Solution:
The value of R is estimated from the leftmost magnitude, 500Ω, or you could have just used the given value. Finding the impedance, Z, of the leftmost circuit, yields an expression for real and imaginary parts, as

$$Re\{Z\} = \frac{1}{\frac{1}{R} + R(2\pi f C)^2},$$

and

$$Imag\{Z\} = 4\pi\sqrt{f}k_2 + \frac{2\pi f C}{\frac{1}{R^2} + (2\pi f C)^2}.$$

Note that if R is known, then the real part has one unknown (C), and then the imaginary part has one remaining unknown (k_2). So, one method to proceed is to relate the real part to a measurement at one frequency, determine C, and then relate the imaginary part at the same frequency, to find k_2.

For example, at $f = 10^8 Hz$, the magnitude of the impedance is estimated as 270, and the phase is estimated as 305 degrees. The real part is $270\cos(305 * 2\pi/360)$ and the imaginary part is $270\sin(305 * 2\pi/360)$. The calculated value of $C = 4.75pF$, and the estimated value of $k_2 = 8e - 5$.

(c.) If the leads of the resistor are each $1.25cm$ long, and are AWG 26 copper, why doesn't your model account for the lead resistance that appeared in problem 1? **Solution:** The lead resistance is very small compared to the nominal resistance of 500Ω.

3. Consider the design of a bandpass filter, with center frequency $f_c = 2.44GHz$ and a Q-factor of 24, for use in a Bluetooth chip. If the load resistance is fixed at 100Ω, find the size of the inductances L for both a series resonance and a parallel resonance circuit. Which implementation would you choose, and why?

Solution: The expressions for the quality factor for series and parallel resonance circuits are

$$Q = \frac{\omega_0 L}{R}, \quad \textit{series resonance}$$

$$Q = \frac{R}{\omega_0 L}, \quad \textit{parallel resonance}$$

where $\omega_0 = 2\pi f_c$, and $R = 100\Omega$. From these equations $L = 156nH$ for the series resonance circuit, and $L = 0.66nH$ for parallel resonance circuit. Thus, the parallel resonance implementation would require the smaller inductor to provide the required Q with the specified load resistance.