Chapter 1

INFORMATION THEORETIC ORGANIZATION PRINCIPLES FOR AUTONOMOUS MULTIPLE-AGENTS

Deniz Erdogmus, Jose C. Principe and Rajvignesh Thogulua
Computational NeuroEngineering Laboratory
Electrical and Computer Engineering Department
University of Florida, Gainesville, FL 32611

Abstract An increased interest in autonomous swarms and their applications has motivated research in a variety of directions. The possible applications for intelligent self-organizing agents range from military to humanitarian. An important desired feature of self-organizing multiple agents is decentralized decision making for fault-tolerant mission accomplishment capability. In this paper, we propose a principled decentralized organization approach that roots from the concept of information theoretic particle interactions. As an example, the problem of self-organizing a set of multiple agents uniformly over a circular region in a two dimensional space. In addition, variations to the proposed approach will be demonstrated for target tracking and obstacle avoiding tasks.

1. Introduction

The idea of swarm intelligence is biologically inspired by the collective behaviour of insect societies, which produce complex cooperative activities [13, 19]. Swarms found applications in versatile fields including oceanographic sampling [23], communication networks [25], material transportation in hazardous zones [9], and planetary missions [14].

Leader-follower based self-organization strategy had been investigated by many researchers. However, in many applications, this scheme is not desirable due to the possibility of a system failure in the case of a malfunctioning leader. Decentralized control is essential and vital to swarms operating in such scenarios, since it introduces robustness to the system. Some reasons for preferring decentralized approaches include fault-tolerant operation even when some of the agents fail, reduced communication and calculation load for the agents,

especially for the to be leader, and reduced design complexity for the leader agent, as well as uniform design specifications for easy production and system up scaling.

In this paper, we propose a decentralized self-organization approach based on information theoretic interactions between the particles, i.e., the agents in the swarm. In this approach, we form an analogy between the interaction rule between the agents and the physical forces due to the gradient of a potential field in physics. This is the motivation behind calling the agents in the swarm as particles in a potential field. Specifically, since the potential field in the analogy is related to the information theoretic concept of entropy, this field is named as the information potential, thus its gradient in space becomes the information force. The notions of information potential and information forces were first introduced by Principe *et al.* [17] in the context of filter adaptation. In this application to the self-organization of swarms, we assume that each agent need not know its own absolute position as well as the positions of the other agents.

The roots of information theory go back to the seminal work of Shannon (1948). Originally, Shannon was determined to address the quantification of information flow through digital communication channels and understanding the limitations imposed by the representations used for transmitting data. In the following decades, information theory not only influenced greatly the area of communication engineering [21, 8], but it has evolved to be mathematical theory itself [6, 5], which has had tremendous impact on many areas of science and engineering, including biology [26], physics [12], and signal processing [11, 3]. In general, the mathematical information theory deals with the statistical implications of the associated definitions, such as entropy and mutual information [5]. In addition, the question of geometric structures lying under statistical function spaces and their implications on learning and adaptive systems has been a focal point of contemporary research in the field [1].

In this paper, we are specifically interested in the problem of uniformly distributing the agents over a region, selected to be circular in this case. The ideas presented here could be modified to accommodate the application of the principles to the uniform distribution of them to other forms. It is well known in information theory that the distribution that maximizes entropy, defined as the average information or uncertainty, under the constraint of bounded finite support is uniform. Therefore, it is possible to achieve the task of uniformly distributing the agents over a circular region by maximizing the entropy of the particles. Entropy, however, is a function of the continuous distribution function, which needs to be defined smoothly over the region. In this situation, the agents act as samples from a random variable distribution

2. Background on Information Theory

Although information is a relatively simple concept to conceive, its mathematical representation and associated properties had started to be formulated with the groundbreaking work of Shannon (1948). Although the information that a random event provides had been defined before as $\log(1/p)$, where p is the probability of that event occurring, Shannon was the first to define and utilize exclusively the quantity called average information or entropy. Given a set of random events with probabilities $\{p_i\}$, i=1,...,N the average information of these events is given by

$$H_S(\{p_i\}) = -\sum_{i=1}^{N} p_i \log p_i \tag{1}$$

Although by definition this quantity is the expected value of the information over the set of events under consideration, its reducing when the probability mass function (pmf) $\{p_i\}$ approaches a δ-distribution and increasing when it approaches a uniform distribution creates an analogy with the entropy concept in physics, leading to the name entropy. It is therefore, also a measure of the uncertainty about the value of the random variable that has this probability distribution. For continuous random variables, the definition of entropy is easily extended. In that case, the differential entropy of a random variable X with probability density function (pdf) $f_X(.)$ becomes

$$H_S(X) = -\int_{-\infty}^{\infty} f_X(x) \log f_X(x) dx \tag{2}$$

The differential entropy is minimized when the pdf approaches to a δ-train type distribution and is maximized for a uniform density under the finite bounded support constraint [5].

An alternative entropy definition is introduced by Renyi after relaxing some assumptions that the entropy needs to satisfy. Renyi's order-α entropy for X is defined as follows [18].

$$H_\alpha(X) = \frac{1}{1-\alpha} \log \int_{-\infty}^{\infty} f_X^\alpha(x) dx \tag{3}$$

Shannon's entropy is a special case of Renyi's definition corresponding to α=1. In addition, the minimum and maximum of any order entropy appear at the same pdfs as in Shannon's entropy described above. In this paper, we are specifically interested in the maximization of entropy under the finite bounded support constraint, since our objective is to distribute the agents (samples) uniformly over the selected region.

