Northeastern University
College *of* Engineering

Optical Science Northeastern University Laboratory

Biomedical Imaging

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EECE–4649
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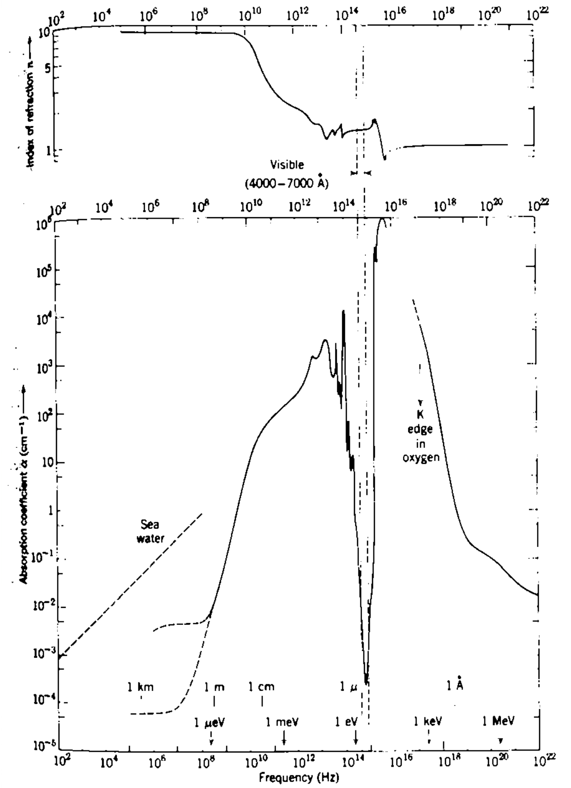
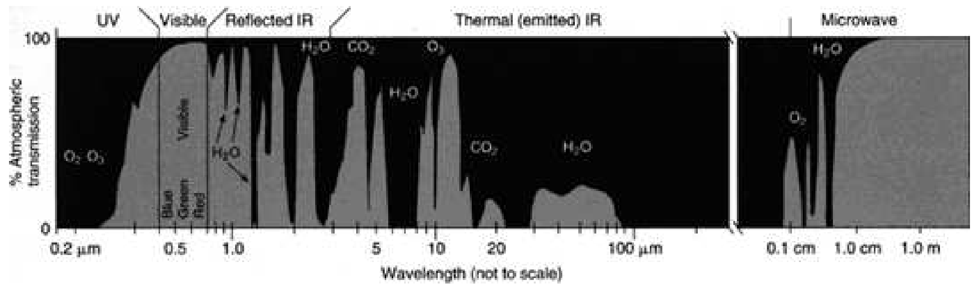
Course Topics

- Background Material
  - Wave Theory. Tissue Properties.
  - Absorption, Scattering, and Reflection
  - Contrast, Resolution, and penetration
- X–Ray, X–Ray CT
- MRI
- Inverse Problems
- Ultrasound 1
  - Microscopy in the laboratory
  - *In–vivo* Microscopy
  - Optical Coherence Tomography
- Endoscopy
- Experimental Techniques

