ECEU574 Wireless Communication Circuits
Midterm Examination Spring, 04

Name: Solutions Duration: 1 hour, 40 minutes

Directions: Complete all 4 problems. Please use both sides of the sheets provided, and secure any additional sheets that you use.

1. Consider the circuit in the following figure.

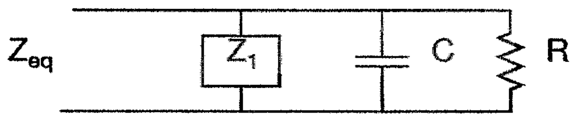


Figure 1: Figure for problem 1.

(1.a.) For any value of resistance R, capacitance C, and frequency ω, find the value impedance Z_1 which makes the equivalent impedance of the circuit, Z_{eq}, purely real. What is the value of Z_{eq} in that case?

$-\frac{1}{Z_1} = j\omega C$, since the three devices are in parallel.

In that case

$$Z_{eq} = R$$

(1.b.) Is it possible to create the impedance Z_1 in part (a.) with a shorted, lossless transmission line of characteristic impedance $Z_0 = R = 50\Omega$, at frequency $\omega = 2\pi\ 7 \times 10^6 Hz$ with $C = 1\mu F$? If not, provide a convincing argument. If so, what is the minimum line length ℓ needed, in meters? You may assume that $c = 2 \times 10^8$ here.

For a shorted t-line

$$Z_L = Z_o \frac{\left(1 - e^{-j\frac{2\pi}{\lambda}2\ell}\right)}{\left(1 + e^{-j\frac{2\pi}{\lambda}2\ell}\right)}$$

$$= Z_o \cdot j \cdot \tan\left(2\pi \frac{\ell}{\lambda}\right)$$

we need

$$\frac{-1}{Z_o \cdot j \tan\left(2\pi \frac{\ell}{\lambda}\right)} = j\omega C$$

or

$$\tan\left(2\pi \frac{\ell}{\lambda}\right) = \frac{1}{Z_o \omega C}$$

$$= \frac{1}{50 \cdot 2\pi\ 7 \cdot 10^6\ 10^{-12}}$$

or

$$\frac{\ell}{\lambda} = \frac{1}{2\pi} \tan^{-1}(3183)$$

$$\frac{\ell}{\lambda} \cong \frac{1}{4}$$

$$\ell = \frac{c}{4f} = \frac{2 \cdot 10^8 m}{4 \cdot 7 \cdot 10^6} = 7.14 \text{ meters.}$$

2. Consider the circuit in the figure below. Note: this circuit is neither the series resonant circuit, nor the parallel resonant circuit, presented in class.

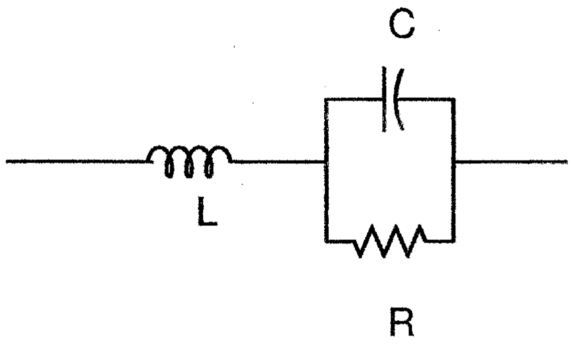


Figure 2: Figure for problem 2.

(a.) Let Z_2 denote the equivalent impedance for the parallel RC combination. Find formulas for the reactance and resistance of Z_2.

$$Z_2 = \frac{\frac{R}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R(1 - j\omega RC)}{1 + (\omega RC)^2}$$

resistance of Z_2: $\dfrac{R}{1+(\omega RC)^2}$

reactance of Z_2: $\dfrac{-\omega R^2 C}{1+(\omega RC)^2}$

(b.) Use your result in (b.) to find an expression for the frequency(ies) at which the entire circuit, including the inductor, is purely resistive. What is the equivalent impedance of the entire circuit in this case? Express the equivalent impedance only in terms of R, L, and C.

since the inductor is in series with Z_2, the circuit is purely resistive when.