3. Nonparametric Estimation of Renyi's Entropy

Estimating the entropy of a random variable requires the knowledge of the underlying pdf. In general, only a finite number of samples are available and the analytical expression for the pdf is unknown. Therefore, we resort to nonparametric estimation methods. First, one needs to obtain an estimate of the pdf. Once this estimate is obtained, it can be plugged in the entropy definition to evaluate the entropy of the random variable whose samples are provided. This approach is referred to as the plug-in estimation method in the entropy estimation literature [2]. One pdf estimation method is particularly useful in the case of estimating Renyi's entropy. Parzen windowing allows the smoothing of the empirical sample distribution through the incorporation of kernels. Suppose we are given N samples $\{x_i\}$, i=1,...,N of the random variable X and the kernel function to be used in the Parzen pdf estimate is $\kappa_\sigma(\cdot)$. The kernel function must be a valid pdf and it is required to be a smooth, continuous and differentiable function for our purposes. Gaussian density is perhaps the most popular choice. The rectangular density (uniform) is also interesting as it corresponds to the sliding histogram density estimation method. Under these circumstances, the estimated pdf of X is [16]

$$\hat{f}_X(x) = \frac{1}{N}\sum_{i=1}^{N} \kappa_\sigma(x - x_i) \tag{4}$$

In the case of multidimensional random vectors, the kernel function needs to be multidimensional. This joint kernel is required to be the product of single-dimensional kernels. Specifically, if the samples are n dimensional, then the joint kernel to be used for joint density estimation must be evaluated by

$$\kappa_\Sigma(x) = \prod_{o=1}^{n} \kappa_{\sigma_o}(x^o) \tag{5}$$

where $\kappa_{\sigma_o}(\cdot)$ is the single-dimensional kernel function for the o^{th} dimension. In the kernel notation, the subscript σ denotes the kernel size or the window length. For example, in the case of Gaussian kernels, the kernel size is usually controlled by the standard deviation.

Now, consider the quadratic entropy from Renyi's family. For α=2, the argument of the log is defined as the (quadratic) information potential. The information potential can be estimated nonparametrically from the samples of X using Parzen windowing with Gaussian samples. Substituting this pdf estimate in the definition, we get

$$\begin{aligned}\hat{V}_2(X) &= \int_{-\infty}^{\infty} \hat{f}_X^2(x)dx = \int_{-\infty}^{\infty} \left(\tfrac{1}{N} \sum_{i=1}^{N} G_\sigma(x - x_i) \right)^2 dx \\ &= \tfrac{1}{N^2} \int_{-\infty}^{\infty} \left(\sum_{i=1}^{N} G_\sigma(x - x_i) \right) \left(\sum_{j=1}^{N} G_\sigma(x - x_j) \right) dx \\ &= \tfrac{1}{N^2} \sum_{j=1}^{N} \sum_{i=1}^{N} \int_{-\infty}^{\infty} G_\sigma(x - x_j) G_\sigma(x - x_i) dx \\ &= \tfrac{1}{N^2} \sum_{j=1}^{N} \sum_{i=1}^{N} G_{\sigma\sqrt{2}}(x_j - x_i)\end{aligned} \tag{6}$$

The important point here is that, in this estimator for Renyi's quadratic entropy, there are no approximations apart from the explicit Parzen window application. In addition, the shape of the kernel is preserved during the integration process. If, in addition to the Parzen pdf estimate, also the sample mean approximation is introduced, then it becomes possible to nonparametrically estimate any order entropy using any suitable kernel function. This is achieved by the equivalent definition of entropy given as an expectation.

$$H_\alpha(X) = \frac{1}{1-\alpha} \log E_X \left[f_X^{\alpha-1}(X) \right] \tag{7}$$

Specifically, for entropy order α and kernel function $\kappa_\sigma(.)$ the plug-in estimator becomes [7]

$$H_\alpha(X) \approx \frac{1}{1-\alpha} \log \frac{1}{N^\alpha} \sum_{j=1}^{N} \left(\sum_{i=1}^{N} \kappa_\sigma(x_j - x_i) \right)^{\alpha-1} \tag{8}$$

Notice in (6) and (8) that the entropy estimate relies on the pair-wise interactions between the samples through the kernel function. Particularly, the kernel function could be regarded as a potential field emanating from the samples to form the total information potential experienced by other samples. For example, in (6), the summation over the index i could be conceived as the superposition of information potential contributions of all other samples to sample j. The summation over j can then be understood as the summation of the potentials of all the particles to determine the overall particle-system potential [17].

4. Information Particles

The information particle interaction idea has been recently introduced [17] and has been successfully utilized in many problems including independent component analysis, nonlinear principal components analysis, and SAR image feature extraction. The principle was generalized into a general particle interaction framework [7], which encompasses the original information particle interaction model for adaptation and self-organization as a special case

corresponding to a specific choice of the particle potential functions. In this section, we will briefly describe the general particle interaction model for self-organization.

Now, suppose that the sample values $\{x_1,\ldots,x_N\}$, correspond to the particle position coordinate vectors in the current analogy. For simplicity, assume we are dealing with a single dimensional space (extension to multi-variable case is trivial). We assume that each particle emanates a potential field. If the potential field that is generated by each particle is $v(\xi)$, we require this function to be continuous and differentiable (except possibly at the origin), and to satisfy the even symmetry condition $v(\xi) = v(-\xi)$; in the multidimensional case, this condition can be changed to become a circular symmetry constraint. With these definitions, we observe that the potential of particle x_j due to particle x_i is $V(x_j|x_i) = v(x_j - x_i)$. The total potential energy of x_j due to all the particles is then given by

$$V(x_j) = \sum_{i=1,i\neq j}^{N} V(x_j|x_i) = \sum_{i=1,i\neq j}^{N} v(x_j - x_i) \tag{9}$$

With analogy to physics, the information force, or the particle interaction force in general, is defined as the gradient of this potential with respect to the particle position (i.e., its value). For particle j, the interaction force due to the potential field emanating from particle i is obtained as

$$F(x_j|x_i) = \frac{\partial V(x_j|x_i)}{\partial x_j} = \left.\frac{\partial v(\xi)}{\partial \xi}\right|_{\xi=(x_j-x_i)} = v'(x_j - x_i) \tag{10}$$

from which the total force acting on particle j is found to be

$$F(x_j) = \sum_{i=1,i\neq j}^{N} F(x_j|x_i) = \sum_{i=1,i\neq j}^{N} v'(x_j - x_i) \tag{11}$$