EM Spectrum



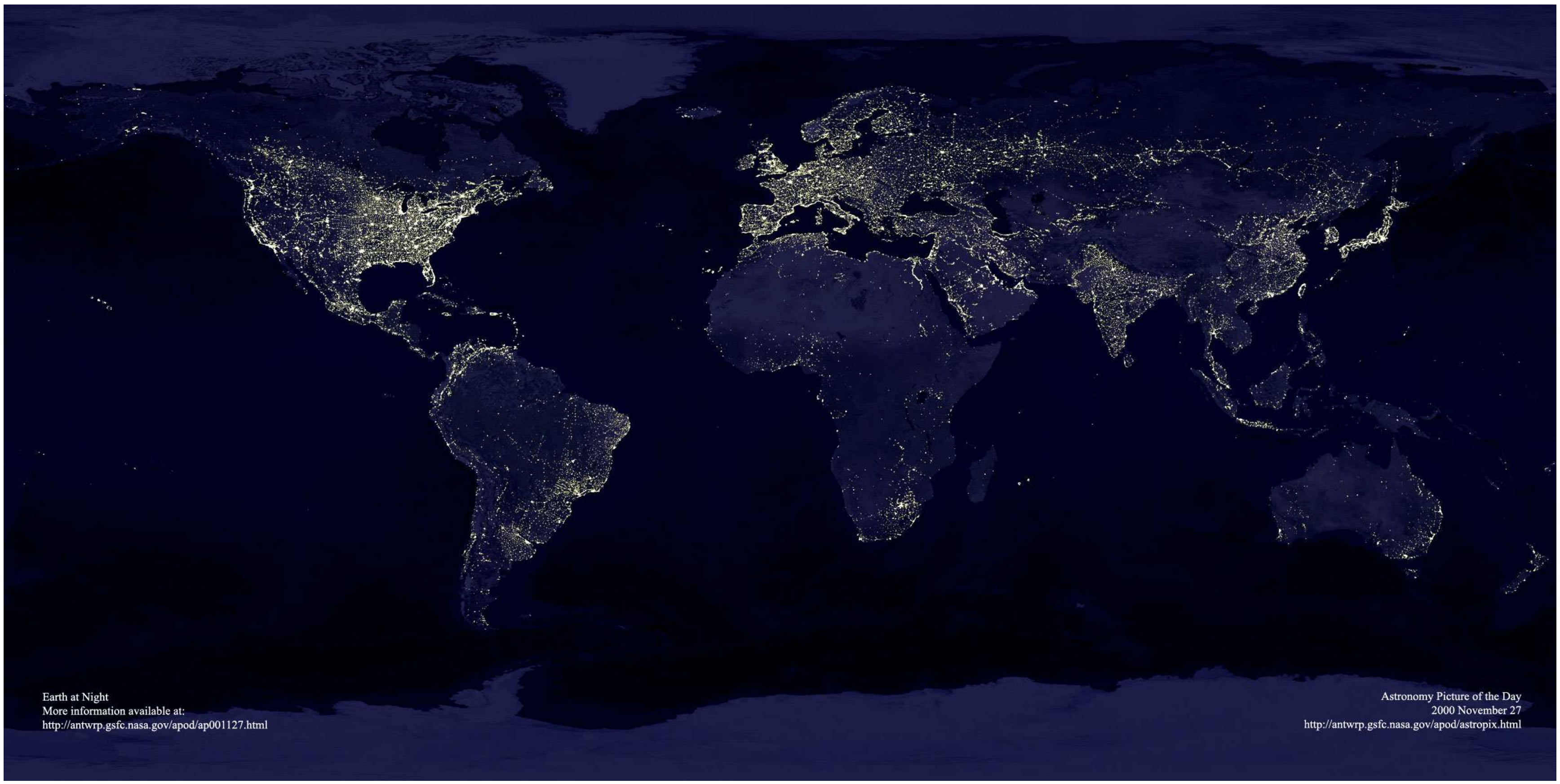
A. Liquid Water



B. Water Vapor

ELECTROMAGNETIC TRANSMISSION. Water strongly absorbs most electromagnetic waves, with the exception of wavelengths near the visible spectrum (A, From Jackson *Classical Electrodynamics*, ©1975). The atmosphere also absorbs most wavelengths, except for very long wavelengths and a few transmission bands (B, NASA's Earth Observatory).

Earthlight



C. Mayhew & R. Simmon (NASA/GSFC), NOAA/ NGDC, DMSP Digital Archive).

Electromagnetic Waves & Electrons

A Wave is a Wave is a Wave (EM, US, Other)

$$E = E_0 e^{j\omega t - kz} \qquad p = p_0 e^{j\omega t - kz}$$

Electromagnetic Waves Interact with Electrons

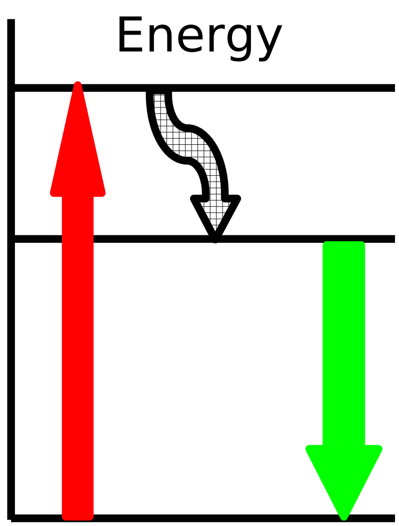
X–Ray	Ultraviolet	Visible/Near–Infrared
picometers	nanometers	nano– to micrometers
Ionization	Orbit to Orbit	Molecules