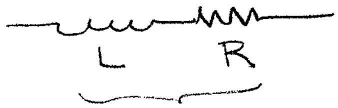
$$\omega_0 L = \frac{\omega_0 R^2 C}{1 + (\omega_0 RC)^2} \Rightarrow \omega_0 = \frac{1}{RC}\sqrt{\frac{R^2C}{L} - 1}$$

at $\omega = \omega_0$ the resistance of Z_2 is the entire impedance. It is given by

$$\frac{R}{1 + (\omega_0 RC)^2} = \frac{L}{RC}$$

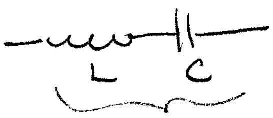
(c.) Let $R = 500\Omega$, $C = 5pF$, and $L = 1.5nH$. Draw an approximate circuit in the small frequency and high frequency cases by letting some, but not all, of the components vanish. Find approximate expressions for the equivalent impedance in each case.

at low frequencies:



$$Z_{eq} \sim R + j\omega L$$

at high frequencies:



$$Z_{eq} \sim j\left(\omega L - \frac{1}{\omega C}\right)$$

(d.) Compare your answers for equiv. imped. in (b.) and (c.). Based only on those answers, would you say that the circuit more closely resembles a series resonant circuit, or a parallel resonant circuit? State your reasoning clearly.

	$\lvert Z_{eq}\rvert^2$
low freq	$(500)^2$
at ω_0	0.36
high	$(\omega L)^2$, large ω.

$\lvert Z_{eq}\rvert^2$ is smallest at the resonant frequency, just like a series resonant circuit

3. It is desired to design a superheterodyne receiver with radio frequency $f_{RF} = 100MHz$, intermediate frequency $f_{IF} = 10MHz$ and audio frequency $f_a = 10kHz$.

(a.) For the VFO and the BFO mixer, specify exactly one oscillator frequency for f_{BFO} and f_{VFO}.

$f_{VFO} = 110MHz$ or $90MHz$

$f_{BFO} = 10.01MHz$ or $9.99MHz$.

(b.) For each answer in (a.), specify the corresponding image frequency.

$f_{V,i} = 120MHz$ or $80MHz$

$f_{B,i} = 10.02MHz$ or $9.98MHz$

(c.) Specify image rejection filters for each mixer, by selecting a center frequency and ~~3dB bandwidth for each.~~ minimum Q for each.

VFO image rejection: $f_0 = 100MHz$, $BW < 40MHz$ $(Q = 2.5)$ or $f_0 = 100MHz$. $BW < 40MHz$

BFO image rejection: $f_0 = 10MHz$. $BW < 40KHz$. $(Q = 250)$ or $f_0 = 10MHz$. $BW < 40.KHz$

4. Consider the schematic for the crystal (the parallel combination) in the following figure. Note that the crystal is being driven by a voltage source with input impedance Z_0.

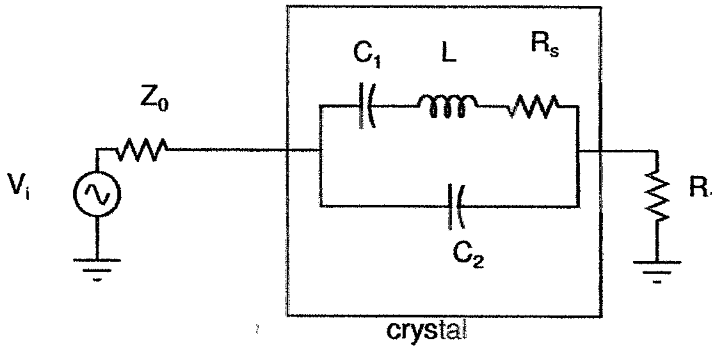


Figure 3: Figure for problem 4.

Usually, the parallel combination has component values which may not be modified by the user. The user may modify the circuit only by changing the external resistor R_1.

$Z_0 = 50\Omega$, and arbitrary R_1

(a.) For $R_s = 0.5\Omega$, $L = 1nH$, $C_1 = 1\mu F$, and $C_2 = 1nF$, find an approximate expression for the resonant frequency.

ignoring C_2, the entire circuit is a series resonant circuit, so

$$\omega_0 \approx \sqrt{\frac{1}{LC}}$$

(b.) Can R_1 be used to adjust the resonant frequency? If so, find the value of R_1 to produce a resonant frequency of $3MHz$. If not, explain what is the purpose of an adjustable R_1.

No, since ω_o is independent of R_1.

The purpose of R_1 is to match impedance at resonance

$Z_0 = R_S + R_1$, for optimal power transfer, and no reflections.