We have assumed that the force applied to a particle by itself is zero by definition. In the case of information potential as defined in (6) or (8), the self-force of a particle is automatically zero, due to the derivative of the kernel function being zero at the origin. In other potential definitions, which might as well be discontinuous at the origin (such as the gravitational field), the force can be defined to be zero at zero distance. Some interesting special cases of particle interaction potentials include the following:

L_p-Norm Type Potential: Consider, for example, the potential function choice of $v(\xi) = \xi^2/(2N^2)$. Then, upon direct substitution of this in (9), and summing these potentials over all particles (which means a summation over j), we obtain the total potential of the particle set as the sample variance. In general,

for potential functions of the form $v(\xi) = |\xi^p|$, where $p > 1$, the total potential becomes

$$V(x) = \sum_{j=1}^{N} \sum_{i=1}^{N} |(x_j - x_i)^p| \tag{12}$$

which is related to the absolute central moments of the particle distribution. Each value of p corresponds to a different choice of the distance metric between the particles from the family of Minkowski norms.

Information Potential: As we have seen in (6), the quadratic information potential corresponds to the choice of Gaussian potential functions. In general, any pdf could be used, leading to more general quadratic-entropy-related particle interaction laws. For non-quadratic-entropy-based interaction laws, the information potential must be defined in consistency with the entropy estimator in (8). In this case, the information force acting on particle j could be written in the following form.

$$\begin{aligned} F_\alpha(x_j) &= \tfrac{\alpha-1}{N^\alpha} \left(\sum_{i=1}^{N} \kappa_\sigma(x_j - x_i) \right)^{\alpha-2} \left(\sum_{i=1}^{N} \kappa'_\sigma(x_j - x_i) \right) \\ &= (\alpha-1) f_X^{\alpha-2}(x_j) \left(\tfrac{1}{N^2} \sum_{i=1}^{N} \kappa'_\sigma(x_j - x_i) \right) \\ &= (\alpha-1) \hat{f}_X^{\alpha-2}(x_j) F_2(x_j) \end{aligned} \tag{13}$$

This reveals the interesting fact that order-α information force is directly related to the quadratic information force, the difference being the scaling factor based on the estimated probability density of particle j. Therefore, it is possible to manipulate how particles in dense and sparse regions of the particle set experience information forces by selecting the entropy order properly. In particular, selecting $\alpha > 2$ will emphasize the forces experienced by particles in dense regions, whereas it will deemphasize the forces acting on the particles in sparse regions. Similarly, for $\alpha < 2$, the forces acting on particles in the sparse regions will be boosted, while the forces on particles in dense regions is reduced.

Inverse-Distance-Squared Potential: This type of potential law is relatively easy to implement using the natural physical behavior of electromagnetic signal intensity. For this interaction law, the potential function is simply $v(\xi) = A/\xi^2$. Since this potential is not defined at the origin, the force at zero distance must be set to zero by definition. The behavior of particles under this interaction law will be similar to the motion of particles under gravitational or electrical charge influence.

5. Self-Organization of Multiple Agents Using Particle Interaction Principles

The application of the particle interaction principles outlined in the preceding sections is quite straightforward. In the multiple-agent setup, we consider each agent (robot) to be a particle and the main task under consideration in this paper is to distribute the agents uniformly over a circular region in two-dimensional space. The circular regions commonly used in self-organizing swarm case studies. For example, Unsal and Bay use a quite restrictive algorithm that spreads the robots uniformly over a circular region, which requires all the robots to know the absolute positions of every robot [24].

In the entropy maximization scheme that is being presented in this paper, spreading of the robots must be counter-acted by a controlling force at the boundary of the desired circular region. Otherwise, the repulsive forces that the robots exert on each other will lead to unbounded spreading of the particles. This can be achieved by comparing the total potential that is measured by a robot with a predetermined threshold, which is a function of the potential function selected, number of robots, and the desired radius. The determination of an analytical function for this threshold is a daunting task. It requires solving a complicated optimization problem similar to sphere packing [4], yet it is different in the sense that the radii of the *spheres* (which is analogous to the interaction radius of each particle at the balance point) need not be equal, nor they are known *a priori*.

Suppose that the potential function generated by each robot is $v(\xi)$, where ξ is the distance to the particle. Then, the potential on robot j due to robot i is simply $v(\mathbf{p}_j - \mathbf{p}_i)$. The overall potential of particle j due to the superposition of all potentials from all the other robots is given by

$$V_j(\mathbf{p}_1, ..., \mathbf{p}_N) = \sum_{i=1, i \neq j}^{N} v(\mathbf{p}_j - \mathbf{p}_i) \tag{14}$$

The interaction force on this particle, also taking into account the direction of the force based on the threshold comparison, is therefore,

$$\begin{aligned} F_j(\mathbf{p}_1, ..., \mathbf{p}_N) &= sign(\gamma - V_j(\mathbf{p}_1, ..., \mathbf{p}_N)) \frac{\partial V_j(\mathbf{p}_1, ..., \mathbf{p}_N)}{\partial \mathbf{p}_j} = \\ &= sign(\gamma - V_j(\mathbf{p}_1, ..., \mathbf{p}_N)) \sum_{i=1, i \neq j}^{N} \frac{\partial v(\mathbf{p}_j - \mathbf{p}_i)}{\partial \mathbf{p}_j} \end{aligned} \tag{15}$$

Since the *sign* function will create a bang-bang type control action, for smoother dynamics, it can be approximated by a sigmoid function, such as *arctan*, in practice. This will reduce unnecessary control actuator fluctuations. If these forces are assumed to be the velocity commands for the robots, then

the overall collective behavior of the particles can be summarized with the following differential equation, where $\dot{\mathbf{p}}$ denotes differentiation with respect to time.