Infrared	Radio Frequency
micrometers	meters (kHz, MHz)
Vibration/Rotation	Nuclear Spins

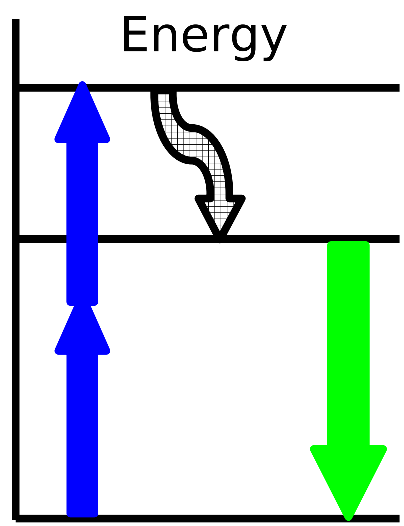
Energy Levels and Transitions

Absorption, ↑   Stimulated Emission, ↓   Spontaneous Emission ↓ nanoseconds?

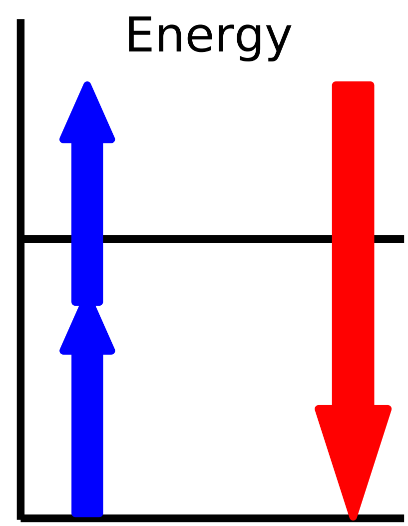
Fluorescence, Harmonic Generation, Mixing, and more



Fluorescence



2–Photon Fluorescence



Second Harmonic

State Populations

Equilibrium:

Boltzman Distribution

$$N(E_n) \propto e^{-E_n/kT} \qquad kT = 25\text{meV} \text{ @ } T = 300K$$

$\lambda = 500$nm	2.5eV	$e^{-E_n/kT} = e^{-98} \approx 10^{-43}$
5μm	250meV	10^{-5}
50μm	25meV	$e^{-1} = 0.37$
5mm	250μeV	0.99

$$E = h\nu = \frac{hc}{\lambda} \text{ Joules} \qquad \frac{hc}{e\lambda} \text{ electronVolts(eV)}$$

Irradiance

Circuits	EM Waves	
$V = IR$	$\left\|\vec{E}\right\| = \left\|\vec{H}\right\| Z$	$Z = \sqrt{\frac{\mu}{\epsilon}}$
V in Volts	$\vec{E}$ in $\frac{\text{Volts}}{\text{meter}}$	
I in Amperes	$\vec{H}$ in $\frac{\text{Amperes}}{\text{meter}}$	
R in Ohms	Z or η in Ohms	
$P = IV$	$\vec{S} = \vec{E} \times \vec{H}$	
	$\left\|\vec{S}\right\| = \left\|\vec{E}\right\| \left\|\vec{H}\right\|$	For Plane Wave ($\vec{E} \perp \vec{H}$)
$P = \frac{V^2}{R}$	$I = \left\|\vec{S}\right\| = \frac{\left\|\vec{E}\right\|^2}{Z}$	Irradiance
P in Watts	I in $\frac{\text{Watts}}{\text{meter}^2}$	

