G205 Fundamentals of Computer Engineering

CLASS 9, Mon. Oct. 6 2003

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M-W, 9:50am-11:30am, 410 Ell

Sets

- Collection of objects
- As important as in math
- Dynamic sets: Change over time
- Basic techniques for representing and manipulating finite dynamic sets
- Best way of implementing a dynamic set depends on the operations to be performed on the set

Elements of a Dynamic Set

- Each element is seen as an object with different fields
- Often one field is identifies as the key
- Non-key fields are satellite data unused in the set implementation
- Often a total ordering is assumed among the keys of a set

Operations on Dynamic Sets

- Two categories
  - Modifying operations: Change the set
  - Queries: Return information about the set
- Modifying operations
  - Insert(S,x): Insert (element pointed by) x in S
  - Delete(S,x): Remove (element pointed by) x from S

Query Operations

- Search(S,k): Returns a pointer x to an element in S such that key[x]=k, or NIL
- Minimum(S): Returns a pointer x to the element of S with the smallest k
- Maximum(S): Similar to Minimum(S)
- Successor(S,x): Returns a pointer to the next larger element in S, or NIL if x is the maximum
- Predecessor(S,x): Similar to Successor(S,x)

Stacks and Queues

- Simple data structures for representing dynamic sets that use pointers
- Delete operation is prespecified
  - Stack: Delete the most recently inserted element (implements LIFO)
  - Queue: Delete the element in the set for the longest time (implements FIFO)

Stacks

- Implementation of a stack with at most n elements with an array S[1…n]
- top[S] maintains the index of the most recently inserted element in the array
- The stack consist of S[1…top[S]]
- When top[S] is 0, the stack is empty
- We do not worry here with stack overflows (top[S] > n)

Stack Operations

```
Stack-Empty(S)
  return top[S] = 0
Push(S,x)                              // Insert
  top[s] = top[s] + 1
  S[top[S]] = x
Pop(S)                                 // Delete
  if Stack-Empty(S) then error "underflow"
     else top[S] = top[S] – 1
  return S[top[S]+1]
```

Queues

- Implementation of a queue with at most n-1 elements with an array Q[1…n]
- head[Q] maintains the index to the head of the queue (the element first to be removed)
- tail[Q] indexes the next location a new element is inserted
- When head[Q]=tail[Q] the queue is empty
- When head[Q]=tail[Q]+1 the queue is full
- (Addresses are “wrapped around”)

Queues Operations

```
Enqueue(Q,x)                          // Insert
  Q[tail[Q]] = x
  if tail[Q]=n then tail[Q]=1
     else tail[Q]=tail[Q]+1
Dequeue(Q)                            // Delete
 x=Q[head[Q]]
  if head[Q]=n then head[Q]=1
     else head[Q]=head[Q]+1
  return x
```

Linked Lists

- Objects are arranged in linear order
- Order is determined by a pointer (not by an index)
- Support all operations on dynamic sets
- Doubly-Linked List implementation: key, prev and next fields
  - Head of the list has no prev element
  - Tail of the list has no next element
- head[L] points to the first element in the list
- If head[L] is NIL, the list is empty

Different Linked Lists

- Doubly linked lists
- Singly linked lists: No prev pointer
- Circular list
  - The prev pointer of the head of the list points to the tail
  - The next pointer of the tail of the list points to the head
- Lists can be sorted or unsorted

Searching a Linked List

- Finds the first element in the list with a given key
- Linear search that returns a pointer: $\Theta(n)$

```
List-Search(L,k)
  x = head[L]
  while x ≠ NIL and key[x] ≠ k do
    x = next[x]
  return x
```

Inserting Into a Linked List

- Insertion at the front of the list: O(1)

```
List-Insert(L,x)
  next[x]=head[L]
  if head[L] ≠ NIL
    then prev[head[L]]=x
  head[L]=x
  prev[x]=NIL
```

Deleting from a Linked List

- Use Search-List to retrieve the element's pointer: $\Theta(n)$

```
List-Delete(L,x)
  if prev[x]≠NIL
    then next[prev[x]]=next[x]
    else head[L]=next[x]
  if next[x] ≠ NIL
    then prev[next[x]]=prev[x]
```

Rooted Trees

- Each tree node is an object with a key field and pointers
- BINARY TREES:
  - Three pointers: left, right and p to the left child, to the right child and to the parent
  - If $p[x] \neq$ NIL then x is the root
  - root[T] is the root of a tree T
  - If root[T] = NIL then the tree is empty

Unbounded Branches Trees

- Left-child, right-sibling representation
- p is the pointer to the parent and root[T] points to the root
- Each node has only two other pointers:
  - left-child[x] points to the leftmost child of x
  - right-sibling[x] points to the sibling of x immediately to the right

Assignments

- Textbook, pages 196—217
- Updated information on the class web page: www.ece.neu.edu/courses/eceg205/2003fa