$$\begin{bmatrix} \dot{\mathbf{p}}_1 \\ \vdots \\ \dot{\mathbf{p}}_j \\ \vdots \\ \dot{\mathbf{p}}_N \end{bmatrix} = \begin{bmatrix} F_1(\mathbf{p}_1, ..., \mathbf{p}_N) \\ \vdots \\ F_j(\mathbf{p}_1, ..., \mathbf{p}_N) \\ \vdots \\ F_N(\mathbf{p}_1, ..., \mathbf{p}_N) \end{bmatrix} \tag{16}$$

Clearly, this control algorithm is designed to guarantee stable convergence to a stationary configuration of the robots in the circular region. This is guaranteed by the repulsive interactions between the particles and the threshold switching of the control command direction. Nevertheless, there are more than one stationary solutions of the dynamical system given in (16). Let $F(\mathbf{p})$ denote the right hand side of (16), where $\mathbf{p}$=[$\mathbf{p}_1$,...,$\mathbf{p}_N$] is the concatenated position vector. The stationary points of (16) are given by the solutions of $F(\mathbf{p})$=$\mathbf{0}$, which is extremely difficult. The local (linearized) stability of these stationary solutions are controlled by the Jacobian matrix

$$\frac{\partial F(\mathbf{p})}{\partial \mathbf{p}} = \begin{bmatrix} \partial F_1(\mathbf{p})/\partial \mathbf{p}_1 & \cdots & \partial F_1(\mathbf{p})/\partial \mathbf{p}_N \\ \vdots & \ddots & \vdots \\ \partial F_N(\mathbf{p})/\partial \mathbf{p}_1 & \cdots & \partial F_N(\mathbf{p})/\partial \mathbf{p}_N \end{bmatrix} \tag{17}$$

In (17), the Jacobian matrix is written in block form, where each of the block entries $\partial F_j(\mathbf{p})/\partial \mathbf{p}_i$ is the Jacobian of each individual force vector with respect to its own particle. These block entries satisfy an interesting structural identity. Notice that the diagonal blocks are easily obtained as (assuming that the *arctan* smoothing function replaces *sign*)

$$\begin{aligned} \frac{\partial F_j(\mathbf{p})}{\partial \mathbf{p}_j} &= \frac{-1}{[1+(\gamma - V_j(\mathbf{p}))^2]} \frac{\partial V_j(\mathbf{p})}{\partial \mathbf{p}_j} \frac{\partial V_j(\mathbf{p})}{\partial \mathbf{p}_j}^T + \arctan(\gamma - V_j(\mathbf{p})) \frac{\partial^2 V_j(\mathbf{p})}{\partial \mathbf{p}_j^2} \\ \frac{\partial F_j(\mathbf{p})}{\partial \mathbf{p}_i} &= \frac{-1}{[1+(\gamma - V_i(\mathbf{p}))^2]} \frac{\partial V_j(\mathbf{p})}{\partial \mathbf{p}_i} \frac{\partial V_j(\mathbf{p})}{\partial \mathbf{p}_j}^T + \arctan(\gamma - V_j(\mathbf{p})) \frac{\partial^2 V_j(\mathbf{p})}{\partial \mathbf{p}_i \partial \mathbf{p}_j} \end{aligned} \tag{18}$$

However, due to the identities

$$\frac{\partial V_j(\mathbf{p})}{\partial \mathbf{p}_i} = -\frac{\partial V_j(\mathbf{p})}{\partial \mathbf{p}_j} \qquad \frac{\partial^2 V_j(\mathbf{p})}{\partial \mathbf{p}_i \partial \mathbf{p}_j} = -\frac{\partial^2 V_j(\mathbf{p})}{\partial \mathbf{p}_j^2} \tag{19}$$

the off-diagonal blocks of the Jacobian matrix become

$$\frac{\partial F_j(\mathbf{p})}{\partial \mathbf{p}_i} = -\frac{\partial F_j(\mathbf{p})}{\partial \mathbf{p}_j} \tag{20}$$

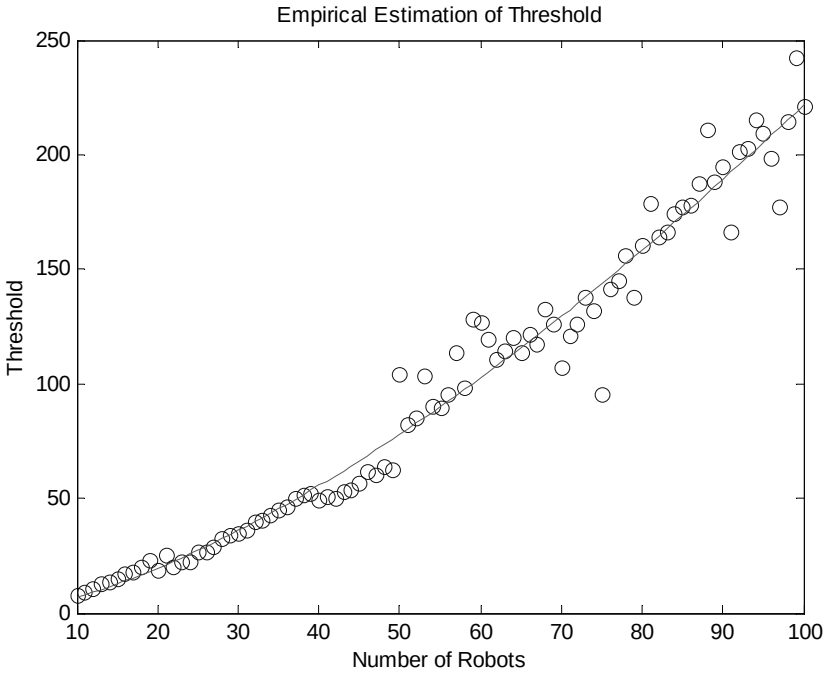

Figure 1.1. Empirical estimation and approximation of threshold

For local stability of the stationary points, the eigenvalues of the Jacobian matrix in (17) must have negative eigenvalues when evaluated at the point of interest.