Absorption

- Absorption Cross–Section, σ_a, and Coefficient, μ_a
  - Absorption Efficiency, Q_a

$$P_a = \sigma_a I = Q_a \pi r^2 I \qquad \mu_a = N_V \sigma_a$$

- Beer's Law for Absorption
  - Power In: $P(z) = I(z) A$
  - Power Absorbed: $dP = I(z) N \sigma_a \quad N$ absorbers
  - Number: $N = N_V \times A dz$
  - Power Out: $P(z + dz) = P(z) - dP$

$$\frac{dP}{dz} = -P(z) \mu_a$$

$$P(z) = P(0) e^{-\mu_a z} \qquad I(z) = I(0) e^{-\mu_a z}$$

Absorption and Scattering

- Absorption Cross–Section, σ_a, and Coefficient, μ_a

$$P_a = \sigma_a I = Q_a \pi r^2 I \qquad \mu_a = N_V \sigma_a$$

- Scattering Cross–Section, σ_s, and Coefficient, μ_s

$$P_s = \sigma_s I = Q_s \pi r^2 I \qquad \mu_s = N_V \sigma_s$$

- Extinction Cross–Section, σ, and Coefficient, μ

$$P_{extinguished} = \sigma I = Q \pi r^2 I \qquad \mu = N_V \sigma$$

$$\sigma = \sigma_a + \sigma_s \qquad \mu = \mu_a + \mu_s$$

- Beer's Law Again

$$\frac{dP}{dz} = -P(z)\,\mu$$

$$P(z) = P(0)\,e^{-\mu z} \qquad I(z) = I(0)\,e^{-\mu z}$$

Scattered Wave

- Where does it go? It's Complicated

$$\mu_s p(\theta,\zeta) \qquad \text{Phase Function}$$

  - Small Particles: Rayleigh Scatter (uniform, $1/\lambda^4$)
  - Large Particles: Forward, independent of wavelength
  - Smooth Surfaces: Fresnel Reflection and Transmission (Impedance Contrast)
- Anisotropy and Transport Scattering Coefficient

$$\mu_s' = (1-g)\mu_s \qquad g = \langle\cos\theta\rangle = \int\int p(\theta,\zeta)\cos\theta d\zeta d\theta$$

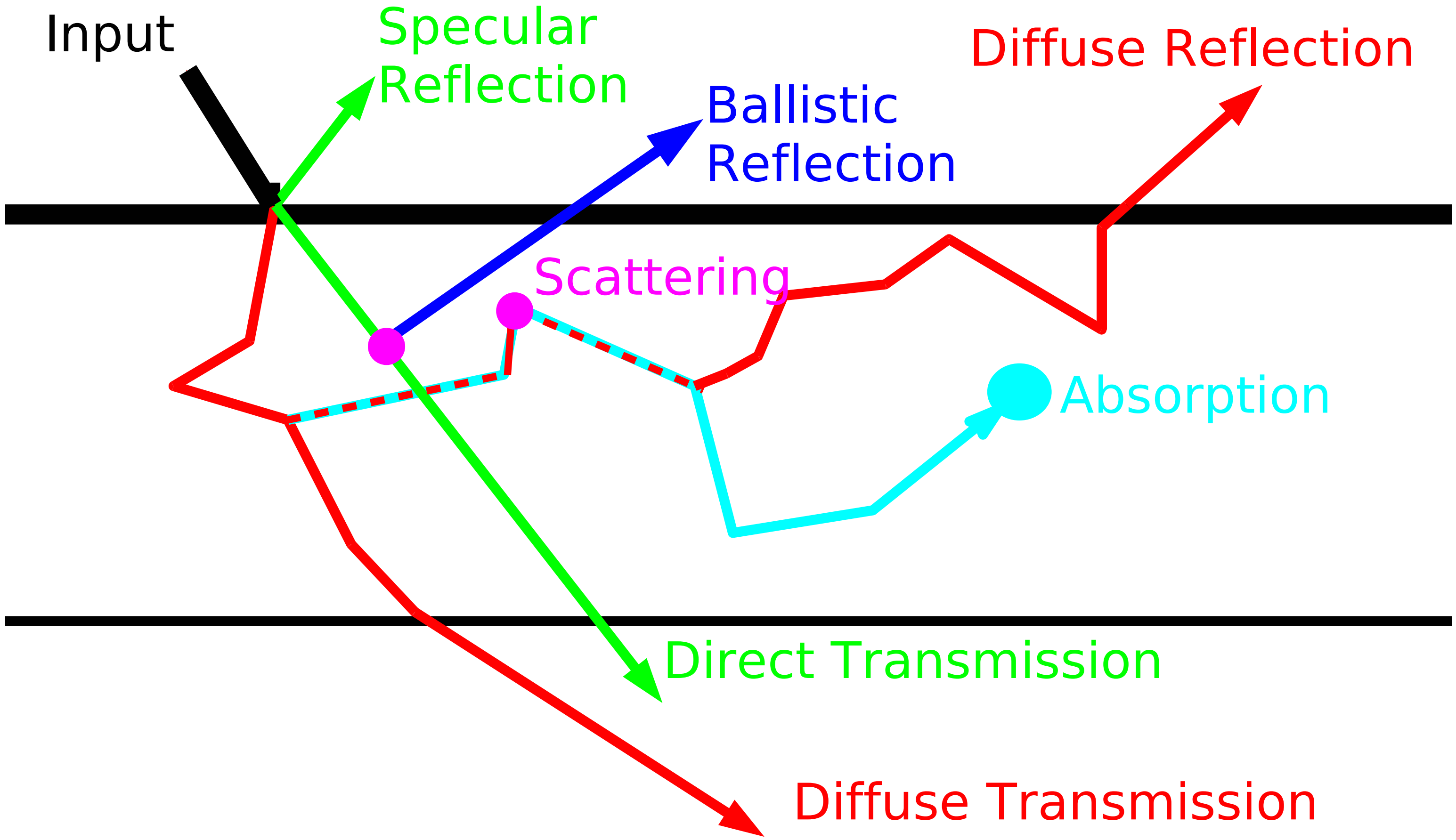
- Detailed Calculations
  - Spheres, Cylinders, *etc.*:
    Mie Scattering; https://omlc.org/calc/mie_calc.html
  - Other Shapes: FDTD, Other

