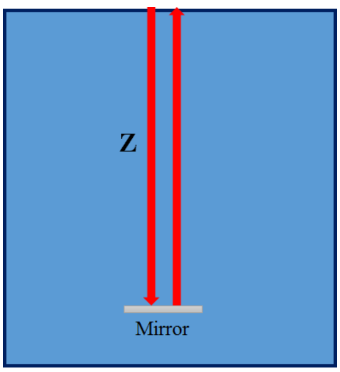
Dialogue of Civilizations 2018 - Biomedical Imaging Homework 1

Due date: 5/22/2017 - Revision v0.3

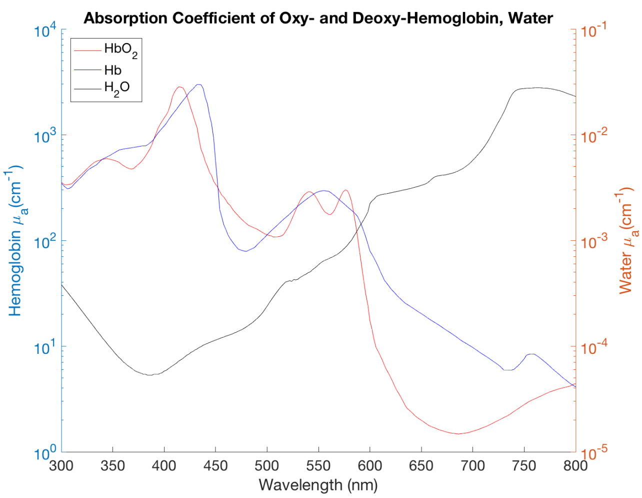
Please solve the following problems and email the solutions to *kercher.e@husky.neu.edu*.

1 Beer's Law

(a) A mirror is immersed in a fluid as shown in Figure 1a. You want to know the maximum depth Z the mirror can be placed below the surface of the fluid such that the reflected light can still be seen by a detector at the surface for red light ($\lambda = 650nm$), green light ($\lambda = 550nm$), and blue light ($\lambda = 450nm$). Find Z at the three wavelengths when the fluid is (i) water and (ii) blood, using the absorption coefficients from the data provided in Figure 1b. What do your results tell you about the usefulness of red vs. blue light when imaging in tissue? Explain your reasoning.



(a)



(b) Data from www.omlc.org

Figure 1: Problem 1a

(b) Two X-ray beams with $\lambda = 0.1nm$ and equal initial intensity I_0 pass through a phantom as shown in Figure 2. The specific parameters of the tissue are provided in

Table 1. Calculate the total attenuation of beams I_1 and I_2 after passing through the phantom. What would this phantom look like on x-ray film?

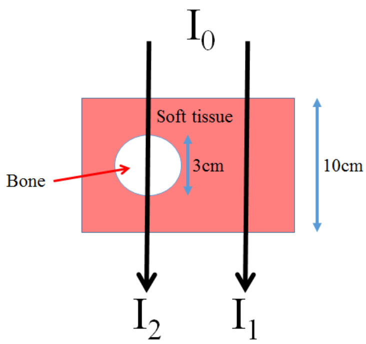


Figure 2: Problem 2b

Table 1: Phantom tissue parameters for problem 1.2

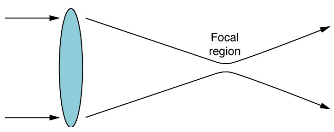
	Soft Tissue	Bone
Mass Absorption Coefficient $[cm^2/g]$	5.38	28.51
Density $[g/cm^3]$	1.06	1.92

2 Diffraction

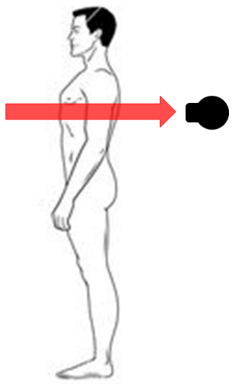
(a) A beam diffracts through a lens with NA $= 0.9$ as shown in Figure 3a. Calculate the beam diameter and depth of focus for green light ($\lambda = 532nm$) and near-infrared (NIR) light ($\lambda = 1152nm$).

(b) Repeat your calculations for a lens with NA $= 0.1$ for both green and NIR light sources. Compare and contrast your results from (a) and (b).