In addition, the information potential of each robot readily provides a Lyapunov energy function that demonstrates asymptotic stability. Specifically, if we let $\{\tau_j\}$, j=1,...,N denote the information potentials of the robots at a particular stationary solution $\mathbf{p}^*$, then the following will be a Lyapunov function.

$$V = \sum_{j=1}^{N} |V_j(\mathbf{p}) - \tau_j| \tag{21}$$

An important point to mention at this point is that, the particle interaction model that is presented here requires each robot to know only the total potential that it experiences at its current position and its gradient with respect to its position. A variety of implementations to extract this required information could be devised.

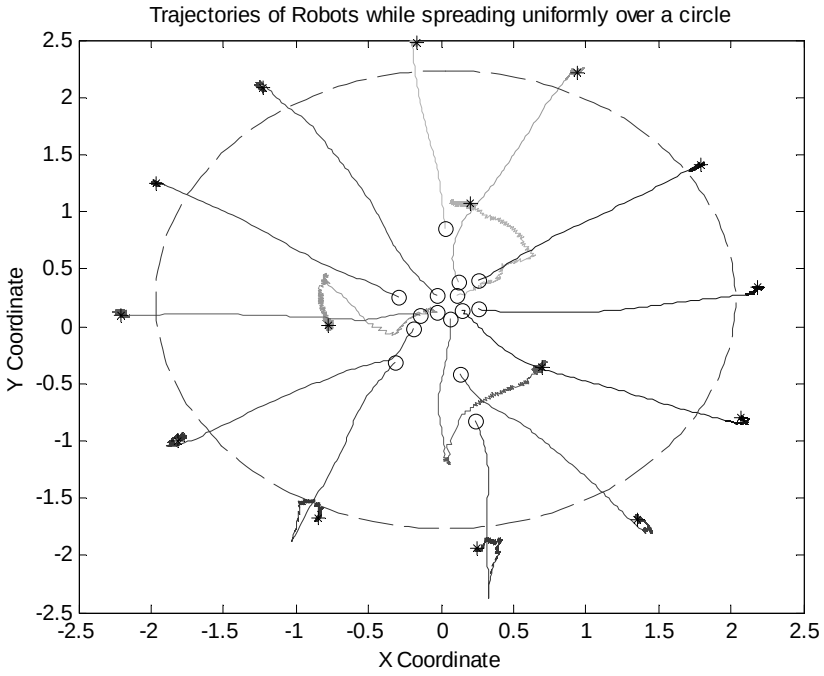

Figure 1.2. Trajectories of robots while spreading themselves uniformly over a circle.

6. Case Study Using a Particular Implementation

In this section, we investigate the performance of the particle interaction self-organization model for a particular choice of the potential function. Specifically, we will concentrate on the inverse-distance-squared type potentials. Under this assumption, the potential field that emanates from a particle obeys the following rule as a function of distance, d.

$$v(d) = A/d^2 \tag{22}$$

Self-Organizing in a Circular Region: For a given number of robots, it is possible to approximately compute the threshold that will yield a unit radius. Due to the potential function in (22), we expect from symmetry and scalability that $\gamma(A, r, N) = A\gamma(1, 1, N)/r^2$. Using 10 Monte Carlo simulations for each of the N values (number of robots), we have empirically determined $\gamma(1, 1)$ to be approximately in the following form.

$$\gamma(1, 1, N) = aN^b \tag{23}$$

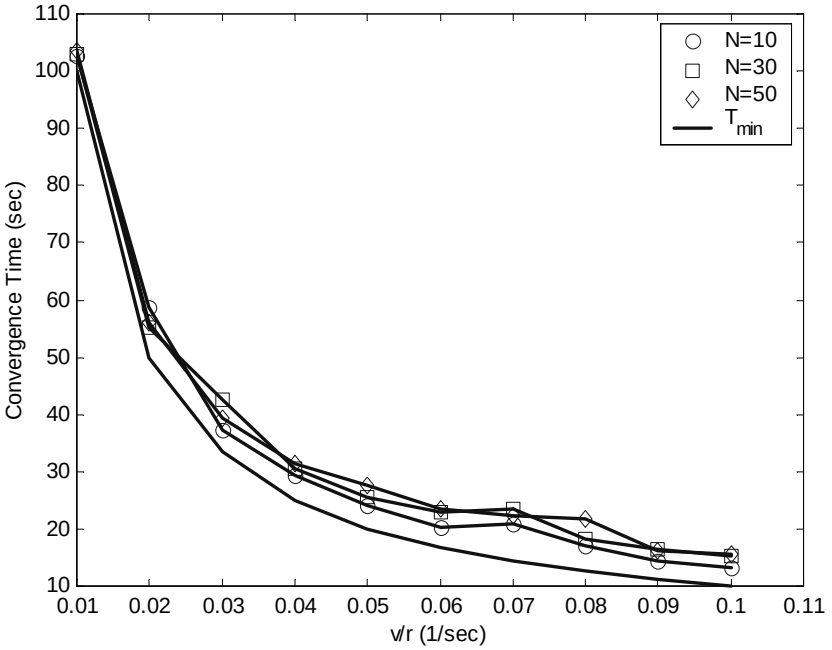

Figure 1.3. Average time of convergence versus maximum-speed-to-desired radius ratio for three collective sizes. Tmin denotes the approximate time it takes for the robots to spread to the desired radius.

From the experimental data, which is shown in Figure 1.1, the coefficients [a,b] = [0.21128,1.5107] using least-squares. In the Monte Carlo simulations, the integration time step was assumed to be 0.03s. Better estimates of these coefficients could be obtained using a smaller time step and more Monte Carlo simulations. The estimation variance is particularly high for large number of robots, as we can see from Figure 1.1.

