Homework Set 1, Problem 2

Here we explore the assumptions and results associated with the Poisson distribution. Start with an unknown distribution function;

\[ P(n, T), \]

which is the probability of detecting \( n \) photons in a time interval \( T \). Now, assume a short additional time, \( dT \), during which we can expect either one or no photons. The probability of detecting one photon during this time is \( \alpha dt \).

(a) Write an expression for \( P(n + 1, T + dT) \), in terms of \( P(n, t) \) and \( \alpha \).

(b) Now arrange this expression as a differential equation with everything involving \( P(n + 1, ?) \) on the left and \( P(n, ?) \) on the right.

(c) Now, if you are feeling courageous, solve this differential equation. If not, substitute in

\[ P(n, t) = e^{-\bar{n}t} \frac{\bar{n}^n}{n!}, \]

and find an equation for \( \bar{n} \).

(d) Plot the distribution for different values of \( \bar{n} \), in powers of 10, from \( 10^{-2} \) to \( 10^3 \). You may wish to use logarithmic scales on some of these plots.

(e) Show that

\[ \sigma_n = \sqrt{\bar{n}}. \]