Scattering and Absorption

- Multiple Scattering
  - More Chances for Exit (T or R)
  - More Chances for Absorption
- Light Diffusion
- Albedo

$$W = \frac{\text{Scattering}}{\text{Extinction}} = \frac{\mu_s}{\mu_s + \mu_a}$$

Waves Interactions



(and of course, emission)

Transmission

- Fresnel Reflection
- Absorption: Beer's Law

$$P(z) = P(0)\, e^{-\mu z}$$

- Phase
  - Speed $= c/n$ where n is the index of refraction
  - Time = distance/speed
  - Phase $= 2\pi \times$ Time/period

$$\phi = 2\pi \frac{\int n d\ell}{\lambda}$$

- Refraction: Snell's Law

$$n \sin\theta = n'\theta'$$

Diffraction

- Angular Divergence

$$\alpha \approx \frac{\lambda}{D}$$

- Spot Size

$$d \approx \frac{\lambda}{D} z$$

- Numerical Aperture

$$d \approx \frac{\lambda}{NA} \qquad NA = n \sin\theta \approx \frac{z}{(D/2)}$$

$$\Delta z \approx \frac{n\lambda}{NA^2}$$

Example: $\lambda = 10^{-6}\text{m}$ $\qquad D = 10^{-4}\text{m}$
$\alpha = 10^{-2}\text{radians}$ $\qquad d = 10^{-2}\text{m}$ @ 1m $\quad (NA \approx 5 \times 10^{-5})$

What is Imaging?

- Wave Source(s)
- Path to Target (Diffraction, Absorption, Scattering)
- Target Contrast (Absorption, Scattering, Fluorescence, Life-time, Phase, *etc.*)
- Path from Target (Diffraction, Absorption, Scattering)
- Detector(s)
- Signal Processing (Inverse Problem)
- Decision

Decision, Decisions

Suppose we Can Find Probability Density of Detection and False Alarm. How do we (1) Make Good Decisions with a Measurement and (2) Compare Different Instruments?

- Cost Function
  - Set a Threshold and Compute "Cost"
  - Vary the Threshold to Minimize Cost

$$C = C_{miss} P_{disease} \left(1 - P_{Detection}\right) + C_{false} \left(1 - P_{disease}\right) P_{false}$$

  - But How do we Decide the Costs?
- Receiver Operating Characteristic
  - Plot with Threshold as a Parameter
    - Probability of False Alarm (1-specificity) Horizontal
    - Probability of Detection (sensitivity) Vertical
  - Measure Area Under Curve?