In this particular implementation, the potential field is assumed to be communicated using a coded RF signal. The transmitted signal power is denoted by A in (22). In order to improve robustness to noise and jamming, we assume that the signal is modulated using a pseudo-noise sequence, specifically an m-sequence [10]. Signals coming from different robots can be distinguished at the receiver side using the autocorrelation properties of m-sequences [22]. If we assume that each robot is equipped with a grid of antennas measuring the potential, the gradient of the potential can also be approximately obtained using the first order difference approximation for differentiation.

A sample self-organization simulation using N=15 robots is presented in Figure 1.2. Here, the m-sequences are of length 127, and the desired radius is

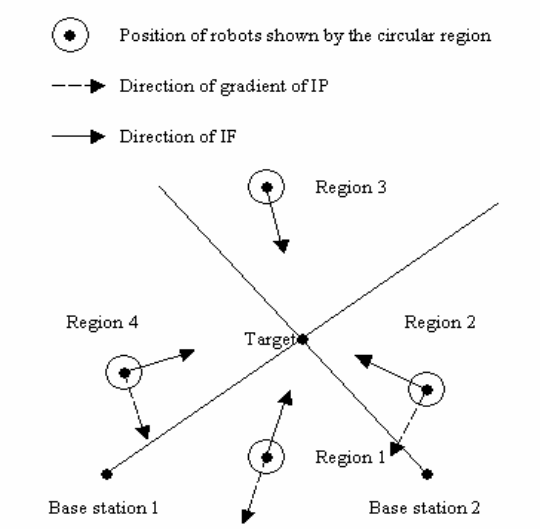

Figure 1.4. Guiding of robots towards a target using interaction forces (IF) with base stations. IF vector is a rotated version of the gradient of the information potential (IP) due to the base stations.

r=2. The robots are assumed to have a 3x3 antenna grid and the integration time step is once again 0.03s.

In order to get an idea on how much time it takes for the self-organization to be achieved under the described settings, we have conducted a series of Monte Carlo simulations. In these simulations, however, we have realistically restricted the magnitude of the velocity vector of each robot in order to avoid very large displacements in position in a very short time. The convergence time is measured in seconds using the distance of the farthest particle to the average position, where convergence is assumed to be achieved when the fluctuations of this quantity in time reduces to insignificant levels. The average convergence times for three different sizes of collectives are shown in Figure 1.3 as a function of the maximum-speed-to-desired-radius ratio. In all simulations, the initial positions of all robots are selected from a two-dimensional normal distribution highly concentrated around the origin. Therefore, approximately desired-radius/maximum-speed seconds are spent just for the robots to reach the boundaries of the desired circle, which is denoted by T_{min} in Figure 1.3. The additional time is spent on self-organizing into the uniform distribution scheme. As expected, as the number of agents in the collective increase, the time required for this organization stage increases.

Target Tracking Using Interactions with Base Stations: Our assumption about the knowledge base of the robots was that they only had access to their

relative distances, but not to their own or each other's absolute position in an *inertial* coordinate frame. This assumption limits the self-achievability of homing to a target or target tracking. In some scenarios, it is not realistic to assume that some beacon signal is transmitted from the target location [15]. It is possible to modify the line-of-sight guidance principle, however, to lead the robots to a desired location.

We assume that each base has access to the following information: line-of-sight angle to the centre of the robot collective, and line-of-sight angle to the target (even if there is no physical line-of-sight). Each base station, is then assumed to transmit an m-sequence coded direction bit selected from {-1,+1} to indicate which direction the robots should move towards in order to meet the target. The base stations can be distinguished by the robots if they are assigned different signature sequences. The amount of interaction force between each robot and the base station of interest can again be calculated using similar ideas to the inter-robot interaction forces. The robot-base interaction force is calculated in a similar manner to that between the robots; however, its direction is not only changed by 180^0, but depending on the combination of bits from the base stations, it can be rotated 90^0 in either direction also. In the two-dimensional scenario, two base stations suffice to guide the robots towards the target.

In order to demonstrate this guidance algorithm that is based on interactions between the robots and the bases, we present a couple of snapshots of the robot positions during the course of homing onto a stationary target in Figure 1.5. This guidance, however, works as well for moving targets. In this operation mode, each robot experiences a command that is the superposition of the inter-robot and robot-base interaction forces.

Avoiding Obstacles Using Interaction Forces Based on Visual Feedback: Suppose that each robot is equipped with a simple camera for visual detection of obstacles on its trajectory. Let the view-interval of the camera be aligned with the velocity vector (i.e., there are no attack and sideslip angles between the frontal body direction and the velocity vector. Based on the location, the area of the obstacle in the camera view, and its rate of growth, the robot can calculate an interaction force to move itself away from the obstacle while deviating from its current trajectory minimally. A sample situation is depicted in Figure 1.6. The robot could try rotating its velocity vector to avoid the obstacle as quickly as possible. If the obstacle does not occupy pixels on both sides of the view frame origin (center), there is no need for a course change, since the current velocity already avoids the obstacle.

In a two-dimensional motion space, the obstacle will appear as a line segment in the view, whose length increases gradually as the robot approaches it. Assume that the robot is moving towards an obstacle whose length in the frame increases according to the similarity principles. In particular, the