(c) A patient goes the doctor to get a chest X-ray (Figure 3b). Assuming the X-ray beam has a wavelength of $\lambda = 1nm$, how small does the beam diameter need to be so that the depth of focus covers the whole body? Please state any assumptions you make about the size of the body.



(a) Problem 2.1 & 2.2



(b) Problem 2.3

Figure 3

3 More Diffraction

The double slit experiment is a common introductory physics experiment that demonstrates the wave-like properties of light. Two slits of width b sit a distance a apart from each other (Fig. 4); light travels through the slits and creates an interference pattern on a screen placed a distance z away. This irradiance pattern as a function of the incident angle θ is given by

$$I(\theta) = 4I_0 \frac{\sin^2(\beta)}{\beta^2} \cos^2(\alpha) \tag{1}$$

where,

$$\alpha = \frac{1}{2} ka \sin\theta \quad \& \quad \beta = \frac{1}{2} kb \sin\theta \tag{2}$$

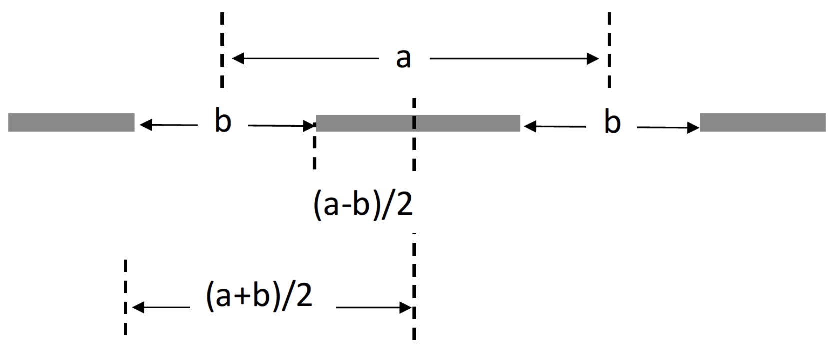


Figure 4: Problem 3

Using Matlab, plot the irradience pattern of red light ($\lambda = 650nm$) on a screen (*i.e.*, as a function of x) 2 meters away from the slits for $a = 5mm$ and $b = 150\mu m$.

4 Reflection

Fresnel reflection is the reflection of light at the intersection of two media with differing indices of refraction. As the result of this mismatch in the index of refraction, some of the incident light transmits through and some is reflected off the surface. The amount of reflection depends on both the angle of incidence and the polarization state of the light. Use the Matlab code `fresnel.m` posted on the course website to answer the following problems. This code calculates the reflection and transmission coefficients for S- and P-polarization states, given the incident angle and index of refraction ratio n_2/n_1.

(a) Unpolarized light is directed at a soft tissue sample with $n = 1.4$. Calculate the reflection as a function of incident angle θ for soft tissue at a boundary with (a) air and (b) water, as shown in Figure 5. Plot your results on **one** graph **Hint:** The total reflection for unpolarized light is $R = \frac{(\rho_s^2+\rho_p^2)}{2}$.

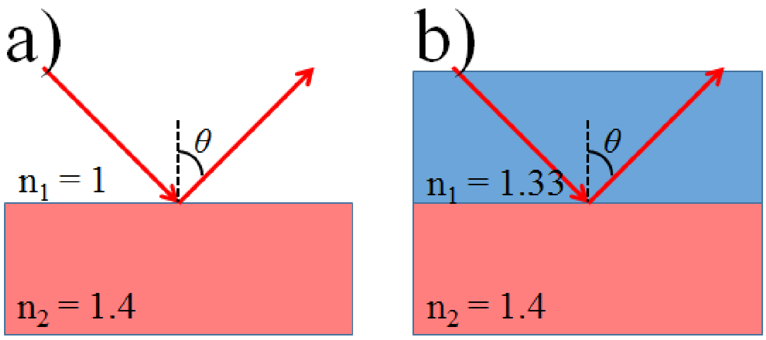


Figure 5: Problem 3.1

(b) An index-matched oil is placed on the soft tissue sample. Re-calculate the reflection as a function of incident angle and describe what happens. In what scenario might this phenomenon be useful? You do not need to plot these results.