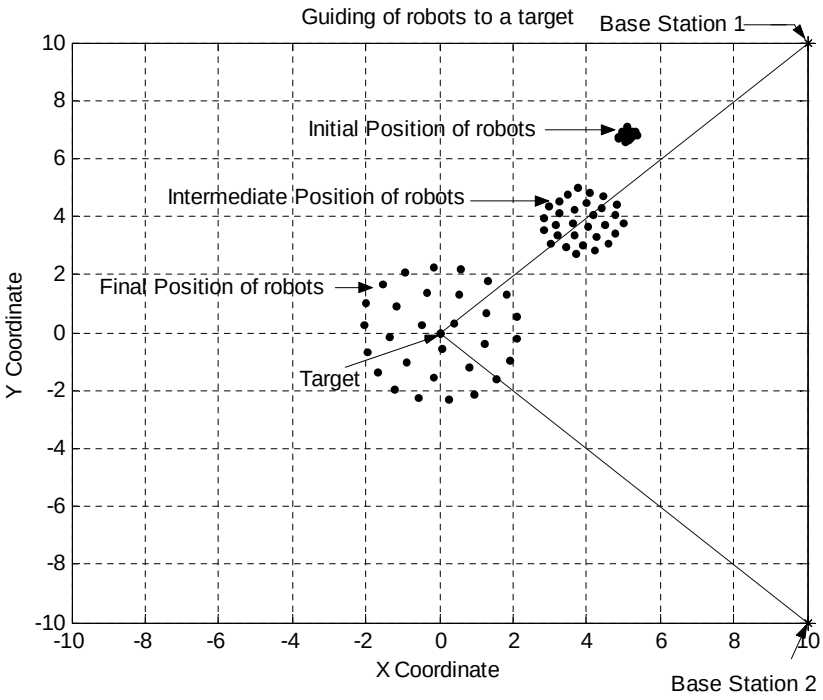

Figure 1.5. Guiding the robots towards a stationary target using robot-base interaction forces in conjunction with inter-robot interaction forces to achieve uniform spreading.

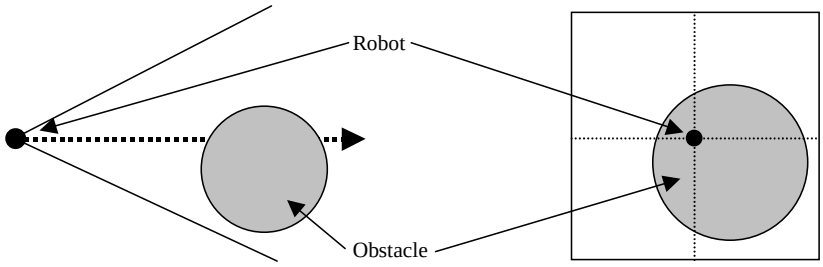

Figure 1.6. On the left, the robot sees an obstacle in its view area (boundaries denoted by solid lines) while moving along its current velocity vector (denoted by dotted arrow). On the right, the area that the obstacle covers in the view area becomes larger.

length of the obstacle L_1 and L_2 at two distances D_1 and D_2 are related by $L_1/D_1 = L_2/D_2$ (assuming that the obstacle fully remains in the view frame at both positions). Therefore, the distance to the obstacle can be estimated as D= *cL*, where c is some proportionality constant. Once again, assuming an inverse-distance-squared type interaction law between the robot and the obsta-

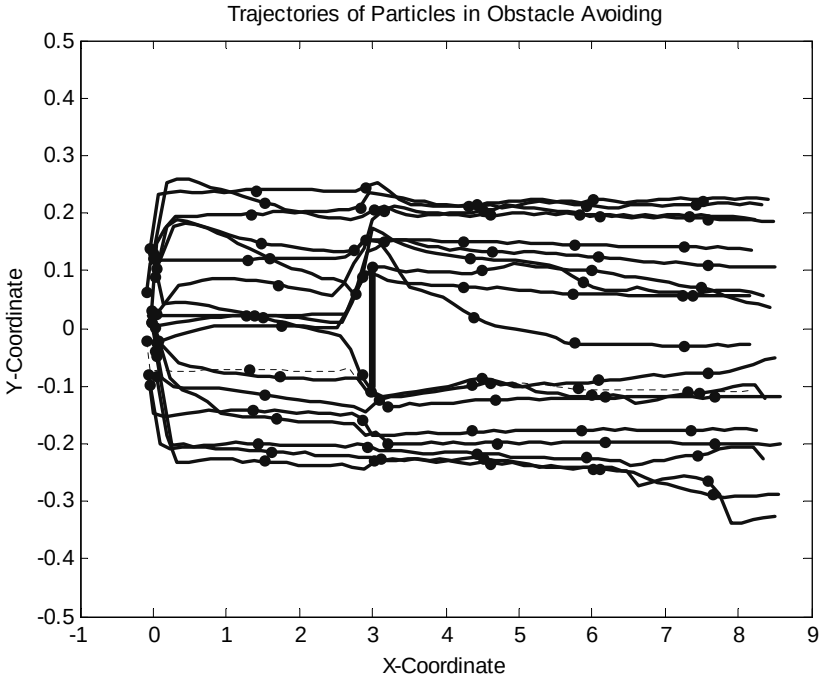

Figure 1.7. Snapshots of robot positions (dots) at various instances, while moving towards right along the dotted trajectories, which are determined by the interactions between the robots and the obstacle located at $X = 3$.

cle, the interaction force for the robot due to the obstacle in sight is obtained to be

$$F_{obst} = \frac{\min(L_{left}, L_{right})}{\max(L_{left}, L_{right})} \left(\frac{2}{c^2 L^3} \right) \mathbf{R}\dot{\mathbf{p}} \tag{24}$$

where $\mathbf{R}$ is a rotation matrix such that the direction of the force is rotated 90^0 right or left towards the minimum of L_{left} and L_{right}. These last two quantities are the lengths of the portions of the obstacle on the left and right side of the current velocity vector $\dot{\mathbf{p}}$, respectively. In simplest terms, the interaction law defined in (24) is equivalent to the obstacle emitting a signal whose power decays as the inverse of the distance, which is in turn used by the robot as the magnitude of the interaction force. The direction of this force is then found based on how this obstacle is located in the view of the robot.

A sample trajectory plot for a group of agents under the interaction law given in (24) when they encounter an obstacle is shown in Figure 1.7. In the mean time, the robots continue to interact among themselves to move into the circular formation. This example demonstrates how the interaction model can

be applied to the problem of obstacle avoiding in a simple experimental setup. More complicated interaction laws can possibly designed depending on the design parameters of the robots.

7. Conclusions

Research on swarm robots is becoming increasingly popular due to numerous applications where such self-organizing cooperative systems can be useful. Such cooperative task management requires extensive communication and resources. Since the resources available to the agents in such a collective are limited by the design parameters, self-organization into a certain orientation should be achieved with as little resources as possible.

In this paper, we proposed a self-organization principle based on particle interactions through a predefined interaction law. Specifically, for the problem of uniformly distribution over a predefined region, these interaction laws are firmly connected to the maximization of swarm entropy. Nevertheless, similar interaction principles can be derived to achieve other tasks, such as target tracking and obstacle avoiding, as we have demonstrated here.

It is remarkable that the simple principle of particle interactions can be applied to the problem of collective behavior control of multiple agents for achieving a variety of tasks. Although the initial idea started out as a purely self-organization scheme based on maximization of system entropy in a fixed volume, it has been observed through the examples shown here that the applicability of the underlying principles, i.e., particle interaction laws, are more general than this simple special case. It has been shown to be possible that every task can be formulated in this framework where the agents in the swarm as well as any external entity that is supposed to have an effect on the collective behavior can be regarded as a particle. The behavior of the agents are solely defined by the pair-wise interactions they experience with every other particle in the system, let it be a target, a base, or an obstacle. The potential function, which defines the interactions between pairs of particles, can be designed to suit the needs of the task under consideration, as well as the available hardware and information. Thus the designer has great flexibility in this respect.

In this paper, we have made certain assumptions about the knowledge and hardware base of the robots in the swarm, and designed interaction laws specifically suited to these schemes. This was necessary for illustration purposes; however, the principle behind the presented self-organization and cooperative control is quite general, as discussed above. Therefore, it is possible to design swarms of robots with other capabilities and come up with corresponding particle interaction laws for cooperative behavior. The details of such designs can only be completed after the specifications of the design parameters are provided.

Acknowledgments

This work was partially supported by NSF grant ECS-9900394.

References

[1] S. I. Amari. *Differential–Geometrical Methods in Statistics*. Springer-Verlag, 1985.

[2] J. Beirlant, E.J. Dudewicz, L. Gyorfi, and E.C. van der Meulen. Nonparametric entropy estimation: An overview. *International Journal of Mathematical and Statistical Sciences*, 6:17–39, 1997.

[3] A. Cichocki and S.I. Amari. *Adaptive Blind Signal and Image Processing*. Wiley, 2002.

[4] J.H. Conway and N.J.A. Sloane. *Sphere Packings, Lattices and Groups*. Springer-Verlag, 1988.

[5] T. Cover and J. Thomas. *Elements of Information Theory*. Wiley, 1991.

[6] I. Csiszar and J. Korner. *Information Theory: Coding Theorems for Discrete Memoryless Systems*. Academic Press, 1981.

[7] D. Erdogmus, J.C. Principe, L. Vielva, and D. Luengo. Potential energy and particle interaction approach for learning in adaptive systems. In *Proc. ICANN'02*, pages 456–461, Madrid, Spain, 2002.

[8] R.M. Fano. *Transmission of Information: A Statistical Theory of Communications*. MIT Press & John Wiley & Sons, Inc., 1961.

[9] V. Genovese, P. Dario, R. Magni, and L. Odetti. Self-organizing behaviour and swarm intelligence in a pack of mobile miniature robots in search of pollutants. In *Proc. IEEE/RSJ Int. Conf. Intelligent Robots and Systems*, pages 1575–1582, Raleigh, NC, 1992.

[10] S. Haykin. *Communication Systems*. Wiley, 3 edition, 1994.

[11] A. Hyvarinen, J. Karhunen, and E. Oja. *Independent Component Analysis*. Wiley, 2001.

[12] E.T. Jaynes. Information theory and statistical mechanics. *Physical Review*, 106:620–630, 1957.

[13] C.R. Kube and E. Bonabeau. Co-operative transport by ants and robots. *Robotics and Automation Systems*, pages 85–101, 2000.

[14] D.P. Miller. Multiple behaviour-controlled micro-robots for planetary surface missions. In *Proc. IEEE Int. Conf. Systems, Man and Cybernatics*, pages 281–292, Los Angeles, CA, 1990.

[15] H.V.D. Parunak and S. Brueckner. Entropy and self-organization in multi-agent systems. In *Proc. Int. Conf. Autonomous Agents*, pages 124–130, Montreal, Canada, 2001.

[16] E. Parzen. On estimation of a probability density function and mode. In *Time Series Analysis Papers*. Holden-Day, Inc., 1967.

[17] J.C. Principe, D. Xu, and J. Fisher. Information theoretic learning. In S. Haykin, editor, *Unsupervised Adaptive Filtering*, volume I, pages 265–319. Wiley, 2000.

[18] A. Renyi. *Probability Theory*. American Elsevier Publishing Company Inc., 1970.

[19] M. Resnick. *Turtles, Termites and Traffic Jams: Explorations in Massively Parallel Microworlds*. MIT Press, 1997.

[20] C.E. Shannon. A mathematical theory of communications. *Bell Systems Technical Journal*, 27, 1948.

[21] C.E. Shannon and W. Weaver. *The Mathematical Theory of Communication*. University of Illinois Press, Urbana, IL, 1964.

[22] R. Thogulua, D. Erdogmus, and J.C. Principe. Self-organization of multiple agents using information theoretic interactions. In *Proceedings of ICONS'03*, Faro, Portugal, 2003. To appear.

[23] R.M. Turner and E.H. Turner. Organization and reorganization of autonomous oceanographic sampling networks. In *Proc. IEEE Int. Conf. Robotics and Automation*, pages 2060–2067, Leuven, Belgium, 1998.

[24] C. Unsal and J.S. Bay. Spatial self-organization in large populations of mobile robots. In *Proc. IEEE Int. Symp. Intelligent Control*, pages 249–254, Columbus, OH, 1994.

[25] T. White and B. Pagurek. Towards multi-swarm problem solving in networks. In *Proc. Int. Conf. Multi-agent Systems*, pages 333–340, 1998.

[26] H. Yockey. *Information Theory and Molecular Biology*. Cambridge University Press, 1992.